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Base Rock Vibration Characteristics Analysis from Surface Ground Motion

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ABSTRACT
A computer program is developed assuming underlying soil as horizontally layered and one-dimensional. Soil material non-linearity is considered in this problem. One dimensional shear wave equation is solved. Space is discretized by Finite Element method and time is discretized by Central Difference method. Two parameter hyperbolic model and Masing rule are used to obtain the dynamic stress-strain behaviour. The number of soil layers, the properties of different soil layers such as density, initial shear modulus, reference strain and the properties of rigid base rock such as density and shear modulus are used as input data. Record of the earthquake which occurred at El-Centro in 1940, is used here as surface acceleration data. The output of the program is dynamic stress-strain behaviour in different soil layers, variation of secant shear modulus with time in different soil layers, deformation in different soil layers and base rock acceleration of the soil layer. The soil properties data assumed here is soft layer in the upper portion and comparatively stiffer layer in the bottom layer. The base rock layer is the stiffest layer. The peak base rock acceleration is found greater than the peak surface acceleration. Hysteresis loop is maintained in each layer of the soil column for obtaining dynamic stress-strain behaviour. Variation of secant shear modulus with time is fluctuating. The surface deformation is found greater than the base rock deformation.
Keywords **-** Central Difference method, Finite Element Method, Hyperbolic Model, Secant Shear Modulus, Hysteresis Loop.

I. INTRODUCTION

One of the most important applications of the theory of structural dynamics is in analyzing the response of structures to ground shaking caused by an earthquake. Earthquakes may be defined as the vibration of earth produced by the rapid release of energy. The energy radiates in form of waves in all directions from its source (focus). Another definition is trembling or shaking of the ground caused by the sudden release of energy stored in the rocks beneath the earth surface. The actual cause for earthquake occurance is the great forces acting deep in the earth, these forces put a stress on the rock which may bend or change in volume (strain). The rock can deform until it reaches the ultimate capacity of the rock and a rupture (rock break) occurs. Most earthquakes are also produced from active faults. Surface ground motion can be measured by different instruments. But it is important to know the base rock vibration for determining different soil properties.

II. Research Objectives

The main objective of the study is to develop a computer program for determining base rock vibration from surface earthquake response data. The program requires several input data. The length of each soil layer, the number of soil layer, the properties of soil layer, the properties of rigid bed rock and the time step of input acceleration are required as input data. The properties of soil layer is characterized by density, initial shear modulus & reference strain. The properties of rigid bed rock are characterized by density and shear modulus. The output of the program are several soil parameters such as deformations with time, variations of secant modulus of each layer with time and the stress-strain history of each layer. The output are in graphical form. The specific objectives are-

- To determine the base-rock vibration from surface vibration.
- To determine different soil parameters such as dynamic stress-strain behaviour, variations of secant modulus with time at different soil layers.

III. MATERIALS AND METHODS

A Matlab (MATLAB 7.5.0(R 2007b)) based computer program was modified to estimate the rigid base rock acceleration from surface acceleration. For this reason, global mass matrix and global stiffness matrix and force matrix were modified according to the formulation. The soil column was assumed as one-dimensional and one dimensional shear wave equation was solved. The space was discretized by finite element method and time was discretized by central difference method. The soil material non-linearity was considered and hyperbolic model and Masing's rule were used. The input data was several soil properties such as density, initial shear modulus and shear strain.

The earthquake which was occurred at El-Centro, 1940 was used as the surface acceleration data. The rigid bedrock property which is governed by density and initial shear modulus was also required as input. The output of the program was dynamic stress-strain behaviour, variation of secant modulus with time in each layer, displacement history at each layer and base rock acceleration.

