

## Model Building And Cascade Compensation Of Angle Servo Control System

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### ABSTRACT

Control theory is foundation to construct the control system and Automation technologies tools are material ensure to construct the control system. The control system experiments are essential in the field of development and research performance by using mathematical model building and cascade compensation. From this point of the design, Angle servo control system emphasizes with mathematical model building and cascade compensation both in theoretical foundation and experimental techniques. In this report, we are analyzed the working principle and function of components in system. To learn how to measure the transfer functions of all components and prepare how to calculate performance parameters using step response curve. In the end of experiment, we gain the values of response curves of the angle servo system and also studied calculation reading as well.

**Keywords** - Control system, Angle servo, Cascade, Mathematical model building, Experiment, Simulation

### I. INTRODUCTION

Over recent years, servo systems are now being widely applied in industrial sector. [1] The angle servo system is made of synchro, phase sensitive demodulation circuit, position regulator, speed regulator, current regulator, PWM (Pulse Width Modulation) circuit, power amplifier, DC motor and DC tachometer generators, system block diagram is shown as following fig.1 [2]. Synchro: to invert mechanical angle difference into voltage signal. Here it is measurement and comparison components.

Phase sensitive demodulation circuit: to transmit AC signals associated with the phase into a DC signal. Position regulator, speed regulator, current regulator: to improve system performance. PWM circuit: to modulate the DC Signal modulated into square wave signal. PA (power amplifier): to amplify the signal controlled further so that the system has enough power to drive motor rotation. Implementation motor: to invert the electrical signal into mechanical motion, and push the load movement.

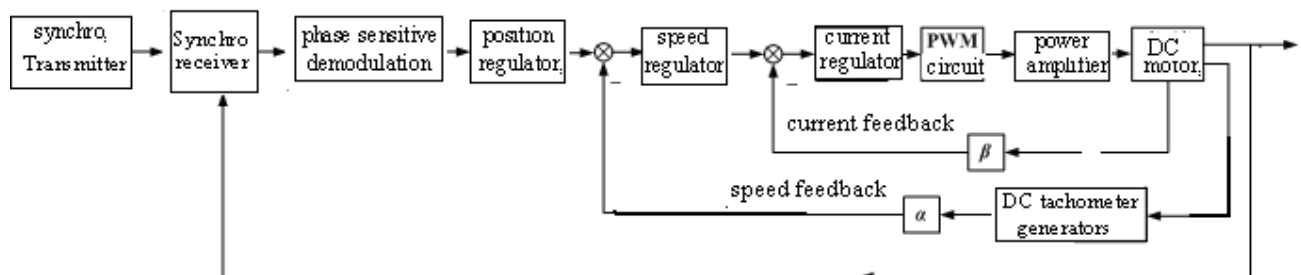


Fig. 1: Angle Servo System Block Diagram

### II. WORKING PRINCIPLE OF SYSTEM

Synchro transmitter is regarded as system input; synchro receiver and the DC torque motor are connected by coupling [6], so that the position of DC motor can be detected by rotating synchro transmitter, the system is added a certain angle, so that a offset angle  $\Delta\theta$  between the transmitter and receiver of synchro is brought, and offset angle  $\Delta\theta \neq 0$ , the receiver output voltage  $E_{scm}\sin\Delta\theta \neq 0$ . Output AC voltage signal is inverted into a DC signal by phase-sensitive demodulator; the DC

signal is used to drive DC motor rotation after throughout position regulator, speed regulator, current regulator, PWM circuitry, power amplifier. DC motor will rotate towards the direction that can eliminate error until the system error is zero.



Fig. 2 angle servo system physical graphic

### III. INPUT AND OUTPUT PROPERTIES AND WORKING PRINCIPLE OF SYSTEM COMPONENTS

#### Control synchro

Control synchro is usually used with a pair. Principle diagram is shown in fig. 3.

