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Design of Fractional Order Pi$^{\lambda}$d$^{\mu}$ Controller for Liquid Level Control of a Spherical Tank Modeled As a Fractional Order System

Rajinikanth Reddy.Mettu[1], Ramesh.Ganta[2]
[1]*Pg student, EEE, Gudlavalleru engineering college,Gudlavalleru,A.p, India*
[2]*Assistant professor, EEE, Gudlavalleru engineering college,Gudlavalleru,A.p, India*

Abstract
Chemical process presents many challenging control problems due to their nonlinear dynamic behavior, uncertain and time varying parameters, constraints on manipulated variable, interaction between manipulated and controlled variables, unmeasured and frequent disturbances, dead time on input and measurements. Control of liquid level in a spherical tank is nonlinear due to the variation in the area of cross section of the level system with change in shape. In this paper, Fractional Order Proportional Integral Derivative (FOPID) controller is designed for a liquid level control of a spherical tank which is modeled as a fractional Order System. The performance of FOPID controller is compared with the traditional Integer Order PID (IOPID) controller designed for the same spherical tank which is modeled as a First Order Plus Dead Time (FOPDT) system. The FOPID controller is designed following a set of imposed tuning constraints, which can guarantee the desired control performance and the robustness to the loop gain variations. From the simulation results presented, it is observed that the designed fractional order controller works efficiently, with improved performance comparing with the integer order controller.
***Keywords*—** Spherical tank, Fractional calculus, Fractional Order System (FOS), Fractional Order Proportional Integral Derivative (FOPID) controller, Integer Order PID Controller (IOPID).

I. INTRODUCTION

The concept of extending classical integer order calculus to non-integer order cases is by no means new. For example, it was mentioned in [1] that the earliest systematic studies seem to have been made in the beginning and middle of the 19th century by Liouville, Riemann, and Holmgren. The most common applications of fractional order differentiation can be found in [2]. Fractional calculus is a mathematical topic with more than 300-year history, but the application in physics and engineering has been recently attracted lots of attention. During past three centuries, this subject was with mathematicians, and only in last few years, this was pulled to several (applied) fields of engineering, science and economics. In engineering particularly in process control, fractional calculus is becoming a hot topic in both modeling and control area.

A commonly used definition of the fractional differointegral is the Riemann-Liouville definition

$$aD_t^{\propto} f(t) = \frac{1}{\Gamma(m-\propto)} \left(\frac{d}{dt}\right)^m \int_{\propto}^{t} \frac{f(\tau)}{(t-\tau)^{1-(m-\propto)}}\, d\tau \quad (1)$$

for $m-1 < \propto < m$ where $\Gamma(\cdot)$ is the well-known Euler's gamma function. An alternative definition, based on the concept of fractional differentiation, is the Grunwald-Letnikov definition given by

$$aD_t^{\propto} f(t) = \lim_{h\to 0} \frac{1}{\Gamma(\propto)h^{\propto}} \sum_{k=0}^{(t-\propto)/h} \frac{\Gamma(\propto+k)}{\Gamma(k+1)} f(t-kh) \quad (2)$$

One can observe that by introducing the notion of fractional order operator $aD_t^{\propto} f(t)$, the differentiator and integrator can be unified.

Another useful tool is the Laplace transform, a fractional order differential equation, provided both the signals $u(t)$ and $y(t)$ at $t = 0$, can be expressed in a transfer function form as

$$G(s) = \frac{a_1 s^{\propto_1} + a_2 s^{\propto_2} + \cdots + a_{m_a} s^{\propto_{m_a}}}{b_1 s^{\beta_1} + b_2 s^{\beta_2} + \cdots + b_{m_B} s^{\beta_{m_B}}} \quad (3)$$

where $(a_m, b_m) \in R^2, (\propto_m, \beta_m) \in R_+^2, \forall\, (m \in N)$

Fractional derivatives provide an excellent instrument for the description of memory and hereditary properties of various materials and processes. This is the main advantage of fractional derivatives in comparison with classical integer order models, in which such effects are in fact neglected. The significance of fractional order control is that it is a generalization of classical integral order control theory, which could lead to more adequate modeling and more robust control performance. The advantages of fractional derivatives become apparent in modeling mechanical and electrical properties of real materials, as well as in the description of rheological properties of rocks, and in many other fields [3].

