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Modulation and Parsvels Theorem For Generalized Fractional Fourier Transform

V. D. Sharma
Mathematics Department, Arts, Commerce and Science College, Amravati, 444606 (M.S.) India

Abstract
The fractional Fourier transform (FrFT) was first introduced as a way to solve certain classes of ordinary and partial differential equations arising in quantum mechanics. FrFt has established itself as a powerful tool for the analysis of time-varying signals, especially in optics. FrFt has found applications in areas of signal processing such as repeated filtering, fractional convolution and correlation, beam forming, optimal filter, convolution, filtering and wavelet transforms, time frequency representations. In this paper operational calculus for Generalized fractional Fourier transform are obtained.
Keywords: Generalized function, Fractional Fourier transform, Fourier transform, signal processing, Test function space.

I. Introduction

The Fourier transform is one of the most important mathematical tools used in Physical optics, linear system theory, signal processing and so on [1]. The conventional Fourier transform can be regarded as a $\frac{\pi}{2}$ rotation in the time-frequency plane and the FrFT performs a rotation of signal to any angle. Moreover, fractional Fourier transform serves as an orthonormal signal representation for chirp signal. The Fractional Fourier transform is also called rotational Fourier transform or angular Fourier transform in some documents. The FrFT is also related to other time-varying signal processing tools such as Wigner distribution, Short time Fourier transform, Wavelet transform [2].
In our previews work we have defined in [3, 4] as,

1.1. Conventional fractional Fourier transform:

The FrFT with parameter α of $f(x)$ denoted by $FrFT\{f(x)\}$ performs a linear operation given by

$$FrFT\{f(x)\} = F_\alpha(u)$$

$$= \int_{-\infty}^{\infty} f(x)\, k_\alpha(x,u) \qquad \cdots\cdots\cdots (1.1)$$

Where

$$k_\alpha \quad (x,u) = \sqrt{\frac{1-icot\alpha}{2\pi}}\; e^{\frac{i}{2sin\alpha}[(x^2+u^2)cos\propto -2xu]} \qquad \cdots\ \cdots\cdots (1.2)$$

1.2 The test function space E:

An infinitely differentiable complex valued smooth function φ on R^n belongs to $E(R^n)$ if for each compact set $I \subset S_a$, where $S_a = \{x; x \in R^n, |x| \le 0, a > 0\}$, I∈Rn.

$$\gamma_{E,p}(\varphi) = \sup_{x \in I} \left| D_x^p \ \varphi(x) \right| < \infty, \quad \text{where p=1, 2, 3.....}$$

Thus $E(R^n)$ will denote the space of all $\varphi \in E(R^n)$ with suppose contained in S_a. Note that the space E is complete and therefore a Frechet space. Moreover we say that f is a fractional Fourier transformable if it is a member of E^*, the dual space of E.

$$FrFT\{f(x)\} = F_\alpha(u) = \langle f(x), K_\alpha(x,u) \rangle\ ,$$

where

$$K_\alpha(x,u) = C_{1\alpha}\, e^{iC_{2\alpha}[(x^2+u^2)\cos\alpha - 2xu]}$$

FrFT has many applications in solution of differential equations, optical beam propagtioan and spherical mirror resonators, optical diffraction theory, quantum mechanics, statistical optics, signal detectors, pattern recognition, space image recovery etc. [5, 6].
In the present work Generalization of the FrFT is presented by kernel method. Scaling property, Modulation theorem, and Parsvels theorem is proved for Generalized Fractional Fourier transform.

II. Distributional Fractional Fourier Transform

The Distributional fractional Fourier transforms of $f(x,y) \in E(R^n)$ can be defined by

$\cdots\cdots\cdots (3.1)$

$\cdots\cdots\cdots (3.2)$

$$\text{where } C_{1\alpha} = \sqrt{\frac{1 - i\cot\alpha}{2\pi}}, \qquad C_{2\alpha} = \frac{1}{2\sin\alpha}$$

The right hand side of (3.1) has a meaning as the application of $f \in E^*$ to $k_\alpha(x,u) \in E$.