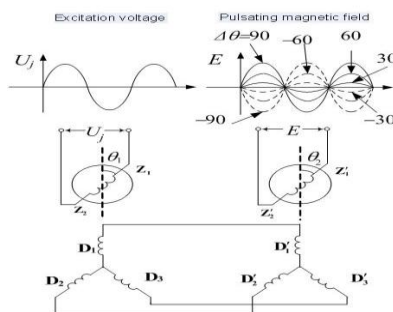


Fig. 3: Control Synchro Principle diagram

The angle between transmitter excitation winding and D1 phase of the stator is represented with  $\theta_1$ , and the angle between receiver output winding and D1' phase of the stator is represented with  $\theta_2$ , then the induced electric potential generated by the output windings of receiver is  $E = E_{scm} \cos(\theta_1 - \theta_2) = E_{scm} \sin \Delta\theta$ , where  $\Delta\theta$  is offset angle, when  $\Delta\theta$  is small:  $E_{scm} \sin \theta \approx E_{scm} \Delta\theta$ , which means that the relationship between input and output of synchro is linear, at this time, the transfer function of synchro can be written as:

$$G(s) = \frac{E(s)}{\Delta\theta(s)} = E_{scm} = KZ \quad (1)$$

#### Phase sensitive demodulation circuit

In the servo system, the output signal of synchro is 400Hz AC signal, the implementation component is DC torque motor, so the AC signal must be inverted into DC signal by demodulation circuit. Demodulation circuit diagram is shown as followed in fig.4.

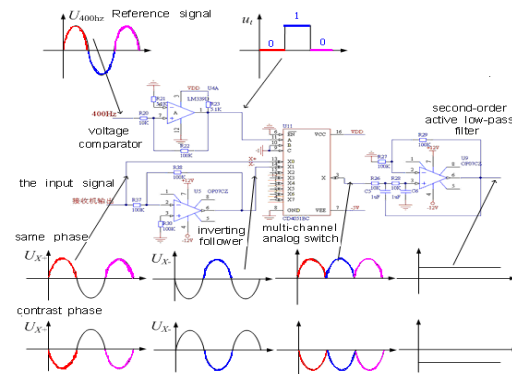


Fig. 4: Phase sensitive demodulation circuit principle

Phase sensitive demodulation circuit is made of voltage comparator, inverting follower, multi-channel analog switch and second-order active low-pass filter. The input signal of demodulation circuit is output of control synchro receiver; reference signal is 400Hz sine signal, its frequency is same to output signal of control synchro receiver. When reference signal pass by voltage comparator, it is inverted into pulse signal, its output is 0 or 1, and this signal is regarded as selection signal of multi-channel analog switch, when its output is 0,  $x_0(x+)$  is selected, when its output is 1,  $x_1(x-)$  is selected, where,  $x+$  and  $x-$  represent separately input and output of inverting follower. When input signal pass by multi-channel analog switch, it is inverted into direct signal with high frequency AC components, so phase sensitive demodulation circuit needs low-pass filter to filter high frequency AC components. Here second-order active low-pass filter is used, through above circuits; the AC signal is inverted into DC signal. Waves of all signals from different components are showed as figure 6. In figure 6, when input signal of demodulation circuit and reference signal has same phase, which means output voltage of control synchro is positive, the output DC voltage of low-pass filter is positive; when input signal of demodulation circuit and reference signal has contrast phase, which means output voltage of control synchro is negative, the output DC voltage of low-pass filter is negative too.

In order to obtain high accuracy of voltage comparator, inverting follower and multi-channel analog switch in phase sensitive demodulation circuit are 1, so transfer function of demodulation circuit is just decided by second-order active low-pass filter: [3]

$$G(s) = \frac{U_s(s)}{E(s)} = \frac{K_s \omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad (2)$$

#### PWM (Pulse Width Modulation) circuit

The principle of PWM circuit, through a series of pulse width modulation, to obtain the

required equivalent waveform (including the shape and amplitude) is called for PWM [7].

In PWM circuit, saw tooth wave and DC voltage are compared in input of voltage comparator, and then square wave signals with different pulse width are output. Square wave signals with different pulse width are showed as Figure 5.

