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Method for Solving Unbalanced Assignment Problems Using Triangular Fuzzy Numbers

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Abstract:
In this paper, we proposed the fuzzy salesman and areas by Hungarian unbalanced assignment problems. In this problems C_{ij} denotes the cost for assigning the salesman to the areas. The cost is usually deterministic in nature. In this paper $(\widetilde{C_{ij}})$ has been considered to be triangular and trapezoidal numbers denoted by $(\widetilde{C_{ij}})$ which are more realistic and general in nature. For finding the optimal assignment, we must optimize total cost this problem assignment. In this paper first the proposed balanced assignment problem and it is formulated to the crisp assignment problem in the linear programming problem form and solved by using Hungarian method and using Robust's ranking method (3) for the fuzzy numbers. Numerical examples show that the fuzzy ranking method offers an effective tool for handling the fuzzy unbalanced assignment problem.
Key words: Fuzzy sets, fuzzy unbalanced assignment problem, Triangular fuzzy number, Trapezoidal fuzzy number, ranking function.

I. INTRODUCTION

The unbalanced assignment problem is a special type of linear programming problem in which our objective is to assign number of salesman's to number of areas at a minimum cost(time). The mathematical formulation of the problem suggests that this is a 0-1 programming problem and is highly degenerate all the algorithms developed to find optimal solution of transportation problem are applicable to unbalanced assignment problem. However, due to its highly degeneracy nature a specially designed algorithm. Widely known as Hungarian method proposed by Kuhn (1) is used for its solution.

In this paper, we investigate more realistic problem and namely the unbalanced assignment problem, with fuzzy costs $(\widetilde{C_{ij}})$. Since the objectives are to minimize the total cost (or) to maximize the total profit. Subject to some crisp constraints, the objective function is considered also as a fuzzy number. The methods are to rank the fuzzy objective values of the objective function by some ranking method for fuzzy number to find the best alternative. On the basic of idea the Robust's ranking methods (3) has been adopted to transform the fuzzy unbalanced assignment problem to a crisp one so that the conventional solution methods may be applied to solve unbalanced assignment problem by R.PanneerSelvam (17). The idea is to transform a problem with fuzzy parameters to a crisp version in the LPP from and solve it by the simple method other than the fuzzy unbalanced assignment problem other applications and this method can be tried in project scheduling maximal flow, transportation problem etc.

Lin and Wen solved the unbalanced assignment with fuzzy interval number costs by a labeling algorithm (4) in the paper by sakawa et.al (2), the authors dealts with actual problems on production and work force assignment in a housing material manufacturer and a sub construct firm and formulated tow kinds of two level programming problems. Chen (5) proved same theorems and proposed a fuzzy unbalanced assignment model that considers all individuals to have some skills. Wang (6) solved a similar model by graph theory. Dubois and for temps (7) surveys refinements of the ordering of solutions supplied by the ,max-min formulation, namely the discrimin practical ordering and the lexinain complete preordering, different kinds of transportation problem are solved in the articles (8,10,12,14,15). Dominance of fuzzy numbers can be explained by many ranking methods (9,11,13,16) of these, Robust's ranking method (3) which satisfies the properties of compensation, linearity and additive. In this paper we have applied Robust's ranking technique (3).

II. PRELIMINARIES

In this section, some basic definitions and arithmetic operations are reviewed. Zadeh (16) in 1965 first introduced fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life.

Definition: 2.1

A fuzzy set is characterized by a membership function mapping elements of a domain, space or universe discourse X to the unit interval [0,1], is A= $\{X, \mu_A(X): x \in X\}$ Here: $\mu_A: x \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(X)$ is called the membership value of $x \in X$ in the fuzzy set A, these membership grades are often represented by real numbers ranging from [0,1]

Definition: 2.2

A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exists at least one $x \in X$ such that $\mu_A(X) = 1$

Definition: 2.3

A fuzzy set A is convex if and only if for any $x_1, x_2 \in X$, the membership function of A satisfies the inequality

$$\mu_A\{\lambda x_1 + (1-\lambda)x_2\} \geq min\{\mu_A(x_1), \mu_A(x_2)\}, 0 \leq \lambda \leq 1$$

Definition .2.4 (Triangular fuzzy number)

For a triangular fuzzy number A(x), it can be represented by $A\ (a,b,c;1)$ with membership function $\mu\ (x)$ given by

$$\mu(x) = \begin{cases} (x-a)/(b-a) & ;a \leq x \leq b \\ 1 & ;\ x = b \\ (c-x)/(c-b) & ;b \leq x \leq c \\ 0 & ;\ Otherwise \end{cases}$$

Definition:2.5 (Trapezoidal fuzzy number)

