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Inversion Formula for Generalized Two-Dimensional Fractional Sine Transform

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Abstract
The theory of integral transform is presented a direct and systematic technique for presentation of classical and distributional theory. In this paper,we have proved inversion formula and uniqueness theorem for generalized two dimensional fractional Sine transform.
Keywords -Fractional Fourier transforms, fractional Sine transform, fractional Cosine transform, Fourier sine transform.

I. Introduction

In the literature there are numerous integral transform and widely used in physics astronomy as well as in engineering. In order to solve differential equation, the integral transform were extensively used and thus there are several work on the theory and application of the integral transform such as Laplace, Fourier, Mellin & Hankel, Cosine and Sine transform to name but a few. In the sequence of these transform, Pei Soo-Chang redefined the fractional Sine and fractional Cosine transform based on fractional Fourier transform in 2001. [1, 2]

FRST is extension of Sine transform and it has been widely used in domain of digital signal processing and image processing.

1.1Two dimensional generalized fractional Sine transform

Two dimensional fractional Sine transform with parameter αf(x, y) denoted by $F_s^{\alpha}(x,y)$ perform a linear operation given by the integral transform.

$F_s^{\alpha}\{f(x,y)\}(u,v) =$
$\int_0^{\infty}\int_0^{\infty} f(x,y)K_{\alpha}(x,y,u,v)\,dx\,dy$(1.1)

Where the kernel,

$$K_s^{\alpha}(x,y,u,v)\sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}e^{i(\theta-\frac{\pi}{2})}\sin(cosec\alpha.ux).\sin(cosec\alpha.vy).$$

1.2 The test function space E

An infinitely differentiable complex valued function $\emptyset$ on R^n belongs to $E(R^n)$ if for each compact set$I \subset S_{a,b}$, where,

$$S_{a,b} = \{x,y: x,y \in R^n, |x| \le a, |y| \le b, a > 0, b > 0\}, I \in R^n$$

$$\gamma_{E_{p,q}}(\emptyset) = \sup_{x,y}\left|D_{x,y}^{p,q}\emptyset(x,y)\right| < \infty$$ Where, p, q =1, 2, 3….

Thus $E(R^n)$ will denote the space of all$\emptyset \in E(R^n)$ with support contained in $S_{a,b}$

Note that the space E is complete and therefore a Frechet space. Moreover, we say that f is a fractional Cosine transformable, if it is a member ofE^*, the dual space of E.

In this paper, we introduce generalized two dimension fractional sine transform and Proved inversion formula and uniqueness theorem.

II. Distributional two-dimensional fractional Sine transform

The two dimensional distributional fractional Sine transform of $f(x,y) \in E^*(R^n)$ defined by

$$F_s^{\alpha}\{f(x,y)\} = F^{\alpha}(u,v) = \langle f(x,y), K_{\alpha}(x,y,u,v)\rangle \quad (2.1)$$

$$K_s^{\alpha}(x,y,u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}e^{i(\theta-\frac{\pi}{2})}\sin(cosec\alpha.ux).\sin(cosec\alpha.vy) \quad (2.2)$$

Where , RHS of equation (2.1) has a meaning as the application of $f \in E^*$ to $K_{\alpha}(x,y,u,v) \in E$.

III. Inversion formula for generalized twodimensional fractional Sine transform

If two dimensional fractional Sine transform is given by

$$F_s^{\alpha}\{f(x,y)\}(u,v)$$
$$=\int_0^{\infty}\int_0^{\infty} f(x,y)\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}} e^{i(\theta-\frac{\pi}{2})}\sin(cosec\alpha.ux)\sin(cosec\alpha.vy)\,dx\,dy$$

Then its inverse f(x, y) is given by

$$f(x,y)=\frac{4}{\pi^2}\int_0^{\infty}\int_0^{\infty} F_s^{\alpha}(u,v)\overline{K_\alpha}(x,y,u,v)dudv$$

Where, $\overline{K_\alpha} = e^{i(\theta-\frac{\pi}{2})}\ e^{\frac{-i}{2}(x^2+y^2+u^2+v^2)cot\alpha}\left(\sqrt{\frac{1-icot\alpha}{2\pi}}\right)^{-1} cosec^2\alpha\sin(cosec\alpha ux)\sin(cosec\alpha vy)$

Solution:

$$F_s^{\alpha}(u,v)=\sqrt{\frac{1-icot\alpha}{2\pi}}\int_0^{\infty}\int_0^{\infty} f(x,y)e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}} e^{i(\theta-\frac{\pi}{2})}\sin(cosec\alpha.ux)\sin(cosec\alpha.vy)dxdy$$

$$F_s^{\alpha}(u,v)e^{\frac{-i}{2}(u^2+v^2)}=C_k\int_0^{\infty}\int_0^{\infty} f(x,y)e^{\frac{i(x^2+y^2)cot\alpha}{2}} e^{i(\theta-\frac{\pi}{2})}\sin(cosec\alpha.ux)\sin(cosec\alpha.vy)dxdy$$