IV. PROBLEM FORMULATION

In earthquake-prone regions, the principal problem of structural dynamics that concern structural engineers is the behavior of structures subjected to earthquake-induced motion of the base of the structures. The displacement of the ground is denoted by u_g, the total (or absolute) displacement of the mass by u^t, and the relative displacement between the mass and ground by u. At each instant of time these displacements are related by

$$u^t(t) = u(t) + u_g \qquad (1)$$

Both u^t and u_g refer to the same inertial frame of reference and their positive directions coincide.

The equation of dynamic equilibrium is

$$f_I + f_D + f_S = 0 \qquad (2)$$

f_D = The damping force = $c\mathring{u}$

f_S = The lateral force = ku

Using the above relationship we find,

$$m\ddot{u} + c\mathring{u} + ku = -m\ddot{u}_g(t) \qquad (3)$$

The basic dynamic equilibrium equation for determining ground response is,

$$[\,m\,]\{\,\ddot{u}\,\} + [\,c\,]\{\,\mathring{u}\,\} + [\,k\,]\,\{\,u\,\} = -\,\{\,M'\,\}\ddot{u}_g \qquad (4)$$

Here, [m] = Global Mass Matrix; { ü } = Acceleration Matrix; [c] = Global Dampin Matrix; { ů } = Velocity Matrix; [k] = Global Stiffness Matrix; { u } = Deformation Matrix; { M' } = Force Matrix; $\ddot{u}_g$ = Base Rock Acceleration.

The basic equilibrium equation for determining ground response in matrix form is given below-

$$\begin{bmatrix} m_{11} & m_{12} & \cdot & \cdot & \cdot & m_{1n} \\ m_{21} & m_{22} & \cdot & \cdot & \cdot & m_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ m_{n1} & m_{n2} & \cdot & \cdot & \cdot & m_{nn} \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \ddot{u}_n \end{Bmatrix} + \begin{bmatrix} c_{11} & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & 0 \end{bmatrix} \begin{Bmatrix} \mathring{u}_1 \\ \mathring{u}_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \mathring{u}_n \end{Bmatrix} + \begin{bmatrix} k_{11} & k_{12} & \cdot & \cdot & \cdot & k_{1n} \\ k_{21} & k_{22} & \cdot & \cdot & \cdot & k_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ k_{n1} & k_{n2} & \cdot & \cdot & \cdot & k_{nn} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ u_n \end{Bmatrix} = -\begin{Bmatrix} M'_1 \\ M'_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ M'_n \end{Bmatrix} \ddot{u}_g \qquad (5)$$

Here, n = number of element

The basic equilibrium equation for determining ground response from base vibration is therefore obtained from the previous equations. The equation is therefore as follows-

$$\begin{bmatrix} m_{11}-\frac{M'_1}{M'_n}m_{n1} & m_{12}-\frac{M'_1}{M'_n}m_{n2} & \cdots & m_{1(n-1)}-\frac{M'_1}{M'_n}m_{n(n-1)} \\ m_{21}-\frac{M'_2}{M'_n}m_{n1} & m_{22}-\frac{M'_2}{M'_n}m_{n2} & \cdots & m_{2(n-1)}-\frac{M'_2}{M'_n}m_{n(n-1)} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ m_{(n-1)1}-\frac{M'_{n-1}}{M'_n}m_{n1} & m_{(n-1)2}-\frac{M'_{n-1}}{M'_n}m_{n2} & \cdots & m_{(n-1)(n-1)}-\frac{M'_{n-1}}{M'_n}m_{n(n-1)} \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \ddot{u}_n \end{Bmatrix}$$

$$+ \begin{bmatrix} c_{11} & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & 0 \end{bmatrix} \begin{Bmatrix} \mathring{u}_1 \\ \mathring{u}_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \mathring{u}_n \end{Bmatrix}$$