III. Scaling Property

$$FrFT\{f(ax)\} = \frac{1}{a}\{FrFT\, g(x)\}(u), \quad \text{where} \quad g(x) = e^{\frac{i}{2}\left\{\left(\frac{1-a}{a}\right)\left[\left(\frac{1+a}{a}\right)\tau^2 \cot\alpha - 2\,u\, cosec\ \alpha\tau\right]\right\}} f(\tau)$$

Proof:

$$FrFT\{f(x)(u)\} = C_{1\alpha}\int_{-\infty}^{\infty} f(x) e^{iC_{2\alpha}[(x^2+u^2)\cos\alpha - 2xu]}\, dx$$

$$FrFT\{f(ax)\}(u) = C_{1\alpha}\int_{-\infty}^{\infty} f(ax) e^{\frac{i}{2sin\alpha}[(x^2+u^2)\cos\alpha - 2xu]}\, dx$$

putting $ax = \tau, \quad x = \frac{\tau}{a}$

$$\therefore a\, dx = d\tau$$

$$FrFT\{f(ax)\}(u) = C_{1\alpha} e^{\frac{iu^2\cot\alpha}{2}}\int_{-\infty}^{\infty} e^{\frac{i}{2}\cot\alpha\left(\frac{\tau}{a}\right)^2 - i\,u\left(\frac{\tau}{a}\right) cosec\ \alpha} f(\tau)\frac{d\tau}{a}$$

$$= \frac{C_{1\alpha}}{a} e^{\frac{iu^2\cot\alpha}{2}}\int_{-\infty}^{\infty} e^{\frac{i}{2}\left[1+\frac{1-a^2}{a^2}\right](\tau)^2\cot\alpha}\; e^{-i\,u\ cosec\ \alpha\left[1+\frac{1-a}{a}\right]\tau} f(\tau)d\tau$$

$$= \frac{C_{1\alpha}}{a} e^{\frac{iu^2\cot\alpha}{2}}\int_{-\infty}^{\infty} e^{\frac{i}{2}\tau^2\cot\alpha - i\,u\,\tau\ cosec\ \alpha}\; e^{\frac{i}{2}\left[\frac{1-a^2}{a^2}\right]\tau^2\cot\alpha - i\,u\left(\frac{1-a}{a}\right)cosec\ \alpha\ \tau} f(\tau)d\tau$$

$$= \frac{1}{a}\int_{-\infty}^{\infty} C_{1\alpha}\, e^{\frac{i}{2sin\alpha}[(\tau^2+u^2)cos\propto - 2\tau u]}\; e^{\frac{i}{2}\left[\frac{1-a^2}{a^2}\right]\tau^2\cot\alpha - i\,u\left(\frac{1-a}{a}\right)cosec\ \alpha\ \tau} f(\tau)d\tau$$

$$= \frac{1}{a}\left\{FrFT\ e^{\frac{i}{2}\left[\frac{1-a^2}{a^2}\right]\tau^2\cot\alpha - i\,u\left(\frac{1-a}{a}\right)cosec\ \alpha\ \tau} f(\tau)\right\}(u)$$

$$= \frac{1}{a}\{FrFT\ \ g(x)\}(u)$$

where $\quad g(x) = e^{\frac{i}{2}\left[\frac{1-a^2}{a^2}\right]\tau^2\cot\alpha - i\,u\left(\frac{1-a}{a}\right)cosec\ \alpha\ \tau} f(\tau)$

$$= e^{\frac{i}{2}\left\{\left(\frac{1-a}{a}\right)\left[\left(\frac{1+a}{a}\right)\tau^2\cot\alpha - 2\,ucosec\ \alpha\ \tau\right]\right\}} f(\tau)$$