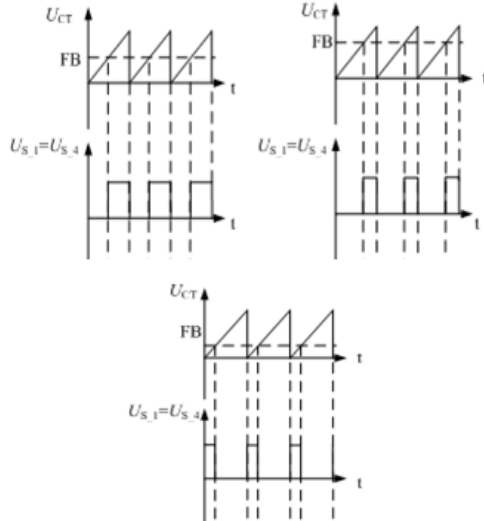


Fig. 5: Square wave signals with different pulse width in PWM circuit

Positive pulse width is the ratio of the total pulse width is called the duty cycle D. in figure 8, U represents input DC voltage of PWM circuit, tH, tL represent positive and negative pulse width, then the duty cycle  $D = t_H / T$ ,  $T = t_H + t_L$ , output voltage of PWM circuit  $U_{out} = UD$ .

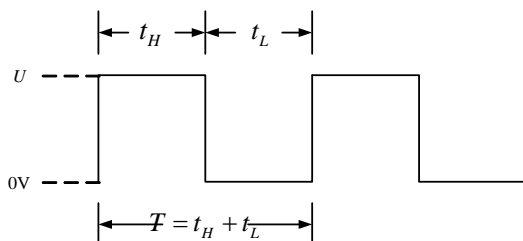


Fig. 6: Square wave signals with different pulse width in PWM circuit

From above analysis, output voltage of PWM circuit is proportional to the duty cycle D, and if this output voltage is amplified and added to

both sides of DC motor, then speed of DC motor will be adjusted.

**PWM circuit in experiment**

In experiment, TL494 PWM chip is used, FB-side is input of the PWM circuit, S\_1, S\_2, S\_3, S\_4 are outputs of the PWM circuit, and the signals will be sent to DC motor drive circuit.

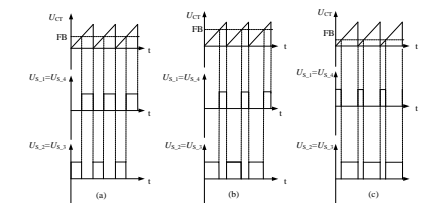
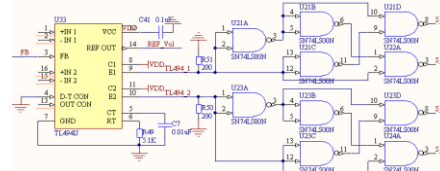


Fig. 7: PWM circuit and output waves in experiment

The transfer function of PWM can be regarded as proportional link:

$$G(s) = \frac{U_{PWM}(s)}{U_s(s)} = K_{PWM} \quad (3)$$

**PA ( power amplifier )**

The role of power amplifier in circuit is to amplify the signal amplitude output by forward class circuit. In experiment, IR2110 chip is used as power amplifier chip, details of chip can be got by searching handbook. Pins of chip are shown as following fig.8.