$$+ \begin{bmatrix} k_{11}-\frac{M'_1}{M'_n}k_{n1} & k_{12}-\frac{M'_1}{M'_n}k_{n2} & \cdots & k_{1(n-1)}-\frac{M'_1}{M'_n}k_{n(n-1)} \\ k_{21}-\frac{M'_2}{M'_n}k_{n1} & k_{22}-\frac{M'_2}{M'_n}k_{n2} & \cdots & k_{2(n-1)}-\frac{M'_2}{M'_n}k_{n(n-1)} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ k_{(n-1)1}-\frac{M'_{n-1}}{M'_n}k_{n1} & k_{(n-1)2}-\frac{M'_{n-1}}{M'_n}k_{n2} & \cdots & k_{(n-1)(n-1)}-\frac{M'_{n-1}}{M'_n}k_{n(n-1)} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ u_n \end{Bmatrix}$$

$$= \begin{bmatrix} -\left(m_{1n}-\frac{M'_1}{M'_n}m_{nn}\right)\ddot{u}_n-\left(k_{1n}-\frac{M'_1}{M'_n}k_{nn}\right)u_n \\ -\left(m_{2n}-\frac{M'_2}{M'_n}m_{nn}\right)\ddot{u}_n-\left(k_{2n}-\frac{M'_2}{M'_n}k_{nn}\right)u_n \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ -\left(m_{(n-1)n}-\frac{M'_{n-1}}{M'_n}m_{nn}\right)\ddot{u}_n-\left(k_{(n-1)n}-\frac{M'_{n-1}}{M'_n}k_{nn}\right)u_n \end{bmatrix} \qquad (6)$$

The mass matrix can be constructed by the soil element properties such as number of element, element length, density etc. The problem is analyzed as one dimensional problem. The mass matrix is determined by the model boundary value problem. In order to formulate the finite element model based on weak form, it is not necessary to decide a priori the degree of approximation of U^e. The model can be developed for an arbitrary degree of interpolation:

U= $U^e = \sum_{j=1}^{n} u_j^e \psi_j^e(x)$ (7)

Where, ψ_j^e are the Lagrange interpolation functions of degree (n-1). In matrix notation, the linear equations can be written as $[K^e]\{u^e\}=\{f^e\}+\{Q^e\}$ (8)

The matrix $[K^e]$ is called the coefficient matrix in structural mechanics applications. The column vector $\{f^e\}$ is the source vector or force vector in structural mechanics application. The co-efficient matrix and column vector are

$$[K^e] = \frac{a_e}{h_e}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + \frac{c_e h_e}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix} \quad (9)$$

$$\{f^e\} = \frac{q_e h_e}{2}\begin{Bmatrix}1\\ 1\end{Bmatrix} \quad (10)$$

The first portion of $[K^e]$ is $\frac{a_e}{h_e}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$ which is element stiffness matrix and the second portion of $[K^e]$ is $\frac{c_e h_e}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$ which is element mass matrix. For 2-node element connected in series, the element mass matrix and element stiffness matrix is then assembled into global mass and global stiffness matrix. The global mass matrix and global stiffness matrix is then modified according to the equation to obtain the base vibration from ground response.

V. Results

The input surface acceleration and surface deformation are presented below in graphical form-

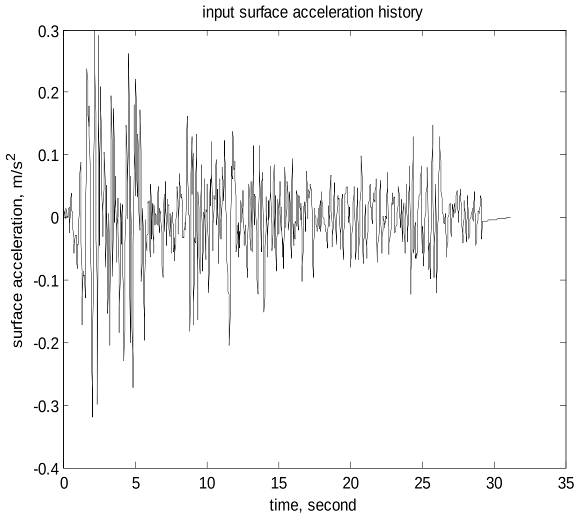

Figure 1: Input surface acceleration with time.