IV. Parseval's Identity for Fractional Fourier transform

If $FrFT\{f(x)\} = F_\alpha(u)$ and $FrFT\{g(x)\} = G_\alpha(u)$ then

i. $\int_{-\infty}^{\infty} f(x)\overline{g(x)}dx = \int_{-\infty}^{\infty} F_\alpha(u)\overline{G_\alpha(u)}du$

ii. $\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty}\left|\overline{F_\alpha(u)}\right|^2 du$

Proof: By definition of

$$FrFT\{g(x)\}(u) = G_\alpha(u) = C_{1\alpha} e^{\frac{i}{2}u^2\cot\alpha}\int_{-\infty}^{\infty} e^{\frac{i}{2}x^2\cot\alpha - i\,u\,x\ cosec\ \alpha}\, g(x)dx \qquad \cdots\ \cdots\cdots(4.1)$$

using inversion formula of $FrFT\{f(x)\}$

$$g(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{sin\alpha}\sqrt{\frac{2\pi}{1 - icot\alpha}}\; e^{-\frac{i}{2sin\alpha}[(x^2+u^2)cos\propto - 2xu]}\, G_\alpha(u)du$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{i}{2}x^2\cot\alpha}\frac{1}{sin\alpha\sqrt{1 - icot\alpha}}\int_{-\infty}^{\infty} e^{-\frac{i}{2}u^2\cot\alpha + ix\ u\ \ cosec\ \ \alpha}\, G_\alpha(u)du$$

Taking complex conjugate we get

$$\overline{g(x)} = \frac{1}{\sqrt{2\pi}} e^{\frac{i}{2}x^2\cot\alpha}\frac{1}{sin\alpha\sqrt{1 + icot\alpha}}\int_{-\infty}^{\infty} e^{\frac{i}{2}u^2\cot\alpha - ix\ u\ \ cosec\ \ \alpha}\,\overline{G_\alpha(u)}du$$

Now

$$\int_{-\infty}^{\infty} f(x)\overline{g(x)}dx = \int_{-\infty}^{\infty} f(x)dx\left(\frac{1}{\sqrt{2\pi}} e^{\frac{i}{2}x^2\cot\alpha}\frac{1}{sin\alpha\sqrt{1 + icot\alpha}}\int_{-\infty}^{\infty} e^{\frac{i}{2}u^2\cot\alpha - ix\ u\ \ cosec\ \ \alpha}\,\overline{G_\alpha(u)}du\right)$$

$$= \frac{1}{sin\alpha\sqrt{1 + icot\alpha}}\int_{-\infty}^{\infty}\overline{G_\alpha(u)}du\frac{1}{\sqrt{1 - icot\alpha}}\left(\sqrt{\frac{1 - icot\alpha}{2\pi}}\int_{-\infty}^{\infty} e^{\frac{i}{2}x^2\cot\alpha} e^{\frac{i}{2}u^2\cot\alpha - ixu\ \ cosec\alpha}\, f(x)dx\right)$$

$$= \frac{1}{sin\alpha\sqrt{1+icot^2\alpha}}\int_{-\infty}^{\infty}\overline{G_\alpha(u)}\left(\sqrt{\frac{1-icot\alpha}{2\pi}}\int_{-\infty}^{\infty}e^{\frac{i}{2sin\alpha}[(x^2+u^2)cos\propto-2xu]}f(x)dx\right)du$$

$$= \frac{1}{sin\alpha\sqrt{1+icot^2\alpha}}\int_{-\infty}^{\infty}\overline{G_\alpha(u)}F_\alpha(u)du$$

$$= \frac{1}{sin\alpha\ cosec\ \alpha}\int_{-\infty}^{\infty}\overline{G_\alpha(u)}F_\alpha(u)du$$

$$\int_{-\infty}^{\infty}f(x)\overline{g(x)}dx = \int_{-\infty}^{\infty}\overline{G_\alpha(u)}F_\alpha(u)du \qquad \cdots\cdots\cdots(4.2)$$