For a trapezoidal fuzzy number A(x), it can be represented by $A\ (a,b,c,d;1)$ with membership function $\mu\ (x)$ given b

$$\mu(x) = \begin{cases} (x-a)/(b-a) & ;a \leq x \leq b \\ 1 & ;\ b \leq x \leq c \\ (d-x)/(d-c) & ;c \leq x \leq d \\ 0 & ;\ Otherwise \end{cases}$$

Definition:2.6 (K-cut of a trapezoidal fuzzy number)

The K-cut of a fuzzy number A(x) is defined as A(K) = $\{x;\mu(x) \geq k; k \in [0,1]\}$

Definition: 2.7 (Arithmetic Operations)

Addition of two fuzzy numbers can be performed as

$$(a_1,b_1,c_1,) + (a_2,b_2,c_2) = (a_1+a_2, b_1+b_2, c_1+c_2,)$$

Addition of two trapezoidal fuzzy numbers can be performed as

$$(a_1,b_1,c_1,d_1) + (a_2,b_2,c_2,d_2) = (a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2)$$

III. Unbalanced Assignment problem to change into balanced Assignment Problem

The number of rows (areas) is not equal to the number of columns (salesmans) then the problem is termed as unbalanced assignment problem then this problem into change balanced assignment problem as follows necessary number of dummy row (s) / column(s) are added such that the cost matrix is a square matrix the values for the entries in the dummy row (s) / column(s) are assumed to be zero.

IV. Robust's Ranking Techniques Algorithms

The assignment Problem can be stated in the form of $n\ X\ n$ cost matrix $[C_{ij}]$ of real numbers as given in the following

	Salesman 1	Salesman2	....	Salesman j	....	Saleman N
Area 1	C_{11}	C_{12}	....	C_{1j}	....	C_{1n}
Area 2	C_{21}	C_{22}	....	C_{2j}	....	C_{2n}
⋮	⋮	⋮	⋮	⋮	⋮	⋮
Area N	C_{n1}	C_{n2}	⋮	C_{nj}	⋮	C_{nn}

Mathematically assignment problem can be stated as

Minimize

$$Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

$$subject\ to \quad \sum_{i=1}^{n} x_{ij} = 1$$

$$\sum_{j=1}^{n} x_{ij} = 1$$

$$x_{ij} \in [0,1] \rightarrow (1)$$

Where

$$x_{ij} = \begin{cases} 1\,; & if\ the\ i^{th} area\ is\ assigned\ the\ j^{th} saleman \\ 0\,; & Otherwise \end{cases}$$

ie the decision variable denoting the assignment of the area i to job j. $(\tilde{C_{ij}})$ is the cost of assigning the j^{th} salesman to the i^{th} area. The objective is to minimize the total cost of assigning all the salesmen's to the available persons (one salesman to one area)

When the costs or time $(\tilde{C_{ij}})$ are fuzzy numbers, then the total costs becomes a fuzzy number.

$$Z^{V^*} = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij} X_{ij}$$

Hence it cannot be minimized directly for solving the problem. We defined by the fuzzy cost Co-efficient into crisp ones by a fuzzy number ranking method. Robust's ranking technique (3) which satisfies compensation linearity, and additivity properties and provides results which are consistence with human intuition. Given a convex fuzzy number $(\tilde{c})$, the Robust's Ranking index is defined by

$$R(\tilde{c}) = \int_0^1 0.5\,(C_K^L, C_K^U)dk \ \ where\ (C_K^L, C_K^L)$$

is the K-level cut of the fuzzy number $(\tilde{c})$

In this paper we use this method for ranking the objective values. The Robust's ranking index $R(\tilde{c})$ gives the representative value of the fuzzy number$(\tilde{c})$ it satisfies the linearity and additive property:

If $\hat{P} = l\hat{E} + m\hat{y}$ and $\hat{Q} = S\hat{C} - t\hat{N}$ where l, m, s, t are constant then we have $R\hat{p} = lR(\hat{E})$ +n R$(\hat{y})$ and $R\hat{\varphi} = SR(\hat{C})$- tR$(\hat{N})$ on the basis of this property the fuzzy assignment problem can be transformaed into a crisp assignment problem linear programming problem from. The ranking technique of the Robust's is

If R$(\hat{G}) \leq R(\hat{H})$ then $\hat{G} \leq H^V$

ie min $\{\hat{G}, H^V\} = \hat{G}$ from the assignment problem (1) with fuzzy objective function.

$$min\ Z^{V^*} = \sum_{i=1}^{n}\sum_{j=1}^{n} R\,(\hat{C}_{ij})X_{ij}$$

We apply Robust's ranking method (3) (using the linearity and assistive property) to get the minimum objective value Z^{V^*} from the formulation.

$$R(Z^{V^*}) = \min z = \sum_{i=1}^{n}\sum_{j=1}^{n} R\,(\hat{C}_{ij})X_{ij}$$

Subject to

$$\sum_{i=1}^{n} x_{ij} = 1$$

$$\sum_{j=1}^{n} x_{ij} = 1$$

$$x_{ij} \in [0,1] \rightarrow (2)$$

Where

$$x_{ij} = \begin{cases} 1\,; & if\ the\ i^{th} area\ is\ assigned\ the\ j^{th} saleman \\ 0\,; & Otherwise \end{cases}$$

Is the decision variable denoting the assignment of the area i to j^{th} salesman. $(\tilde{C_{ij}})$ is the cost of designing the j^{th} job to the i^{th} area. The objective is to minimize the total cost of assigning all the salesman to the available areas.

