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Unknown Input Full Order Observer Construction Using Generalized Matrix Inverse

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ABSTRACT
In this paper a design methodology is proposed to provide a constructive solution to the problem of designing a full order observer for linear time invariant systems subjected to unknown disturbances. Necessary conditions for existence of unknown input observers are stated and solved using generalized matrix inverse. The effect of unknown disturbance present in the system is eliminated from the observer by proper selection of gain parameter. Simulation is carried out and results are discussed to illustrate the proposed procedure.
Keywords: Full Order Observer, Generalized Matrix Inverse, Linear Time Invariant System (LTI), Missile autopilot, Unknown Input Observer (UIO).

I. INTRODUCTION

The inputs which act on control system are classified into two classes- the control inputs which can be manipulated by the designer and the disturbance inputs over which the designer has no control .The conventional observers that assume all the inputs to the system are accessible cannot be used for the estimation of states of an LTI system subjected to both accessible and inaccessible inputs. So a new branch of observer, the unknown input observer (UIO) was developed for the reconstruction of states of such systems. In 1969 Basile first introduced the concept of observability of system in presence of unknown inputs [1]. Fuyu yang and Richard W. Wilde [2] proposed a construction method of unknown input full order observer using the classical Luenberger observer structure assuming that no prior information is available about the unknown disturbances. Darouach **et** *al* [3] extended Yang and Wild results and reduced the design procedure of full order observer with unknown inputs to a standard one where inputs are known. Two different approaches for construction of unknown input full order observer discussed in [2],[3] had been developed using straightforward matrix operations in[4] and these procedures are simple and direct compared to previous approaches**.** Stefen Hui and Stanislaw H.Zak [5] constructed full order and reduced order unknown input observer using projection operator approach. In [6] it was shown that the design problem of full order observer for linear systems with unknown inputs can be reduced to a simplified form where the unknown input vector does not interfere in the observer equations. In [7] G. Das and T.K. Ghoshal proposed a construction method of reduced order observer using generalized matrix inverse. In [8] it was shown that generalized matrix inverse is not harder than matrix multiplication. The observer design method using generalized matrix inverse [7] has prominent advantages over Luenberger observer. A comparison between the reduced order Luenberger observer and the Das & Ghoshal observer [7] is given in [9]. In [10] Das & Ghoshal observer is extended and used for the construction of unknown input reduced order observer. In [11] a detailed comparative study between two construction methods of UIO using projection operator approach and generalized matrix inverse is done and is shown that the later method is simpler and more effective. In [12] Das & Ghoshal observer had been extended to full-order observer using the principle of generalized matrix inverse. But if there is any unknown inputs or disturbances present, then the estimated states of full order observer may not converge to the system states. In this paper, it will be established how the full order observer [12] can be modified to incorporate disturbances and can estimate the system states without any knowledge of the unknown input.

Notations: R denotes the field of real numbers; $m{\times}n$ will be used to represent the dimension of a matrix with m rows and n columns; A^g denotes the Moore-Penrose generalized inverse of the matrix A; A^T indicates the transpose of the matrix A and I denotes the identity matrix of appropriate dimension.

II. MATHEMATICAL PRELIMINARIES

If $A \in R^{m\times n}$ is a matrix and a matrix $A^g \in R^{n\times m}$ exists that satisfies the four conditions below,

$$AA^g = (AA^g)^T \tag{1}$$
$$A^g A = (A^g A)^T \tag{2}$$
$$AA^g A = A \tag{3}$$
$$A^g AA^g = A^g \tag{4}$$

Then the matrix A^g is called the Moore-Penrose generalized matrix inverse of A and is unique for each A . If a system of linear equation is given by,

$$Ax = y \tag{5}$$

Where $A \in R^{m\times n}$ is a known matrix, $y \in R^{m\times 1}$ is a known vector and $x \in R^{n\times 1}$ is an unknown vector.
Then eqn. (5) is consistent if and only if,

$$AA^g y = y \tag{6}$$

If eqn. (5) is consistent then general solution of eqn. (5) is given by,

$$x = A^g y + (I - A^g A)v \tag{7}$$

([13] Graybill 1969 p.104).
Where $v \in R^{n\times 1}$ denotes an arbitrary vector having elements as arbitrary functions of time.

III. PROBLEM FORMULATION

Consider an LTI system described by

$$\dot{x} = Ax + Bu + E_d w, x_0 = x(0) \tag{8}$$

$$y = Cx \tag{9}$$

Where $x \in R^{n\times 1}$ is the unknown state vector; $u \in R^{n\times 1}$ is known input vector; $w \in R^{1\times 1}$ is unknown input vector and the corresponding coefficient matrix $E_d \in R^{n\times 1}$; $y \in R^{m\times 1}$ denotes the output vector. The matrixes A, B, C are known and have appropriate dimensions. We assume that the pair {A, C} is completely observable which implies the simultaneous solution for eqn. (8) and eqn. (9) for x is unique when x_0, u and y are given. Our objective is to design a full order unknown input observer for the system described by eqn. (8) and (9).