The paper is organized as follows. The section I give a brief introduction about fractional calculus. The section II brief about the mathematical modeling of the nonlinear spherical tank process. In section III the integer order PID controller is tuned based on Cohen-coon tuning method for the spherical tank which is modeled as first order plus dead time system and subsequently in section IV, implementation of fractional $pi^{\lambda}d^{\mu}$ controller for the spherical tank modeled as fractional order system is presented. The simulated results for IOPID controller with integer order system and FOPID controller with fractional order system given in section V. finally conclusion is given in section VI.

II. Mathematical Modelling of Spherical Tank

A spherical time system, shown in figure 1 is essentially a system with non linear dynamics. The spherical time system has a maximum height of 1.5 meter, maximum radius of 0.7 meter. The level of the tank at any instant is determined by using the mass balance equation given below:

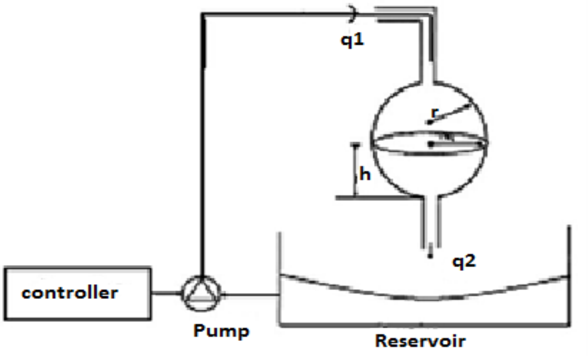

Fig: 1 Schematic diagram of Spherical Tank process

Let,
q_1 (t) – inlet flow rate to the tank in m^3/sec
q_2 (t) – outlet flow rate of the tank in m^3/sec
H- Height of the spherical tank in meter.
r – Radius of tank in meter (0.75 meter).
Xo – thickness of pipe in meter (0.06 meter).
Inlet water flow from the pump q_1(t)

Using law of conservation of mass the non-linear plant equation is obtained.
For the spherical tank,

$$q_1(t) - q_2(t) = A(h_1)\frac{dh_1}{dt} \tag{4}$$

Where $A = \pi r^2{}_{surface}$

Radius on the surface of the fluid varies according to the level (height) of fluid in the tank. Thus let this radius be known as $r_{surface}$.

$$r_{surface} = \sqrt{2rh_1 - h_1^2}$$

$$A = \pi\,(2rh_s - h_s^2) \tag{5}$$

And

$$q_2(t) = a\sqrt{2g\,(h - x_0)} \tag{6}$$

Where
$a = \pi\,(x_o/2)^2$
Solving (5) and (6) in equation (4) for linearizing the non linearity in spherical tank gives rise to a transfer function of

$$Q_1(s) - \frac{a\sqrt{2g}\,H(s)}{2\sqrt{h_s - x_0}} = \pi(2rh_s - h_s^2)SH(s)$$

Rearranging

$$\frac{H(s)}{Q(s)} = \frac{1}{\pi(2rh_s - h_s^2)s + \frac{a\sqrt{2g}}{2\sqrt{h_s - x_0}}} \tag{7}$$

Applying the steady state conditions values the linearized plant transfer function is given by

$$P(s) = \frac{H(s)}{Q(s)} = \frac{0.5897}{s + 0.004} \tag{8}$$

III. TUNING OF PID CONTROLLER

A proportional–integral–derivative controller (PID controller) is a generic control loop feedback mechanism (controller) widely used in industrial control systems– a PID is the most commonly used feedback controller. A PID controller takes an "error" value as the difference between a measured process variable and a desired set point. The controller attempts to minimize the error by adjusting the process control inputs. The proportional, integral, and derivative terms are summed to calculate the output of the PID controller. Defining u(t) as the controller output, the final form of the PID controller in time domain is:

$$u(t) = K_p[e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d\frac{d}{dt}e(t)]$$