Since R $(\tilde{C_{ij}})$ are crisp values, this problem (2) is obviously the crisp assignment problem of the form (1) which can be solved by the conventional methods, namely the Hungarian method or simplex method to solve the linear programming problem form of the problem. Once the optimal solution x^* of model (2) is found the optimal fuzzy objective value Z^{V^*} of the original problem can be calculated as

$$(Z^{V^*}) = \sum_{i=1}^{n}\sum_{j=1}^{n} (\tilde{C}_{ij})\ x_{ij}^*$$

V. Numerical Example

Let us consider a fuzzy unbalanced assignment problem with rows representing 4 area A, B, C, D and columns representing the salesman's S_1, S_2, S_3, S_4, S_5 the cost matrix $(\tilde{C}_{ij})$ is given whose elements are triangular fuzzy numbers. The problem is to find the optimal assignment so that the total cost of area assignment becomes minimum.

$$(\tilde{C}_{ij}) = \begin{pmatrix} (3,6,9) & (6,9,11) & (3,6,12) & (3,9,12) & (3,6,9) \\ (3,9,12) & (3,6,9) & (6,9,11) & (6,9,18) & (3,9,12) \\ (6,9,11) & (9,12,15) & (3,6,9) & (3,6,12) & (6,9,18) \\ (3,9,12) & (6,9,11) & (6,9,18) & (3,6,9) & (3,12,15) \end{pmatrix}$$

Solution: The given problem is an fuzzy unbalanced assignment problem so to change into the fuzzy balanced assignment problem as follows

$$(\tilde{C}_{ij}) = \begin{pmatrix} (3,6,9) & (6,9,11) & (3,6,12) & (3,9,12) & (3,6,9) \\ (3,9,12) & (3,6,9) & (6,9,11) & (6,9,18) & (3,9,12) \\ (6,9,11) & (9,12,15) & (3,6,9) & (3,6,12) & (6,9,18) \\ (3,9,12) & (6,9,11) & (6,9,18) & (3,6,9) & (3,12,15) \\ (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) \end{pmatrix}$$

In conformation to model (2) the fuzzy balanced assignment problem can be formulated in the following mathematical programming form

$$\begin{aligned} \min\{\ & R\,(3,6,9)x_{11} + R\,(6,9,11)x_{12} + R\,(3,6,12)x_{13} + r\,(3,9,12)x_{14} + R\,(3,6,9)x_{15} \\ & + R\,(3,9,12)x_{21} + R\,(3{,}912)x_{22} + R(6,9,11)x_{23} + R(6,9,18)x_{24} \\ & + R(3,9,12)x_{25} + R(6,9,11)x_{31} + R(9,12,15)x_{32} + R(3,6,9)x_{33} \\ & + R(3,6,12)x_{34} + R(6,9,18)x_{35} + R(3,9,12)x_{41} + R(6,9,11)x_{42} \\ & + R(6,9,18)x_{43} + R(3,6,9)x_{44} + R(3,12,15)x_{45} + R(0,0,0)x_{51} \\ & + R(0,0,0)x_{52} + R(0,0,0)x_{53} + R(0,0,0)x_{54} + R(0,0,0)x_{55}\ \} \end{aligned}$$

Subject to

$$\begin{aligned} & x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 1 \\ & x_{11} + x_{21} + x_{31} + x_{41} + x_{51} = 1 \\ & x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 1 \\ & x_{12} + x_{22} + x_{32} + x_{42} + x_{52} = 1 \\ & x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 1 \\ & x_{13} + x_{23} + x_{33} + x_{43} + x_{53} = 1 \\ & x_{41} + x_{42} + x_{43} + x_{44} + x_{45} = 1 \\ & x_{14} + x_{24} + x_{34} + x_{44} + x_{54} = 1 \\ & x_{51} + x_{52} + x_{53} + x_{54} + x_{55} = 1 \\ & x_{15} + x_{25} + x_{35} + x_{45} + x_{55} = 1 \\ & x_{ij} \in [0,1] \end{aligned}$$