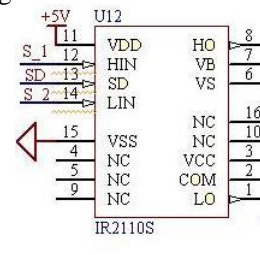


Fig. 8: Pins of IR2110 chip

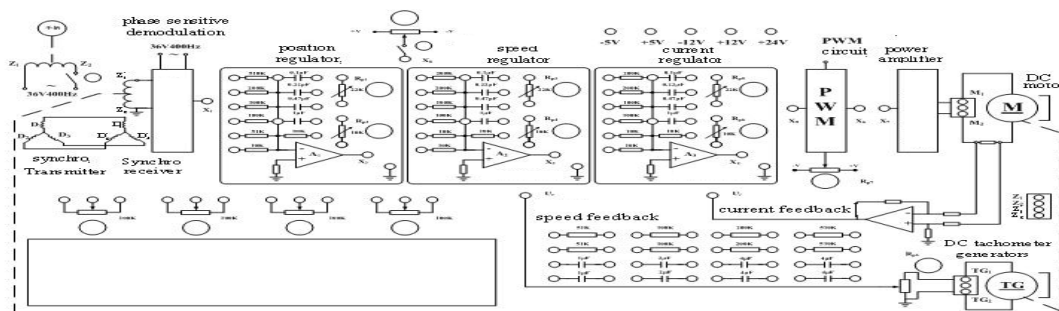


Fig. 9: Experiment system panel

Experimental box panel is shown in figure 9, all input and output interfaces of the system are distributed on the panel.  $U_i, U_v$  are feedback signals of current and speed;  $Z_1, Z_2, Z'_1, Z'_2$  are synchro transmitter excitation winding and the receiver output winding;  $M_1, M_2$  is armature winding of DC motor;  $TG_1, TG_2$  are DC tachometer generator armature winding,  $X_1$  is output of the phase sensitive demodulator;  $X_2$  is output of position regulator;  $X_3$  is output of speed regulator;  $X_4$  is output of current regulator;  $X_5$  and  $X_6$  are the input and output ports of the PWM circuit;  $X_7$  is input of power amplifier ( $X_6$  and  $X_7$  has been connected);  $X_8$  is speed command output, adjustable DC voltage; In Potentiometer and Switches, Potentiometer  $RP_1 \sim RP_6$  are adjustable resistance of position regulator, speed regulate

The transfer function of PA can be regarded as proportional link:

The voltage of DC motor armature

$$G(s) = \frac{U(s)}{U_{PWM}(s)} = K_P$$

#### IV. MATHEMATICAL MODEL BUILDING BY EXPERIMENT METHOD

##### To measure the transfer function of synchro

We have known that the transfer function of synchro can be written as:

$$G(s) = \frac{E(s)}{\Delta\theta(s)} = E_{scm} = K_Z \quad (5)$$

So we just need to measure value of  $K_Z$ .

Steps: Firstly, 400Hz, 36V AC power is supplied, the using an oscilloscope to view Synchro output waveform that should be sine wave. If at this point the angle of the receiver is set to be zero, the offset angle  $\Delta\theta =$  the angle of transmitter. Change the angle of transmitter, the size of the sine wave amplitude will be changed with the mechanical differential angle  $\Delta\theta$  changing.

Secondly, measure the value of  $K_Z$ .

##### Method 1:

An oscilloscope is connected to output termination of the Synchro, then the oscilloscope will display as sine wave; to turn the transmitter, so that sine wave amplitude is adjusted to the maximum, at this time, record the effective value (RMS) of the waveform, this RMS shall be  $K_Z$ .

##### Method 2:

An oscilloscope is connected to output termination of the Synchro, then the oscilloscope will display as sine wave; to turn the transmitter, so that sine wave will change. When offset angle  $\Delta\theta$  is small, to record separately the amplitudes of input and output of synchro using oscilloscope, then the ratio of the amplitudes of input and output is  $K_Z$ .

A. To measure the transfer function of phase

sensitive demodulator

The input waveform of phase sensitive demodulator is sine wave; output waveform is similar to the DC waveform, and the polar of amplitude will change with the changing of angle difference  $\Delta\theta$ .

We have known that the transfer function of phase sensitive demodulator can be written as

$$G(s) = \frac{U_s(s)}{E(s)} = \frac{K_s \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (6)$$

Step1: to measure the time constant  $T_s$ , and overshoot  $\sigma_p$  of demodulator

To cut off the connection of current regulator and PWM circuit and the system is open-loop state; a certain angle is sent by rotating synchro transmitter, and then power is turned on; using the oscilloscope to measure output of phase-sensitive demodulator, the step response curve will be appeared on the oscilloscope, The time constant  $T_s$ , and overshoot  $\sigma_p$  will be got by analyzing the step response curve.

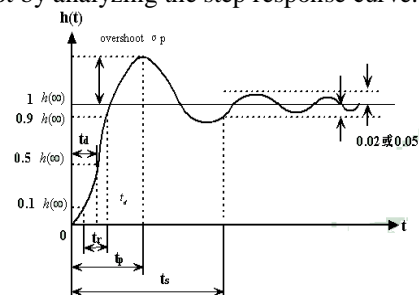


Fig. 10: the relationship between system parameters and the step response curve

The transfer function of phase-sensitive demodulator can be got by calculating the relationship between time merits ( $T_s, \sigma_p$ ) and frequency domains merits ( $\zeta, \omega_n$ ).