Hence proved

ii) Let $f(x)=g(x)$ then $F_\alpha(u)=G_\alpha(u)$ and $\overline{F_\alpha(u)}=\overline{G_\alpha(u)}$ then eq^n (4.2) becomes

$$\int_{-\infty}^{\infty}f(x)\overline{f(x)}dx = \int_{-\infty}^{\infty}F_\alpha(u)\,\overline{F_\alpha(u)}du$$

$$\Rightarrow \int_{-\infty}^{\infty}|f(x)|^2dx = \int_{-\infty}^{\infty}\left|\overline{F_\alpha(u)}\right|^2du$$

V. Modulation property for FrFT

If $FrFT\{f(ax)\}(u)$ denotes generalized fractional Fourier transform of $f(x)$ then

$$FrFT\{f(x)\cos ax\}(u)$$

$$= \frac{C_{1\alpha}}{2}e^{-\frac{i}{2}u^2\cot\alpha\sin\alpha}\{[FrFT\,f(x)e^{i\,u\,a\cot\alpha}](u-a\sin\alpha)+[FrFT\,f(x)e^{-i\,u\,a\cot\alpha}](u+a\sin\alpha)\}$$

Proof:

$$FrFT\{f(x)\}(u) = C_{1\alpha}e^{\frac{i}{2}u^2\cot\alpha}\int_{-\infty}^{\infty}e^{\frac{i}{2}x^2\cot\alpha-i\,u\,x\,cosec\ \alpha}f(x)dx$$

$$FrFT\{f(x)\cos ax\}(u)$$

$$= C_{1\alpha}e^{\frac{i}{2}u^2\cot\alpha}\int_{-\infty}^{\infty}e^{\frac{i}{2}x^2\cot\alpha-i\,u\,x\,cosec\ \alpha}\left(\frac{e^{iax}+e^{-iax}}{2}\right)f(x)dx$$

$$= C_{1\alpha}e^{\frac{i}{2}u^2\cot\alpha}\frac{1}{2}\int_{-\infty}^{\infty}\left\{\left(e^{\frac{i}{2}x^2\cot\alpha-i\,u\,x\,cosec\ \alpha}e^{iax}\right)+\left(e^{\frac{i}{2}x^2\cot\alpha-i\,u\,x\,cosec\ \alpha}e^{-iax}\right)\right\}f(x)dx$$

$$= \frac{C_{1\alpha}}{2}e^{\frac{i}{2}u^2\cot\alpha}\int_{-\infty}^{\infty}\left\{\left(e^{\frac{i}{2}x^2\cot\alpha-i\,(u\,cosec\ \alpha-a)x}\right)+\left(e^{\frac{i}{2}x^2\cot\alpha-i\ x\,(u\,cosec\ \alpha+a)}\right)\right\}f(x)dx$$

$$= \frac{C_{1\alpha}}{2}e^{\frac{i}{2}u^2\cot\alpha}\int_{-\infty}^{\infty}\left\{\left(e^{\frac{i}{2}x^2\cot\alpha-i\,cosec\ \alpha(u-a\sin\alpha)x}\right)+\left(e^{\frac{i}{2}x^2\cot\alpha-i\ x\,cosec\ \alpha(u+a\sin\alpha)}\right)\right\}f(x)dx$$

$$= \frac{C_{1\alpha}}{2}\left\{\int_{-\infty}^{\infty}e^{\frac{i}{2}u^2\cot\alpha}e^{\frac{i}{2}x^2\cot\alpha-i\,cosec\ \alpha(u-a\sin\alpha)x}f(x)dx\right.$$
$$\left.+\int_{-\infty}^{\infty}e^{\frac{i}{2}u^2\cot\alpha}e^{\frac{i}{2}x^2\cot\alpha-i\ x\,cosec\ \alpha(u+a\sin\alpha)}f(x)dx\right\}$$

