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Determination of Hyper Static Moments in Post-Tensioned Concrete Flat Slab

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ABSTRACT
Hyperstatic forces develop in an indeterminate structure. These forces result when the imposed deformations due to prestressing are restrained or prevented by the support conditions of the structure. The presence of these secondary moments involves extra work in the analysis and design of statically indeterminate prestressed concrete structures.

A parametric study has been presented in this paper where three flat slabs of different aspect ratios have been analysed. Based on the observations of this study further problem has been defined.

In this paper, a 3x3 panel flat slab of size 7.2mx5.76m has been selected for the analysis. The ultimate moments due to dead load and live load are balanced totally by cables. Percentage hyperstatic moments developed in comparison to primary moments in all strips are computed. Further balancing of moments to a selected range of 50%, 60% and 70% has been made.

The compressive stress check of all the models at transfer and service stage has been made.

Keywords: Hyperstatic moments, post-tensioned flat slab, concordant profile.

I. INTRODUCTION

Statically indeterminate structures offer greater redundancy. Hyperstatic Forces are generated in an Indeterminate Structure. In an indeterminate structure, the number of secondary forces equals the number of indeterminacies, and the secondary moment is the moment due to all such secondary forces acting simultaneously. The net effect of a tendon is the sum of the primary and the secondary moments. In an indeterminate structure, the restoring secondary moments are generated in such a manner that it would lose contact with one or more supports. Secondary Moments develop in Post-Tensioned Concrete members due to Prestressing forces as a consequence of constraint by the supports to the free movement of the member. Secondary moments are significant and hence must be accounted for in the design of Prestressed Concrete Indeterminate Structures.

(Aalami, Dec 1998)[1]Illustrated hyperstatic action in a two span post-tensioned beam which was cast and stressed prior to installation.

(Bommer, January 2004)[2] Explained the concept and use of secondary forces and moments using the basic engineering principle of equilibrium. **(Guyon, 1951)** [3] introduced the concept of the concordant profile, which is a profile that causes no secondary moments; e_s and e_p coincide. He pointed out that it was possible to move the cable profile by means of a linear transformation in such a way that the line of thrust remains unchanged. Studies were carried out by **Girija S Karanjikar** [4] under the guidance of Dr.K.K.Sangle. on Determination of Hyperstatic Moments in Two-way Post-tensioned Pre-stressed Slab **(2011-12)**. In this study a flat slab of size 8x8 m was analysed for determination of hyperstatic moments. In this analysis the tendons were used to balance 50% of (DL + LL) moments. In this stretching, 11% of hyperstatic moments were developed in comparison to primary moments.

II. METHODOLGY & VALIDATION

For the determination of hyperstatic moments, SAFE software (V12.3.0) is used. SAFE is a tool for designing reinforced and post-tensioned concrete structures based on finite element approach.

To validate the software a prestressed continuous and propped cantilever beam have been analysed and the results obtained by both manual calculations and software are compared.

Propped cantilever rectangular beam of 300 x 500 mm of length 12m is Prestressed by linear cable with eccentricity 100mm having a prestresing force of 293.579 KN to determine the hyperstatic moments developed in this beam due to prestressing. The details of the beam are as given:

b = 300mm, D = 500mm
ρ_c= 23.563 KN/m^3
E = 23.667 x 10^3 N/mm^2
$I= \frac{bD^3}{12}= \frac{0.3\times0.5^3}{12} = 3.125 \times 10^{-3}\ m^4$

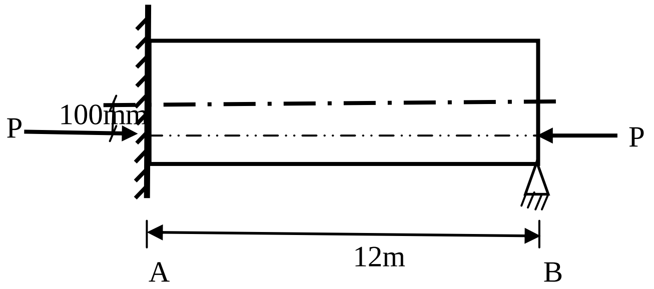

Figure2.1 Propped cantilever beam with straight tendon

Dead Load = ρ_cx b x d
= 23.563 X 0.3 X 0.5 = 3.53 KN/m
Live Load = 2 KN/m
Total load = D.L + L.L = W = 5.534 KN/m

$R_A = \frac{5WL}{8} = \frac{5 \times 5.534 \times 12}{8} = 41.51$ KN

$R_B = \frac{3WL}{8} = \frac{3 \times 5.534 \times 12}{8} = 24.903$ KN

$\Delta_{max} = 0.00541 \frac{WL^4}{EI} = 0.00541 \frac{5.534 \times 12^4}{26.67 \times 10^6 \times 3.125 \times 10^{-3}}$
$= 7.458 \times 10^{-3}$