The transfer function of PID controller in Laplace transform form is given as

$$c(s) = K_p + \frac{K_i}{s} + K_d s$$

To design an integer order PID controller, the integer order model (8) should be approximated into a first order plus dead time (FOPDT) system [4] which is given in the following

$$P(s) = \frac{Ke^{-Ls}}{Ts+1} = \frac{146.62e^{-4.64s}}{237.14s+1} \tag{9}$$

Then according to Cohen-coon tuning method [5],

$$K_p = \left(\frac{1.35}{a}\right) * \left(1 + \frac{0.18*tau}{1-tau}\right);$$

$$a = \frac{K*L}{T};$$

$$tau = \frac{L}{(L+T)};$$

$$K_i = K_p * \frac{(1-0.39*tau)}{(2.5-2*tau)*L};$$

$$K_d = K_p * \frac{(0.37-0.37*tau)*L}{(1-0.81*tau)};$$

An integer order PID (IOPID) controller is designed for (9) as below.

$$c_I(s) = 0.4722 + \frac{0.0410}{s} + 0.8077s \tag{10}$$

IV. FRACTIONAL ORDER $PI^{\lambda}D^{\mu}$ CONTROLLER DESIGN

To enhance the robustness and performance of PID control systems, Podlubny has proposed a generalization of the PID controllers, namely $PI^{\lambda}D^{\mu}$ controllers, including an integrator of order λ and differentiator of order μ (the order λ and μ may assume real noninteger values) [6].
The transfer function of such a controller has the following form

$$c(s) = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu} \quad (\lambda, \mu > 0).$$

The control signal *u* (*t*) can then be expressed in the time domain as

$$u(t) = K_p e(t) + K_i D^{-\lambda} e(t) + K_d D^{\mu} e(t)$$

Using λ=1, μ=0 and λ =0 and μ=1 respectively, corresponding to the conventional PI and PD controllers. The order of the integrator and differentiator add increased flexibility to the controller. Various design methods on the $PI^{\lambda}D^{\mu}$ controllers have been presented in [7]-[10]. It has been shown in these methods that the $PI^{\lambda}D^{\mu}$ controller, which has extra degrees of freedom introduced by λ and μ, provides a better response than the integer-order PID controllers when used both for the control of integer-order systems and fractional-order systems. These are less sensitive to the parameter variation due to the extra two degree of freedom to better adjust the dynamical properties of fractional order systems. Fractional order PID control is a useful control strategy, since it provides five parameters to be tuned as opposed to the three available in ordinary PID control.

Frequency domain design specifications for robust fopid tuning

Frequency domain design of FOPID controllers was proposed by Monje et al. [11] based on a constrained optimization problem. i.e. If P(s) be the model of the process plant, then the objective is to find a controller C(s), so that the open loop system G(s)=C(s)P(s) would meet the following design specifications:

(a) Gain crossover frequency specification:

$$|G(j\omega_{gc})| = |C(j\omega_{gc})P(j\omega_{gc})| = 0 \qquad (11)$$

(b) Phase margin specification:

$$Arg[G(j\omega_{gc})] = Arg[C(j\omega_{gc})P(j\omega_{gc})] = -\pi + \phi_m \qquad (12)$$

(c) Robustness to gain variation (Iso - damping):

$$\left(\frac{d}{d\omega}(Arg[G(j\omega)])\right)_{\omega=\omega_{gc}} = 0 \qquad (13)$$

(d) Noise rejection:

$$|T(j\omega)| = \left|\frac{C(j\omega)P(j\omega)}{1+C(j\omega)P(j\omega)}\right|_{dB} \le A dB \ \forall \omega \ge \omega_t rad/s \qquad (14)$$

$$\Rightarrow |T(j\omega_t)| = A\ dB$$

Where, A is the specified magnitude of the complementary sensitivity function or noise attenuation for frequencies $\omega \ge \omega_t$ rad/s