$$= \frac{C_{1\alpha}}{2}\left\{\int_{-\infty}^{\infty}e^{\frac{i}{2}(u-a\sin\alpha)^2\cot\alpha}e^{i\,u\,a\cos\alpha}e^{\frac{-a^2i}{2}\cos\alpha\sin\alpha}e^{\frac{i}{2}x^2\cot\alpha-i\,cosec\ \alpha(u-a\sin\alpha)x}f(x)dx\right.$$
$$\left.+\int_{-\infty}^{\infty}e^{\frac{i}{2}(u+a\sin\alpha)^2\cot\alpha}e^{-i\,u\,a\cos\alpha}e^{\frac{-a^2i}{2}\cos\alpha\sin\alpha}e^{\frac{i}{2}x^2\cot\alpha-i\ x\,cosec\ \alpha(u+a\sin\alpha)}f(x)dx\right\}$$

$$= \frac{C_{1\alpha}}{2}e^{\frac{-a^2i}{2}\cos\alpha\sin\alpha}\left\{e^{i\,u\,a\cos\alpha}\int_{-\infty}^{\infty}e^{\frac{i}{2}[x^2+(u-a\sin\alpha)^2]\cot\alpha}e^{-i\ x\,(u-a\sin\alpha)cosec\ \alpha}f(x)dx\right.$$
$$\left.+e^{-i\,u\,a\cos\alpha}\int_{-\infty}^{\infty}e^{\frac{i}{2}[x^2+(u+a\sin\alpha)^2]\cot\alpha}e^{-i\ x\,(u+a\sin\alpha)cosec\ \alpha}f(x)dx\right\}$$

$$= \frac{C_{1\alpha}}{2}e^{\frac{-i}{2}a^2\cos\alpha\sin\alpha}\left\{\int_{-\infty}^{\infty}e^{\frac{i}{2}[x^2+(u-a\sin\alpha)^2]\cot\alpha}e^{-i\ x\,(u-a\sin\alpha)cosec\ \alpha}f(x)e^{i\,u\,a\cos\alpha}dx\right.$$
$$\left.+\int_{-\infty}^{\infty}e^{\frac{i}{2}[x^2+(u+a\sin\alpha)^2]\cot\alpha}e^{-i\ x\,(u+a\sin\alpha)cosec\ \alpha}f(x)e^{-i\,u\,a\cos\alpha}dx\right\}$$

$$= \frac{C_{1\alpha}}{2}e^{\frac{-i}{2}a^2\cos\alpha\sin\alpha}\{[FrFT\,f(x)e^{i\,u\,a\cot\alpha}](u-a\sin\alpha)\quad+[FrFT\,f(x)e^{-i\,u\,a\cot\alpha}](u+a\sin\alpha)\}$$

6. If $\{FrFT\{f(x)\}\}(u)$ denotes generalized fractional Fourier transform of $f(x)$ then

$$\{FrFT\{f(x)\}\ sin\ ax\}(u) = (-i)\frac{C_{1\alpha}}{2}e^{\frac{-i}{2}a^2\cos\alpha\sin\alpha}\{[FrFT\,f(x)e^{i\,u\,a\cot\alpha}](u-a\sin\alpha) + [FrFT\,f(x)e^{-i\,u\,a\cot\alpha}](u+a\sin\alpha)\}$$

Proof:

$$[FrFT\,f(x)](u) = C_{1\alpha}e^{\frac{i}{2}u^2\cot\alpha}\int_{-\infty}^{\infty}e^{\frac{i}{2}x^2\cot\alpha - i\,u\,x\,cosec\,\alpha}f(x)dx$$