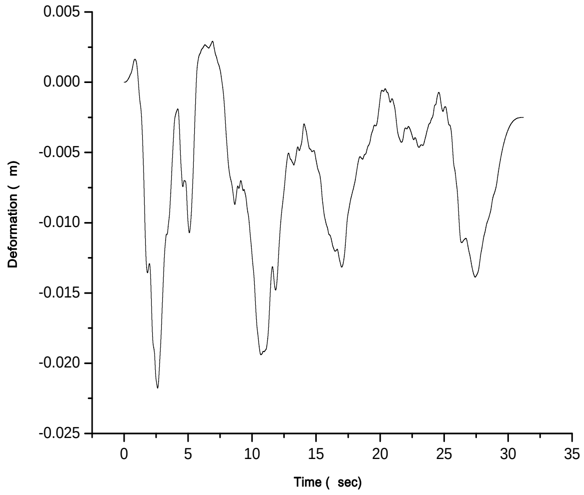

Figure 2: Input surface deformation with time.

The output base rock acceleration and base rock deformation is presented below in graphical form-

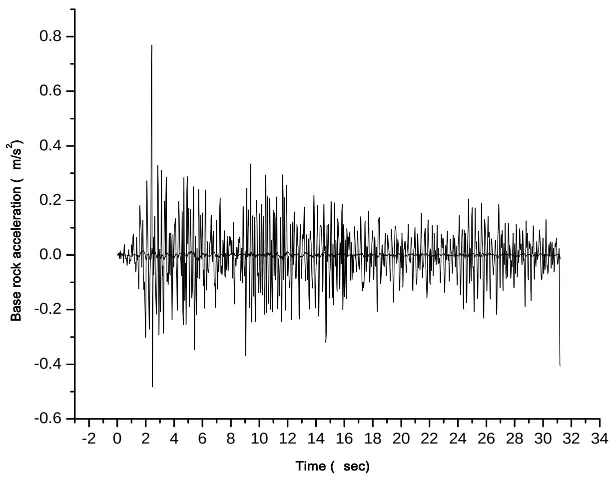

Figure 3: Base rock acceleration with time.

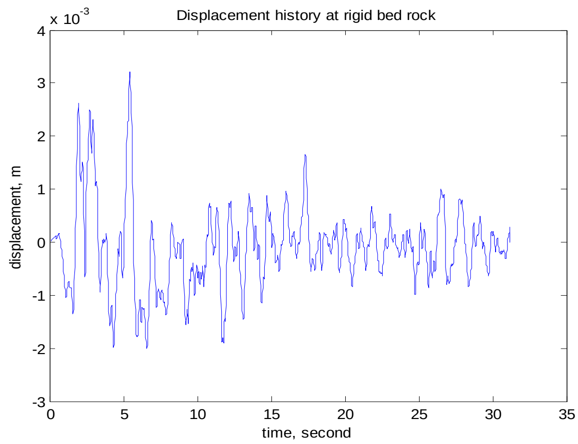

Figure 4: Displacement at rigid bed rock with time.

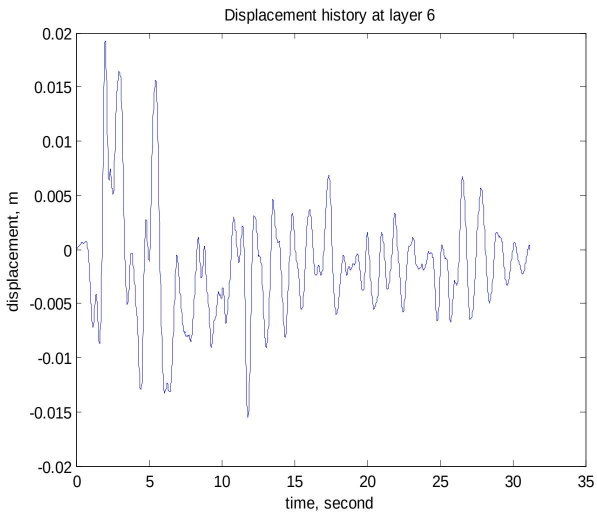

Figure 5: Displacement with time at an intermediate layer.