(e) Sensitivity specification:

$$|S(j\omega)| = \left|\frac{1}{1+C(j\omega)P(j\omega)}\right|_{dB} \le B dB \ \forall \omega \le \omega_s rad/s \qquad (15)$$

Where, *B* is the specified magnitude of the sensitivity function or load disturbance suppression for frequencies $\omega \le \omega_s$ rad/s

The parameters of design problem has been transformed to a problem of solving the system of five nonlinear equations (given by the corresponding design specification) and five unknown parameters P, I, D, λ, μ. Here, MATLAB is a very appropriate tool for the analysis and design of control systems, the optimization toolbox of MATLAB has been used to reach out the best solution with the minimum error. The function used for this purpose is called FMINCON, which finds the constrained minimum of a function of several variables. In this case, the specification in Eq.(11) is taken as the main objective function to minimize, and the rest of specifications ((12)-(15)) are taken as constrains for the minimization, all of them subjected to the optimization parameters defined within the function FMINCON. It solves problems of the form MIN_X F(X) subject to: C (X)≤0, C_{eq} (X)=0, LB ≤ X ≤ UB, where F is the function to minimize; C and C_{eq} represent the nonlinear inequalities and equalities, respectively (nonlinear constraints); X is the minimum looked for; LB and UB define a set of lower and upper bounds on the design variables, X.
The design specifications required for the system are:

- Gain margin=10 db,
- Phase margin, $\theta_m = 60^{\circ}$,
- Robustness to variations in the gain of the plant must be fulfilled
- Noise rejection: $|T(j\omega)|_{db}$ = - 20 dB, ω_t =10 rad/s;
- Sensitivity function: $|S(j\omega)|_{db}$ = - 20 dB, ω_s =0.01 rad/s;

A fractional order model can be obtained for the plant (1) by using "normalize" command as

$$G_{fp} = \frac{147}{71.39s^{0.76} + 0.27s^{0.46}} e^{-2.974s} \qquad (16)$$

Controller design :-

The first step in controller design is choosing a suitable controller structure. The plant exhibits a significant input-output delay. So we consider a smith predictor [12]. The basic structure of a smith predictor is given in figure 2.

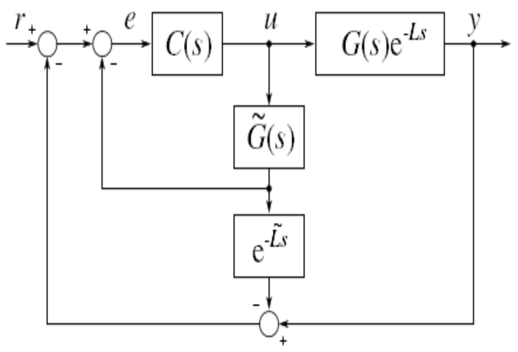

Fig: 2 Smith predictor

The design of controller is carried in two steps. First, we formulate an integer order PID controller, and then optimize the parameters of the obtained controller, including the order of the integrator and differentiator, finally arriving at a fractional order pid controller working with a smith predictor based control scheme.

Conventional PID tuning :-

By using “iopid_tune” tool to approximate the fractional order model by a conventional FOPDT model, and then apply classical tuning formulae to get the PID controller parameters.
The fractional order model is then identified as

$$G(s) = \frac{2097.78}{6815.33s + 1} e^{-1.0875s} \qquad (18)$$

The result of approximation is illustrated in figure 3.

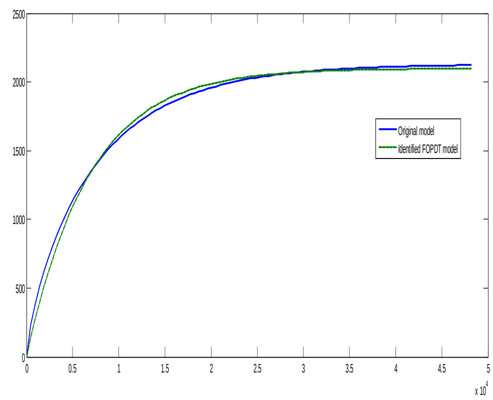

Fig: 3 classical FOPDT model approximation of the fractional order model

By using COHEN-COON tuning method, the obtained PID controller parameters are
K_p=1.4224*E^6 , K_i=1.8607*E^{11}, K_d=1.60921