$M_{positive} = \frac{9WL^2}{128} = \frac{9 \times 5.534 \times 12^2}{128} = 56.03$ KNm

$M_{negative} = \frac{-WL^2}{8} = \frac{-5.534 \times 12^2}{8} = -99.54$KNm

SAFE Result

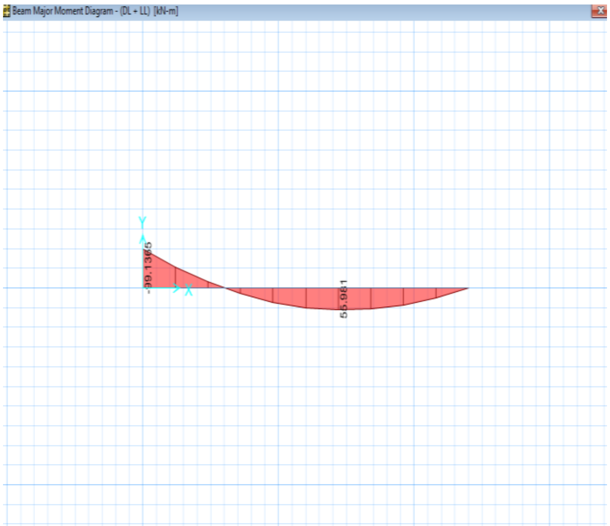

Figure 2.2 Moments due to (DL + LL)

Secondary moment at A = $\frac{3P \times e}{2} = \frac{3 \times 293.579 \times 0.1}{2} =$ 44.03 KNm
Primary moment = P x e = 293.579 x 0.1 = 29.3579 KNm
PT Final = Primary moment – Hyperstatic moment
At A = 44.03 – 23.3579 = 14.67 KNm
At B = 23.3579 KNm

SAFE Result

Figure 2.3 Hyperstatic moments (secondary)

SAFE Result

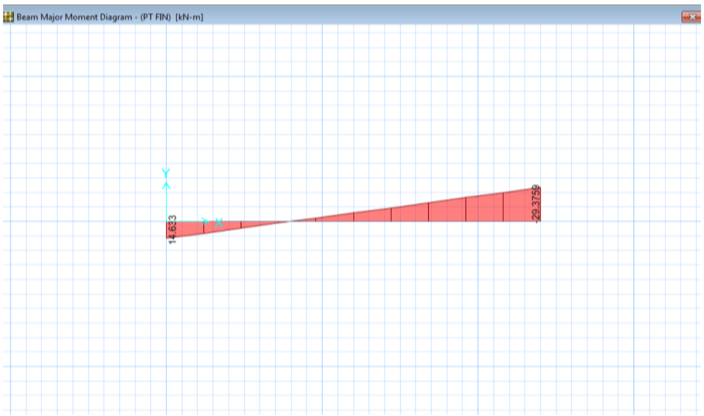

Figure 2.4 PT Final moments

III. PARAMETRIC STUDIES

Three panels with different aspect ratios have been selected for the study. Since, the flat slabs are post-tensioned, only column strips are used. Column strips share higher proportions of the total moment and column strips being stiffer than middle strips, the middle strips are ignored for the model.

A flat slab of 3x3 panel of size 7.2m x 4.8m, 7.2m x 5.76m, 7.2m x 6.4m has been selected for the analysis. The flat slab model has been analysed for dead load and live load moments and then balanced for 50%, 60% and 70% of the moments. The prestressing force is also varied to check the behavioural pattern of hyperstatic moments.

It has been observed that exterior strips share fewer moments than the interior strips, hence values of eccentricity to be provided exceeds more than the limiting value at interior strips. The prestressing force needs to be increased for such cases.

Graphical representation of hyperstatic moments developed in exterior column strip CSA1

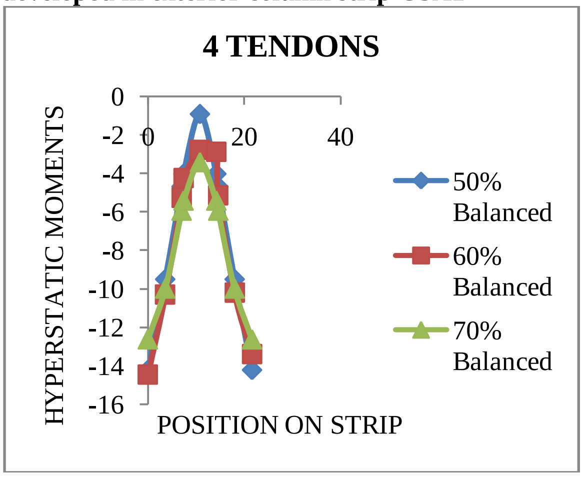

Figure 3.1 Hyperstatic moments for A/R=1.5 using 4 tendons

Graphical representation of hyperstatic moments developed in exterior column strip CSA1