Where, $C_k=\sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{i(\theta-\frac{\pi}{2})}$

$$F_s^{\alpha}(u,v)e^{\frac{-i}{2}(u^2+v^2)}=\int_0^{\infty}\int_0^{\infty} g(x,y)\sin(cosec\alpha.ux)\sin(cosec\alpha.vy)dxdy$$

Where, $g(x,y)=C_k e^{\frac{i(x^2+y^2)cot\alpha}{2}} f(x,y)$

$$F_s^{\alpha}(u,v)e^{\frac{-i}{2}(u^2+v^2)cot\alpha}=[Cg(x,y)](cosec\alpha.u)(cosec\alpha.v)$$

Let $(cosec\alpha.u)=\eta$

$$d\eta=(cosec\alpha.du)$$

Let $(cosec\alpha.v)=\xi$

$$d\xi=(cosec\alpha.dv)$$

$$F_s^{\alpha}(u,v)e^{\frac{-i}{2}(u^2+v^2)cot\alpha}=[Cg(x,y)](\eta,\xi)]$$

Invoking the Sine inversion we can write,

$$F_s^{\alpha}(u,v)e^{\frac{-i}{2}(u^2+v^2)cot\,\alpha}=G(\eta,\xi)$$

The rhs is the Sine transform of g(x, y) with argument η,ξ

$$g(x,y)=\frac{4}{\pi^2}\int_0^{\infty}\int_0^{\infty} G(\eta,\xi)\sin(\eta x)\sin(\xi y)\,d\eta d\xi$$

$$C_k e^{\frac{i(x^2+y^2)cot\alpha}{2}} f(x,y)=\frac{4}{\pi^2}\int_0^{\infty}\int_0^{\infty} F_s^{\alpha}(u,v)e^{\frac{-i}{2}(u^2+v^2)cot\alpha}\sin(\eta x)\sin(\xi y)\,d\eta d\xi$$

$$C_k e^{\frac{i(x^2+y^2)cot\alpha}{2}} f(x,y)=\frac{4}{\pi^2}\int_0^{\infty}\int_0^{\infty} F_s^{\alpha}(u,v)e^{\frac{-i}{2}(u^2+v^2)cot\alpha}\sin(cosec\alpha.ux)\sin(cosec\alpha.vy)cosec^2\alpha dudv$$

$$f(x,y)=\frac{4}{\pi^2}\int_0^{\infty}\int_0^{\infty} e^{\frac{-i(x^2+y^2+u^2+v^2)cot\alpha}{2}} C_k^{-1}F_s^{\alpha}(u,v)\sin(cosec\alpha.ux)\sin(cosec\alpha.vy)cosec^2\alpha dudv$$

$$f(x,y)=\frac{4}{\pi^2}\int_0^{\infty}\int_0^{\infty} F_c^{\alpha}(u,v)\,\widetilde{K_\alpha}(x,y,u,v)dudv$$

Where

$$\widetilde{K_\alpha}(x,y,u,v)=,e^{\frac{-i(x^2+y^2+u^2+v^2)cot\alpha}{2}} e^{i(\theta-\frac{\pi}{2})}\sqrt{\frac{1-icot\alpha}{2\pi}}\sin(cosec\alpha.ux)\sin(cosec\alpha.vy)cosec^2\alpha$$

IV. Uniqueness theorem for generalized two dimensional fractionalSine transform

If $[\,F_s^{\alpha}f(x,y)](u,v)=F(u,v)$, $[\,F_s^{\alpha}g(x,y)\,](u,v)=G(u,v)$ for $0<\alpha\leq\frac{\pi}{2}$ and $supf\subset S_{a,b}$, $supg\subset S_{a,b}$ where, $S_{a,b}=\{x,y: x,y\in R|x|\leq a,|y|\leq b, a>0, b>0\}$

If $F_\alpha(u,v)=G_\alpha$ then f=g in the sense of equality in $D^*(I)$.

Proof: By inversion theorem

$f-g=\frac{4}{\pi^2}\lim_{N\to\infty}\int_{-N}^{N}\int_{-N}^{N}\widetilde{K_\alpha}(x,y,u,v)dudv = 0$

Thus f=g in $D^*(I)$.

V. Conclusion

We have extendedtwo-dimensional fractional Sine transform in the distributional generalized sense, the testing function space and distributional generalized two-dimensional fractional Sine transform is defined. Inversion theorem and uniqueness theorem of generalized two-dimensional fractional Sine transform are proved.

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