Fractional order PID controller design:

The design of fractional order PID controller is carried out using fpid_optim tool. The initial values are given from the conventional PID controller values and the values of λ and μ are set to be 1. For the basic smith predictor model as shown in figure 4, a new model is to be build in the simulation by modifying the basic model. This setup is to evaluate the impact of the difference of the reference and original model on the control system. The resulting model is shown in figure 5.

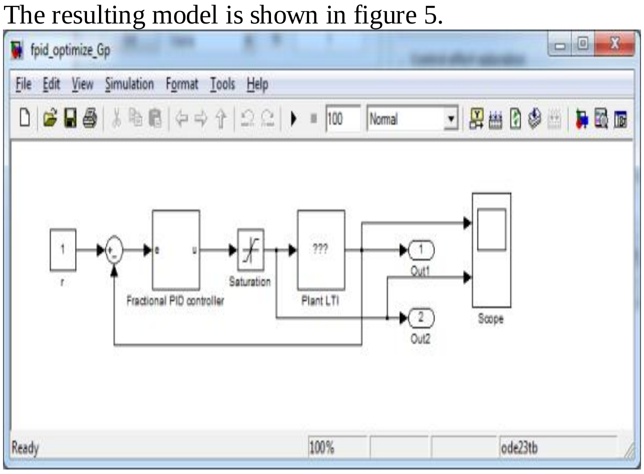

Fig 4: initial model of smith predictor

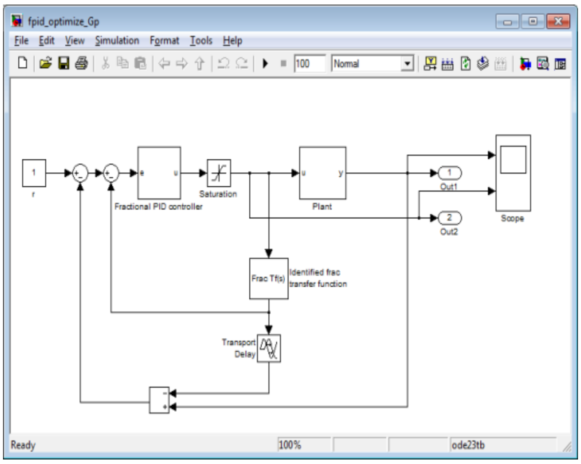

Fig 5: Fractional Smith predictor control system

The identified model is added as a part of the predictor. By giving the frequency domain specifications and by choosing optimization algorithm, the optimization carried out. After some iterations, the obtained fractional $PI^{\lambda}D^{\mu}$ controller to control the system is

$$C_f(s) = 75.1336 + \frac{83.3691}{s^{0.9974}} + 70.9123\, s^{1.0016} \qquad (19)$$

V. SIMULATION RESULTS

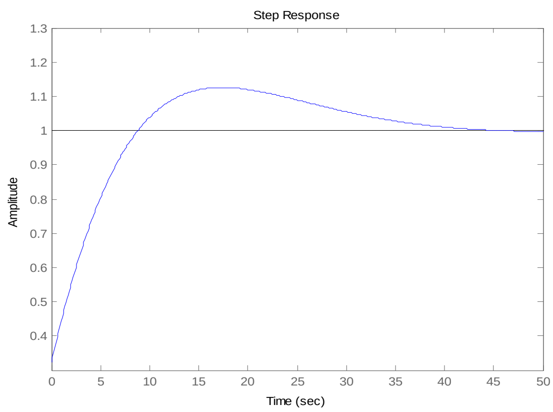

Fig: 6 Simulation of closed loop step response of the integer order model with integer order controller