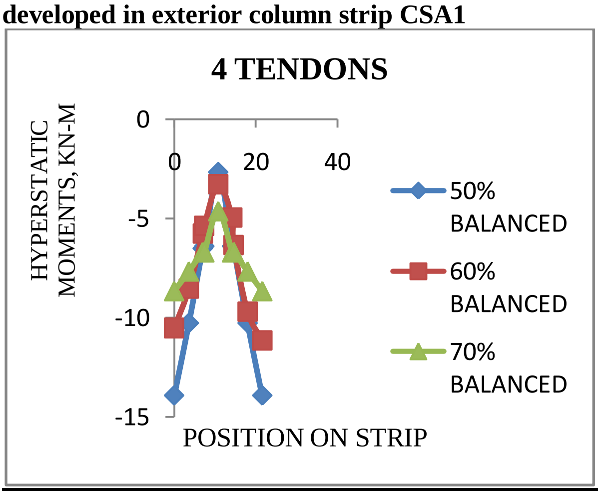

Figure3.2 Hyperstatic moments for A/R=1.25 using 4 tendons

Graphical representation of hyperstatic moments developed in exterior column strip CSA1

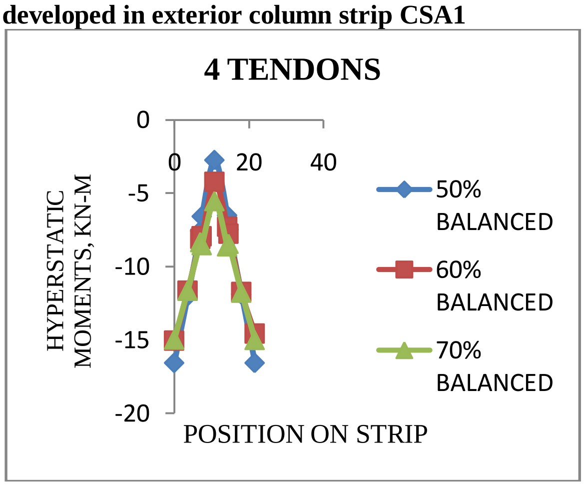

Figure3.3 Hyperstatic moments for A/R=1.125 using 4 tendons

IV. RESULTS AND DISCUSSIONS

From the observations of all panel sizes, 7.2 m×5.76m panel has been selected for further study.

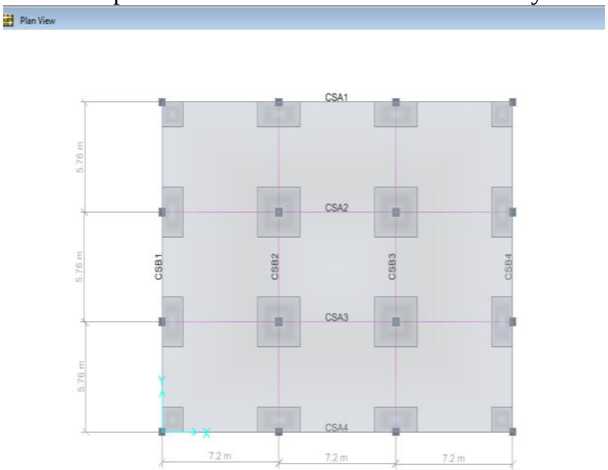

Figure 4.1 Flat slab Model of 3x3 panel of size 7.2m x 5.76m

A three spanned continuous beam has been analysed by balancing 100% of the dead and live load moments. Hyperstatic moments developed in three spanned continuous beam for given loading condition are fairly negligible when moments are fully balanced. It has been analysed for the 1.5(DL+LL) moments.

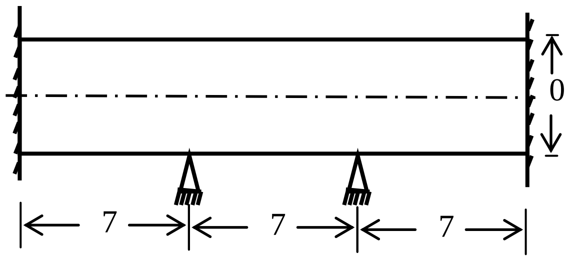

Figure 4.2 Three spanned continuous beam

Beam Details:
Width of beam, b = 300mm
Depth of beam, d = 600mm
Characteristic strength of concrete, f_{ck} = 30Mpa
Dead load = 1Kpa, Live load = 4 Kpa