Step 2: to measure the gain coefficient  $K_s$  of demodulator

The system is open-loop state, and to change the mechanical differential angle  $\Delta\theta$  of synchro, and then to measure the RMS of synchro output voltage and DC output voltage value of demodulator with a multimeter, a set of repeated measurements will get a set of measurement data, and to draw a curve, the slope of the curve shall be gain coefficient  $K_s$  of demodulator.

B. To measure the transfer function of PWM circuit and power amplifier (PA)

We have known that the transfer function of PWM and PA can be regarded as proportional links:

$$G(s) = \frac{U_{PWM}(s)}{U_s(s)} = K_{PWM} \quad (7)$$

$$G(s) = \frac{U(s)}{U_{PWM}(s)} = K_P \quad (8)$$

So in this step, we regard PWM circuit and power amplifier as one component.

Step 1: To view the output waveform of PWM with an oscilloscope, the waveform should be a square wave. Using Rp7 potentiometer can set to the initial zero of PWM circuit, that when the input is zero, the duty cycle D of PWM circuit output waveform is 50%.

Step 2: To view the input waveform of power amplifier (PA) (it is also the output of PWM) with an oscilloscope, the waveform should be a square wave with  $\pm 24V$  amplitude.

Step 3: To make the system be open loop state; to change the input voltage of the PWM circuit and measure the effective value (RMS) of the input voltage of the PWM circuit with a multi meter, at the same time, to measure the average value of output voltage of power amplifier (square wave) with the oscilloscope, repeated several times to get a set of data, draw the curve, the slope of curve shall be multiplying KPWM \*Kp of gains of PWM circuit and power amplifier.

C. To measure the transfer function of DC torque motor

We have known that the transfer function of DC motor can be regarded as inertial link:

$$\frac{\omega(s)}{u_a(s)} = \frac{K_m}{T_m s + 1} \quad (9)$$

The step response method can be used to measure the transfer function of DC motor.

Step1: To make the system be open loop state and connect X8 and X5, which means that input voltage with certain amplitude to the PWM circuit, then DC motor will be driven by PWM and power amplifier circuit and rotate.

Step 2: To turn on the switch from OFF position to ON position, which means that add a step signal to the DC motor. The output of DC motor is speed; the value cannot be measured with the oscilloscope, so the value should be achieved by indirect methods. DC motor and DC tachometer generator are connected using Couplings coaxial connector, so the output of the former is latter's input, so through measuring the output curve of DC tachometer generator, the step response curve of DC torque motor can be obtained, Km and Tm, can be got by analyzing the step response curve.

D. To measure the transfer function of DC Tachometer Generator

As the speed feedback component, DC Tachometer Generator's transfer function is its output slope; it can be obtained from the nameplate:

$$K_C = 2.7V / rad \cdot s^{-1}$$

E. To measure the transfer function of speed regulator and current regulator

As experiment modules, the transfer function of speed regulator and current regulator can be

calculated according to actual analog circuits.

## V. CASCADE COMPENSATION OF ANGLE SERVO CONTROL SYSTEM

According to performance requirements: Adjust-time:  $t_s < 0.2s$ , overshoot:  $\sigma_p < 25\%$ , steady-state error:  $e_{ss} = 0$

Cascade compensation is perhaps the most common control system topology [4]. To design a reasonable cascade compensation link to comprehensive system, and calculates parameters of compensation link. The design method can be frequency response method or root locus method. We analysis the function of cascade compensation network in angle servo control system and observe the results as well [5] Kv=10K Where, K is open-loop gain of inherent characteristics (no any compensation link) of system.