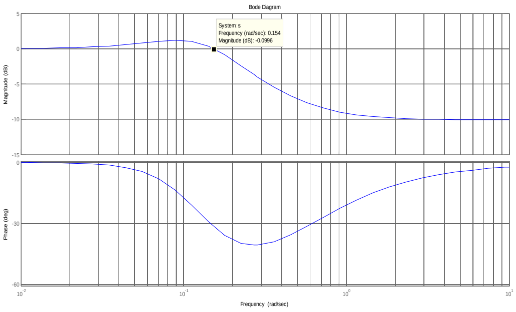

Fig: 7 Bode plot of the integer order model with integer order controller

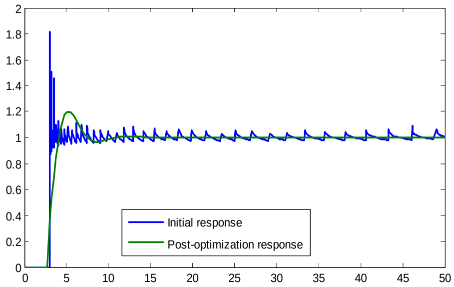

Fig: 8 Comparison of step responses of closed loop fractional order model with integer and fractional controller.

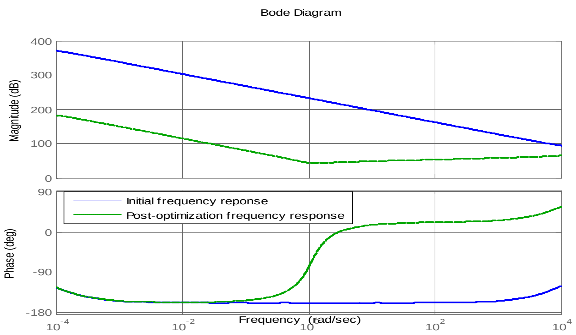

Fig: 9 Comparison of bode diagrams of the fractional order model with fractional order controller and same model with integer order controller

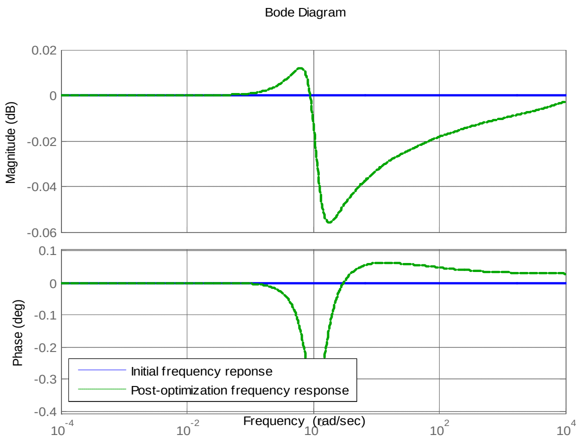

Fig: 10 Noise rejection verification

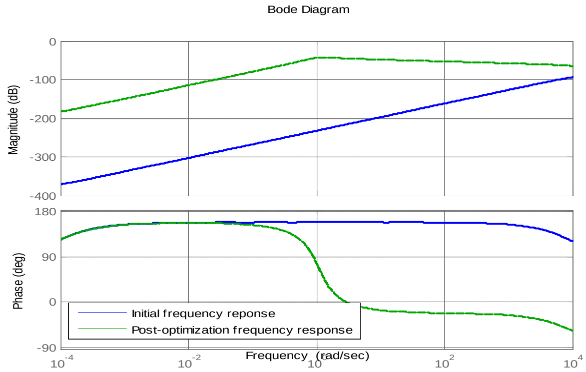

Fig: 11 Sensitivity specifications

VI. CONCLUSION

In this paper, fractional order proportional integral derivative (FOPID) controller was designed for liquid level control in a spherical tank which is modeled as fractional order system. The designed fractional order proportional integral derivative (FOPID) controller can provide better results as compared with the traditional IOPID controller in simulation. The FOPID controller was designed following a set of imposed tuning constraints, which can guarantee the desired control performance and the robustness of the designed controllers to the loop gain variations. From the simulation results, it is observed that the designed fractional order PID controller works efficiently.

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