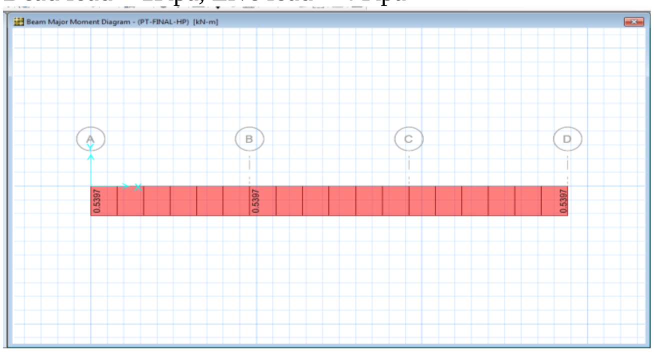

Figure 4.3 Hyperstatic moments

The same concept of balancing the moments applied to 3x3 panel post-tensioned flat slab. The prestressing force to be selected has been altered several times so as to keep the eccentricity obtained in a practical location. The values of eccentricity to be provided should be within the limiting range of slab depth so as to maintain the minimum cover. Hence, the prestressing force has been increased to 4406.3925KN by using 30 unbonded tendons. It has been observed that the hyperstatic moments are not completely eliminated after 100% balancing. It is negligible at the mid span but fairly more at the supports. The compressive stresses at service are satisfactory but it has been observed that the compressive stresses at bottom face of slab during transfer exceed the maximum value. As continuous slab is an indeterminate structure the hyperstatic moments cannot be eliminated totally. Hence, balancing of DL+LL moments to a specific percentage could be a resort to achieve the stresses within allowable limits.

% BALANCED	Strip Text	Span ID Text	Location Text	P x e KNM	PT(HYP) KNM	%HYP MOMENTS
50% balanced	CSA1&4	Span 1	End	-83.7215	5.879	**7.02**
	CSB1&4	Span 1	Start	30.84475	-28.1991	**91.42**
60% balanced	CSA1&4	Span 3	Start	-96.9406	7.8435	**8.09**
	CSB1&4	Span 3	End	35.25114	-33.0642	**93.80**
70% balanced	CSA1&4	Span 3	Start	-114.566	7.3999	**6.46**
	CSB1&4	Span 3	End	44.06393	-41.2303	**93.57**
100% balanced	CSA2&3	Span 2	End	-392.169	-5.584	**1.42**
	CSB1&4	Span 3	End	61.6895	-40.4057	**65.50**

Secondary moments can be eliminated if a concordant profile is achieved. In the design of a statically indeterminate prestressed concrete member, it is not necessary to ensure that the chosen profile is concordant, although this simplifies the calculations. Consideration on limits of magnitude of prestressing force is required as the stress limits should not exceed the allowable limits and the range of eccentricities should also be within the slab depth taking cover into the account. In practice it is found that concordant profiles are not the most economical, but in the design process it is quite useful to start with a concordant profile and then to modify it as necessary.

The same panel has been further balanced for 50%, 60% and 70% of the dead load and live load moments to study the percentage of hyperstatic moments developed with respect to primary moments keeping the prestressing force same in all the strips. 100% balancing of 1.5(DL+LL) moments give 1.42% to 65.5% of hyperstatic moments in comparison to primary moments. The compressive stress at the bottom face of slab for 100% exceeds the maximum value at transfer conditions and is within allowable limits at service stage.50% balancing of 1.5(DL+LL) moments give 7.02% to 91.42% of hyperstatic moments in comparison to primary moments. The compressive stresses at the bottom face of slab for 50% balanced moments is within allowable limits for both transfer and service conditions. 60% balancing of 1.5(DL+LL) moments give 8.09% to 93.8% of hyperstatic moments in comparison to primary moments. The compressive stress at the bottom face of slab for 60% exceeds the maximum value at transfer conditions and is within allowable limits at service stage. 70% balancing of 1.5(DL+LL) moments give 6.46% to 93.57% of hyperstatic moments in comparison to primary moments. The compressive stresses at the bottom face of slab for 70% balanced moments is within allowable limits for both transfer and service conditions. The percentage hyperstatic moments are shown below:

V. CONCLUSIONS

Increase in prestressing force also results in increase in hyperstatic moments. Balancing the moments totally does not eliminate secondary moments.100% balancing of ultimate dead and live load moments yields compressive stresses more than the allowable limits at the transfer stage. Exterior strips share fewer moments than the interior strips, hence values of eccentricity to be provided exceeds more than the limiting value at interior strips. Thus, the prestressing force is increased to 30 unbonded tendons. In practice it is found that concordant profiles are not the most economical, but in the design process it is quite useful to start with a concordant profile and then to modify it as necessary. Determination of cable profile is generally an iterative procedure. Amount of load to be balanced by prestress should be chosen after individual study.

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