### Control System Testing

According to the system performance to design correction network, and then the correction network is accessed to system. To change the mechanical differential angle and observe if the desired targets is achieved using an oscilloscope, if the requirement is not satisfied, we can adjust the parameters of the calibration network with a Minato test method. To make the system be closed loop state, changing transmitter angle and followed by the receiver rotating. At this time, to observe output waveform of demodulation with the oscilloscope. Step response curves testing: turn off the power and to change the transmitter angle (not too large, usually 50 or so), then connect the power supply. At this point the output of phase sensitive demodulation is shown:

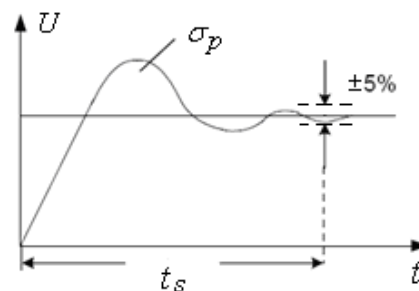


Figure 12: Step response curves of angle servo system

If the above waveform is divergent oscillatory, the power must be immediately turned off, so that to prevent the control circuit burn out and check the reason. By observing the step response curve of the system, and adjust parameters according to the general impact on the system response, repeated Minato try to change the parameters to meet performance requirements, and then to determine the correct link parameters. Before Minato, we learn how the parameter values of the correction link influence the system response. By using the oscilloscope, the system performance is gain with the response curve as

shown in fig. 12.

**VI. ANALYSIS OF EXPERIMENT DATA OBTAINED OPEN-LOOP TRANSFER FUNCTION**

**1. Strike Kz • Ks initial angle  $\theta = 43$  degrees**

Kz-Synchro magnification phase sensitive demodulator Ks-Ts-gain phase sensitive demodulator time constant. Obtained from the experimental curve, the value of Ts = 0.01s

**Table: 1 the given correction link parameters and gain values of step response curve of the system**

	Input(Degree)	output(Degree)	Kz.Ks	Mean
1	53-43	0.38	1.74	1.87
2	63-43	0.697	2	
3	73-43	0.9766	1.87	
4	83-43	1.5864	1.87	

**2. Strike Kpwm • Kp initial angle  $\theta = 43$  degrees**

Kpwm-PWM circuit gain Kp-amplifier gain Tm motor time constant, observed by the oscilloscope, the value of Tm = 0.08s

**Table: 2 the given correction link parameters and gain values of step response curve of the system**

	input	output	Km.Kc	Average
1	-13.556	-55.25	4.08	3.92
2	-10.70	-42.35	3.96	
3	-8.89	-34.09	3.83	
4	8.76	32.97	3.76	
5	10.50	40.54	3.86	
6	12.19	48.79	4.00	

**3. Strike Km • Kc initial angle  $\theta = 43$  degrees**

**Table: 3 the given correction link parameters and gain values of step response curve of the system**

	Input	Output	Kpwm. Kp	Average
1	-9.0006	-13.361	1.48	1.48
2	-8.120	-12.087	1.49	
3	-6.893	-10.286	1.49	
4	-4.556	-6.907	1.52	
5	4.822	6.958	1.44	
6	6.859	9.958	1.45	
7	8.675	12.632	1.46	
8	9.873	14.511	1.47	

Km-motor gain coefficient Kc-speed motor speed voltage ratio, the value of Kc = 2.7v/rad/s plate reading to Km = 1.45

**VII. DESIGN ASPECTS OF SYSTEM CALIBRATION TO MEET THE FOLLOWING INDICATORS:**

Adjust the time ts <0.2s, overshoot  $\sigma$  <0.25%, steady-state error is 0, Open-loop transfer function of the inherent links:  $G(S) = 4 / S \cdot (0.01S + 1) \cdot (0.08S + 1)$ , Cut Frequency  $Wc0 = 3.8$ ,  $Wc = 38.7$  in place, phase margin  $\gamma = -3$  degrees, Indicators Conversion:  $\gamma = 60.8$  degrees  $Wc = 38.7$

So the use of advance correction: as shown with two devices in series (two ahead of providing a total phase margin of 64 degrees), Circuit diagram of a slightly (where  $R1 = 10K$ ,  $C = 4\mu f$ ). Transfer function is -  $Rf (R1 + R2) CS + 1$ ,  $R1 R2CS + 1$ . Corrected final design aspects as: function omitted here (indicating the function parameter size). Three experimental debugging process, summarize tandem correction control law. Experimental process parameter changes are as follows: An advance correction two leading correction after correction correction link is:

- R1 = 10K R1 = 10K unchanged
- R2 = 2.54K unchanged R2 = 2.54K unchanged
- C = 4 -  $2\mu f$  C =  $4\mu f$  unchanged (0.0005s + 1) (0.01s + 1)
- Rf = 60.5K adjust Rf = 39.3k regulation

**Summary:**

① first correction device according to the design of the connection, give step input, output has a larger oscillation, rapid response;

② adjustment Rf, adding step signal becomes smaller when the Rf oscillation found that smaller, but the reaction rate becomes small; increases the oscillation becomes large, the reaction speed. Taken with the oscilloscope to observe a balance, the result does not meet system metrics.