Now we calculate R(3,6,9) by applying Robust's ranking method. The membership function of the triangular fuzzy number (3,6,9) is

$$\mu(x) = \begin{cases} \dfrac{x-3}{3} & ; \ 3 < x < 6 \\ 1 & ; \quad x = 6 \\ \dfrac{9-x}{3} & ; \ 6 < x < 9 \\ 0 & ; \ Otherwise \end{cases}$$

The K-Cut of the Fuzzy number (3,6,9) is

$$(C_K^L, C_K^L) = [(b-a)k + a, c - (c-b)k]$$

$$= [3k+3, 9-3k]$$

= 3k+3+9-3k
= 12

$$R(\tilde{C}_{11}) = R\,(3,6,9) = \int_0^1 0.5\ (C_K^L, C_K^L)\,dk$$

$$= \int_0^1 0.5\,(12)dk \ = \ 6$$

Proceeding similarly the Robust's ranking indices for the fuzzy costs $(\tilde{C}_{ij})$ are calculated as:

$$R\,(C_{12}) = 8.75, R(C_{13}) = 6.75, R(C_{14}) = 8.25, R(C_{15}) = 6$$
$$R\,(C_{21}) = 8.25, R(C_{22}) = 6, R(C_{23}) = 8.75, R(C_{24}) = 10.5$$
$$R\,(C_{25}) = 8.25, R(C_{31}) = 8.75, R(C_{32}) = 12, R(C_{33}) = 6$$
$$R\,(C_{34}) = 6.75, R(C_{35}) = 10.5, R(C_{41}) = 8.25, R(C_{42}) = 8.75$$
$$R\,(C_{43}) = 10.5, R(C_{44}) = 6, R(C_{45}) = 10.5, R(C_{51}) = 0$$
$$R\,(C_{52}) = 0, R(C_{53}) = 0, R(C_{54}) = 0, R(C_{55}) = 0$$

We replace these values for their corresponding $(\tilde{C}_{ij})$ in (3) which results in a convenient assignment problem in the linear programming problem. We solve it by Hungarian method to get the following optimal solution.

$$(\tilde{C}_{ij}) = \begin{pmatrix} 6 & 8.75 & 6.75 & 8.25 & 6 \\ 8.25 & 6 & 8.25 & 10.50 & 8.25 \\ 8.75 & 12 & 6 & 6.75 & 10.50 \\ 8.25 & 8.75 & 10.50 & 6 & 10.50 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Solution: by Hungarian method

$$\begin{pmatrix} \cancel{0} & 2.75 & 0.75 & 2.25 & 0 \\ 2.25 & \boxed{0} & 2.75 & 4.50 & 2.25 \\ 2.75 & 6 & \boxed{0} & 0.75 & 4.50 \\ 2.25 & 2.75 & 4.50 & \boxed{0} & 4.50 \\ \boxed{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{pmatrix}$$

The optimal assignment

$$S_1 \to E, S_2 \to B, S_3 \to C, S_4 \to D, S_5 \to A$$

The optimal total minimum cost
= Rs. 6+6+6+6+0
= Rs. 24

The fuzzy optimal assignment

$$x_{15}^{*} = x_{22}^{*} = x_{33}^{*} = x_{44}^{*} = x_{51}^{*} = 1 \; and \; x_{11}^{*} = x_{12}^{*} = x_{13}^{*} = x_{14}^{*} = x_{21}^{*} = x_{23}^{*} = x_{24}^{*} = x_{25}^{*} = x_{31}^{*} = x_{32}^{*} = x_{34}^{*} = x_{35}^{*} = x_{41}^{*} = x_{42}^{*} = x_{43}^{*} = x_{45}^{*} = x_{52}^{*} = x_{53}^{*} = x_{54}^{*} = x_{55}^{*} = 0$$

The fuzzy optimal assignment.

$$S_1 \to E, S_2 \to B, S_3 \to C, S_4 \to D, S_5 \to A$$

The fuzzy optimal total minimum cost

$$= \widetilde{C_{15}} + \widetilde{C_{22}} + \widetilde{c_{33}} + \widetilde{c_{44}} + \widetilde{c_{51}}$$

$$= R\,(3,6,9) + R\,(3,6,9) + R(3,6,9) + R(3,6,9) + R(0,0,0)$$

= R(12,24,36)

Also we find that R Z^{V^*} = 24

VI. Conclusions

In this paper, the unbalanced assignment costs are considered as imprecise number described by fuzzy numbers which are more realistic and general in nature. Moreover, the fuzzy unbalanced assignment problem has been transformed into crisp assignment problem using Robust's ranking indices (3). Numerical example show that by using method we can have the optimal assignment as well as the crisp and fuzzy optimal total cost. By using Robust's (3) ranking methods we have shown that the total cost obtained is optimal moreover, one can conclude that the solution of fuzzy problems can be obtained by Robust's ranking methods effectively, this technique can also be used in solving other types of problems like, project schedules, transportation problems and network flow problems.

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