$\therefore [FrFT\,f(x)\sin ax](u)$

$$= C_{1\alpha}e^{\frac{i}{2}u^2\cot\alpha}\int_{-\infty}^{\infty}e^{\frac{i}{2}x^2\cot\alpha - i\,u\,x\,cosec\,\alpha}\sin ax\ f(x)dx$$

$$= C_{1\alpha}e^{\frac{i}{2}u^2\cot\alpha}\int_{-\infty}^{\infty}e^{\frac{i}{2}x^2\cot\alpha - i\,u\,x\,cosec\,\alpha}\left(\frac{e^{iax}-e^{-iax}}{2i}\right)f(x)dx$$

$$= \frac{C_{1\alpha}}{2i}e^{\frac{i}{2}u^2\cot\alpha}\int_{-\infty}^{\infty}\left(e^{\frac{i}{2}x^2\cot\alpha - i\,u\,x\,cosec\,\alpha}e^{iax}\right)-\left(e^{\frac{i}{2}x^2\cot\alpha - i\,u\,x\,cosec\,\alpha}e^{-iax}\right)f(x)dx$$

$$= \frac{C_{1\alpha}}{2i}e^{\frac{i}{2}u^2\cot\alpha}\int_{-\infty}^{\infty}\left\{\left(e^{\frac{i}{2}x^2\cot\alpha - i\,(u\,cosec\,\alpha - a)x}\right)-\left(e^{\frac{i}{2}x^2\cot\alpha - i\,x\,(u\,cosec\,\alpha + a)}\right)\right\}f(x)dx$$

$$= \frac{C_{1\alpha}}{2i}e^{\frac{i}{2}u^2\cot\alpha}\int_{-\infty}^{\infty}\left\{\left(e^{\frac{i}{2}x^2\cot\alpha - i\,cosec\,\alpha(u - a\sin\alpha)x}\right)-\left(e^{\frac{i}{2}x^2\cot\alpha - i\,x\,cosec\,\alpha(u + a\sin\alpha)}\right)\right\}f(x)dx$$

$$= \frac{C_{1\alpha}}{2i}\left\{\int_{-\infty}^{\infty}e^{\frac{i}{2}u^2\cot\alpha}e^{\frac{i}{2}x^2\cot\alpha - i\,cosec\,\alpha(u - a\sin\alpha)x}f(x)dx - \int_{-\infty}^{\infty}e^{\frac{i}{2}u^2\cot\alpha}e^{\frac{i}{2}x^2\cot\alpha - i\,x\,cosec\,\alpha(u + a\sin\alpha)}f(x)dx\right\}$$

$$= \frac{C_{1\alpha}}{2i}\left\{\int_{-\infty}^{\infty}e^{\frac{i}{2}(u - a\sin\alpha)^2\cot\alpha}e^{i\,u\,a\cos\alpha}e^{\frac{-a^2 i}{2}\cos\alpha\sin\alpha}e^{\frac{i}{2}x^2\cot\alpha - i\,cosec\,\alpha(u - a\sin\alpha)x}f(x)dx - \int_{-\infty}^{\infty}e^{\frac{i}{2}(u+a\sin\alpha)^2\cot\alpha}e^{-i\,u\,a\cos\alpha}e^{\frac{-a^2 i}{2}\cos\alpha\sin\alpha}e^{\frac{i}{2}x^2\cot\alpha - i\,x\,cosec\,\alpha(u + a\sin\alpha)}f(x)dx\right\}$$

$$= \frac{C_{1\alpha}}{2i}e^{\frac{-a^2 i}{2}\cos\alpha\sin\alpha}\left\{e^{i\,u\,a\cos\alpha}\int_{-\infty}^{\infty}e^{\frac{i}{2}[x^2+(u - a\sin\alpha)^2]\cot\alpha}e^{-i\,x\,(u - a\sin\alpha)cosec\,\alpha}f(x)dx - e^{-i\,u\,a\cos\alpha}\int_{-\infty}^{\infty}e^{\frac{i}{2}[x^2+(u+a\sin\alpha)^2]\cot\alpha}e^{-i\,x\,(u + a\sin\alpha)cosec\,\alpha}f(x)dx\right\}$$