③ adjust other parameters: change R2 effect is not obvious, so remain unchanged, changed C1, the  $4\mu f$  changed to  $2\mu f$ , adding step signal and found that the oscillation was significantly smaller, but fast is not affected. Therefore, under the condition  $C1 = 2\mu f$  adjust Rf1, Rf2, meet the conditions of point obtained, taken with an oscilloscope observation point, the results in addition to the steady state error is slightly outside, to meet the targets. That is, for the adjustment process and summary.

**Records corrected link parameters and system step response curves**

correction parameter tuning the final part: Advance correction means an advance correction device 2

- $R1 = 10K\Omega$   $R1 = 10K\Omega$
- $R2 = 2.5K\Omega$   $R2 = 2.5K\Omega$
- $C = 2\mu f$   $C = 4\mu f$

After tuning correction link:  $G = 23.8 (0.025s + 1) (0.05s + 1) (0.005s + 1) (0.01s + 1)$

The original system, calibration devices, after correction system Bode diagram see table

output curve data record: a step input is  $\Delta\theta \approx 40$  degrees to 50 degrees

Data recorded as follows:

- output stable value: 0.838 overshoot  $\sigma = 0.078/0.838 * 100\% = 9.3\%$
- Maximum:  $0.838 + 0.078 = 0.156$
- Adjust the time  $t_s$ :  $t_s = 0.14s$
- the steady-state error:  $e_{ss} > 0$

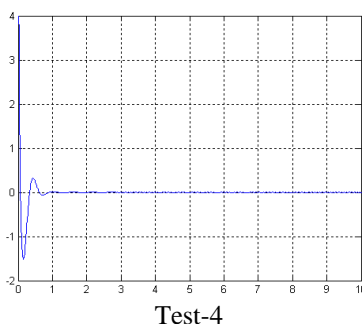
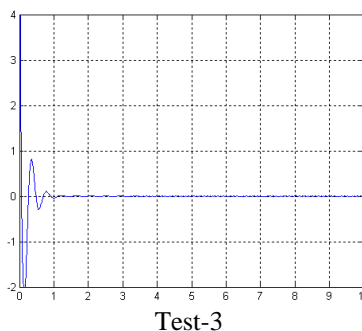
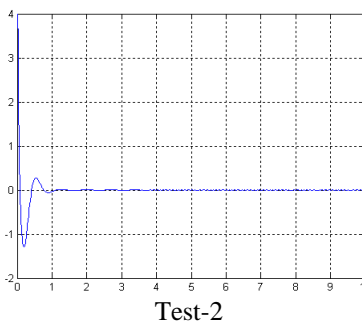
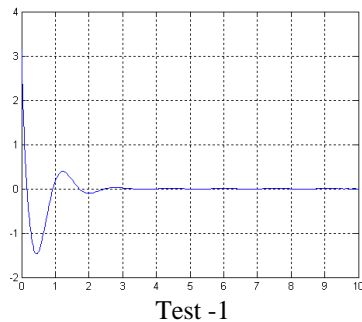


Fig. 13: Test: 1,2,3,4 Show System performance of step response curves of angle servo system

### VIII. CONCLUSION

In conclusion to analysis experimental data and obtain open-loop transfer function of system. And

design correction part of the design and make them satisfy the following performance indicators: Adjust the time  $t_s < 0.2s$ , overshoot  $\sigma_p < 25\%$ , and steady-state error  $e_{ss} = 0$ . According to the debugging process, to conclude cascade compensation control law and to record parameters of calibration link and the step response curve of system, analysis and process the data and curves. Therefore some thinking questions are also appended which describe the detail role of calibration link in the system and basic ideas of the general control system design, and draw a the general correction system block diagram. In the end given correction link parameters and gain values of step response curve of the system are observed.

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