The output stress-strain history and change of secant modulus with time of different layers of soil column are presented below in graphical form-

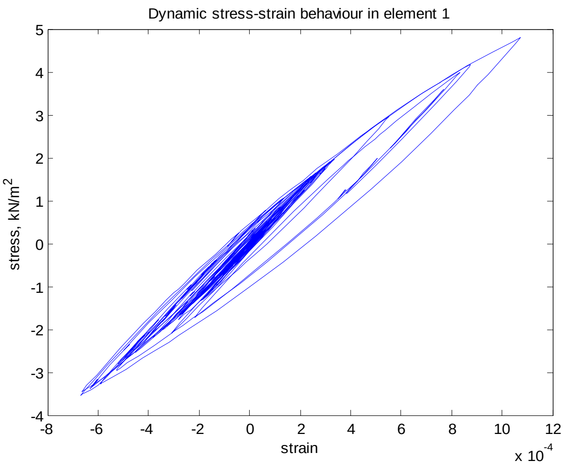

Figure 6: Dynamic stress-strain behaviour in a layer just below the surface layer.

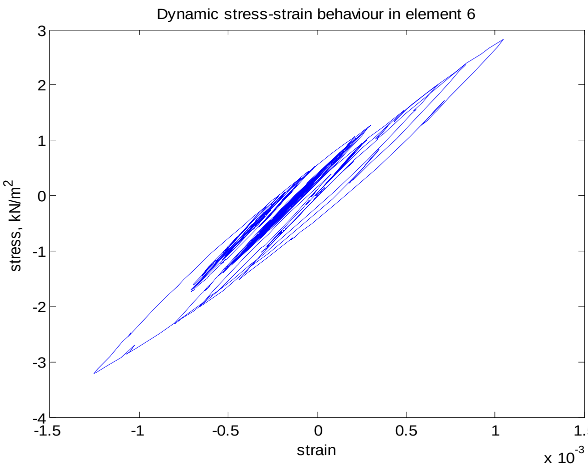

Figure 7: Dynamic stress-strain behaviour in an intermediate layer.

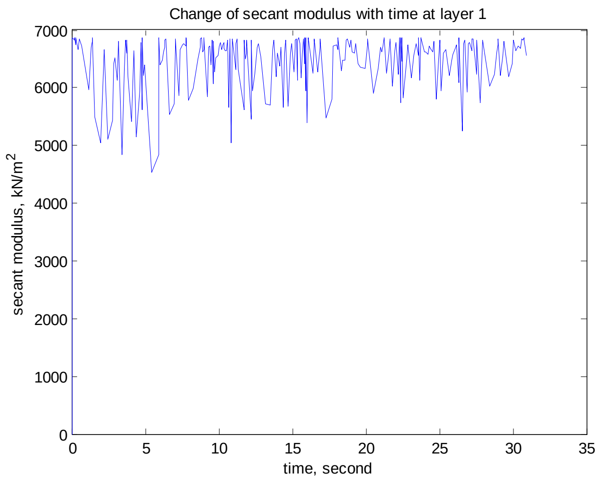

Figure 8: Change of secant modulus with time in a layer just below the surface layer.