$$= \frac{C_{1\alpha}}{2i}e^{\frac{-i}{2}a^2\cos\alpha\sin\alpha}\left\{\int_{-\infty}^{\infty}e^{\frac{i}{2}[x^2+(u - a\sin\alpha)^2]\cot\alpha}e^{-i\,x\,(u - a\sin\alpha)cosec\,\alpha}f(x)e^{i\,u\,a\cos\alpha}dx - \int_{-\infty}^{\infty}e^{\frac{i}{2}[x^2+(u+a\sin\alpha)^2]\cot\alpha}e^{-i\,x\,(u + a\sin\alpha)cosec\,\alpha}f(x)e^{-i\,u\,a\cos\alpha}dx\right\}$$

$$= (-i)\frac{C_{1\alpha}}{2}e^{\frac{-i}{2}a^2\cos\alpha\sin\alpha}\{[FrFT\,f(x)e^{i\,u\,a\cot\alpha}](u-a\sin\alpha) + [FrFT\,f(x)e^{-i\,u\,a\cot\alpha}](u+a\sin\alpha)\}$$

Generalized Fractional Fourier Transform

Sr. No	$f(x)$	$[FrFT\,f(x)](u)$
1.	$f(ax)$	$\frac{1}{a}\ FrFT\left\{e^{\frac{i}{2}\left\{\left(\frac{1-a}{a}\right)\left[\left(\frac{1+a}{a}\right)\tau^2\cot\alpha - i\,ucosec\,\alpha\,\tau\right]\right\}}f(\tau)\right\}du$
2.	$\cos ax\,f(x)$	$\frac{1}{2}\sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{-\frac{i}{2}a^2\cot\alpha\sin\alpha}\{[FrFT\,f(x)e^{i\,u\,a\cot\alpha}](u-a\sin\alpha) + [FrFT\,f(x)e^{-i\,u\,a\cot\alpha}](u+a\sin\alpha)\}$
3.	$\sin ax\,f(x)$	$\frac{(-i)}{2}\sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{-\frac{i}{2}a^2\cot\alpha\sin\alpha}\{[FrFT\,f(x)e^{i\,u\,a\cot\alpha}](u-a\sin\alpha) + [FrFT\,f(x)e^{-i\,u\,a\cot\alpha}](u+a\sin\alpha)\}$
4.	$C_1 f(x) + C_2 g(x)$	$C_1[FrFT\,f(x)](u) + C_2[FrFT\,g(x)](u)$

5.	$e^{-ax} f(x)$	$\sqrt{\frac{1-icot\alpha}{2\pi}}\left\{\left(1+\frac{i}{a}cosec\ \alpha\right)FrFT\left(e^{-ax}xf(x)\right)(u) + \left(\frac{a-i\ cosec\alpha}{a+i\ cosec\alpha}\right)FrFT\left(e^{-ax}f'(x)\right)(u)\right\}$
6.	$f'(x)$	$(-i\cot\alpha)FrFT\{x\, f(x)\}(u) + (i\, u\,\mathrm{cosec}\,\alpha)FrFT\{f(x)\}(u)$
7.	$f(x-x_0)$	$e^{\frac{i}{2}x_0{}^2\cot\alpha - i\, u\, x_0\, cosec\ \alpha}\ FrFT\left\{e^{i\, x\, x_0\, cosec\ \alpha\, f(x)}\right\}(u)$
8.	$e^{iax} f(x)$	$e^{\frac{a^2\sin 2\alpha}{2}} e^{ia(u-a\sin\alpha)\cos\alpha}\ \left\{FrFT\left(f(x)\right)\right\}(u-a\sin\alpha)$

VI. Conclusion

In this paper Modulation theorem, Scaling property, Parsvels theorem is proved for Generalized Fractional Fourier transform.

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