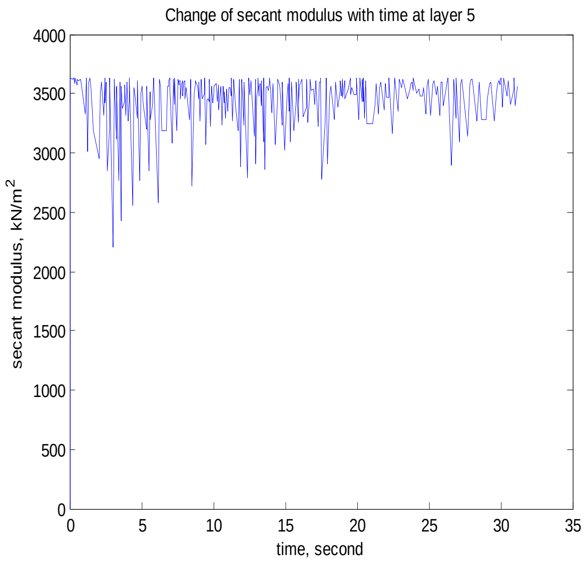

Figure 9: Change of secant modulus with time in an intermediate layer.

VI. Discussions

- From the results we have found that surface acceleration and base rock acceleration varies with time in a very highly irregular manner.
- The soil properties data implies that if we calculate the shear wave velocity then we find that the upper surface layer is soft and the bottom layer is comparatively stiff than the upper layer.
- For the soil condition and the results we have found we can say that the rigid base rock acceleration is higher than the surface acceleration. The displacement is higher at the surface and lower at the rigid base rock.
- The dynamic stress-strain maintains a hysteresis loop.
- The variation of secant modulus shows high frequency fluctuation. High value of secant

modulus represents smaller loops, where smaller value of secant modulus represents smaller loops.

- At each reversal point of the stress-strain history, the corresponding secant shear modulus was determined as the slope of the line joining the present and previous reversal points.
- Due to the high frequency reversal of loading, the variations of the secant modulus show high frequency fluctuations. It is evident that the high values of the secant shear modulus are mainly due to the smaller loops or rapid reversals of the stress-strain history; whereas the smaller values correspond to the bigger loops. Hence the variations of the minima of the history of the change of secant shear modulus are more representative of the material property.
- The dynamic stress-strain behaviour was plotted for each element of the soil column. We have found that for each element of the soil layer it maintains a hysteresis loop. The hysteresis loop represents energy dissipation characteristics which are non-linear. Because of the non-linearity the hysteresis loop does not have rounded corners.

VII. CONCLUSION

In most of the case, we can measure surface acceleration by the instruments which are set on the ground. From the program, we can easily find different soil properties from rigid base rock to surface layer due to an earthquake. The output is very useful in earthquake resistant structure design.

VIII. ACKNOWLEDGEMENTS

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REFERENCES

Journal Papers:

[1] Raheem, S.E.A, Hayashikawa, T., Hashimoto, I. (2003), “Effect of soil-foundation-superstructure interaction on seismic response of cable stayed bridges tower with spread footing foundation”, *Journal of Structural Engineering Vol.49A.*

Books:

[2] Chopra, A.K. (2006), “*Dynamics of Structures*” (Prentice-Hall of India, New Delhi-110 001, ISBN-81-203-2139-1).

[3] Clough, R.W and Penzien, J. (1993), “*Dynamics of Structures*” (Second Edition, McGraw-Hill, Inc, ISBN 0-07-113241-4).

[4] Ishihara, K. (1996),“ *Soil Behaviour in Earthquake Geotechnics*” (Clarendon Press, Oxford , ISBN 019 856224).

Theses:

[5] Hassan, Md. Raquibul, *Estimation of base rock vibration characteristics from surface ground motion. BSc* Thesis, Department of Civil Engineering, Bangladesh University of Engineering and Technology, BUET, 2011.

Proceedings Papers:

[6] Lou, L and Zerva, A.(2004), “Effects of spatially variable ground motions on the seismic response of a skewed , multi-span , RC highway bridge, “*Soil Dynamics and Earthquake Engineering”*, August-October, pp. 729-740 , 11th International Conference on Soil Dynamics and Earthquake Engineering (ICSDEE) : Part 1.