Construction of full-order Unknown Input Observer:

The full order observer in presence of unknown input 'w' can be derived in similar way as given in [7] and is governed by the following equations.
The general solution of eqn. (9) can be expressed as discussed in Das & Ghoshal [7] ,

$$x = C^g y + (I - C^g C)h \tag{10}$$

Where $h \in R^{n\times 1}$ denotes vector whose elements are arbitrary functions of time.

Now from eqn. (8) & eqn. (9) the general solution of $\dot{h}$ is

$$\dot{h} = (I - C^g C)A(I - C^g C)h + (I - C^g C)Bu + (I - C^g C)E_d w + (I - C^g C)AC^g y + C^g Cp \tag{11}$$

And the consistency condition to exist such solution has been simplified as,

$$\dot{y} = CA(I - C^g C)h + CBu + CE_d w + CAC^g y \tag{12}$$

$p \in R^{n\times 1}$ is a vector whose elements are arbitrary functions of time. For simplicity it can be taken as null vector as in [12].
Now combining eqn. (11) & eqn. (12) we get,

$$\dot{\hat{h}} = \{(I - C^g C)A(I - C^g C) - KCA(I - C^g C)\}\hat{h} + \{(I - C^g C)B - KCB\}u + C^g Cp + K\dot{y} + \{(I - C^g C)E_d - KCE_d\}w + \{(I - C^g C)AC^g - KCAC^g\}y \quad 13)$$

If $\hat{h}$ will converge to h then $\hat{x}$ will converge to x.

Observer dynamic equation can be written after introducing a change of variable to eliminate $\dot{y}$

$$\hat{h} = \hat{q} + Ky \tag{14}$$

$$\dot{\hat{q}} = \{(I - C^g C)A(I - C^g C) - KCA(I - C^g C)\}\hat{q} + \{(I - C^g C)AC^g - KCAC^g + (I - C^g C)A(I - C^g C)K - KCA(I - C^g C)K\}y + \{(I - C^g C)B - KCB\}u + \{(I - C^g C)E_d - KCE_d\}w + C^g Cp \tag{15}$$

Observed state vector can be represented as,

$$\hat{x} = (I - C^g C)\hat{q} + (C^g + (I - C^g C)K)y \tag{16}$$

Where $K \in R^{n\times m}$ is an arbitrary matrix, denotes the observer gain.

Condition for Unknown Input Observer:

The unknown input 'w' is present in the observer dynamics. To obtain the state information of the system we must solve the dynamic equation (15) and the solution is possible if the input vector 'w' is known. But the vector 'w' represents the unknown input vector .To eliminate the effect of the unknown input 'w' from the observer dynamics, observer gain K can be designed such that

$$(I - C^g C)E_d - KCE_d = 0 \tag{17}$$

The eqn. (17) is consistent and can be solved for K, if and only if

$$(I - C^g C)E_d - (I - C^g C)E_d(CE_d)^g(CE_d) = 0 \tag{18}$$

The eqn. (18) is the consistency condition of eqn. (17) and also the condition for existence of the UIO. The general solution for K can be expressed as

$$K = (I - C^g C)E_d(CE_d)^g + H(I - (CE_d)(CE_d)^g) \tag{19}$$

Where H is any arbitrary matrix and its dimension is same as dimension of K. Putting the value of K in eqn. (15) we get the observer matrix $(A_1 - HC_1)$
Where,

$$A_1 = (I - C^g C)A(I - C^g C) - (I - C^g C)E_d(CE_d)^g CA(I - C^g C) \tag{20}$$

$$C_1 = (I - CE_d(CE_d)^g)CA(I - C^g C) \tag{21}$$

If pair $\{A_1, C_1\}$ is observable then poles can be placed arbitrarily otherwise the system should be at least detectable. By applying this gain formulation technique, the observer will be able to estimate the system states effectively irrespective of any disturbance present or not. Hence the observer dynamic (eqn. (15)) becomes,

$$\dot{\hat{q}} = \{(I - C^g C)A(I - C^g C) - KCA(I - C^g C)\}\hat{q} + \{(I - C^g C)AC^g - KCAC^g + (I - C^g C)A(I - C^g C)K - KCA(I - C^g C)K\}y + \{(I - C^g C)B - KCB\}u + C^g Cp \tag{22}$$

IV. NUMERICAL EXAMPLE

Here we have considered the state space model of flight path rate demand autopilot in pitch plane as described in [13]. A, B, C matrixes of the state space model are shown below, A=

$$\begin{bmatrix} -\frac{1}{T_a} & \frac{1+\sigma^2\omega_b^2}{T_a} & -\frac{K_b\sigma^2\omega_b^2}{T_a} & -K_b\sigma^2\omega_b^2 \\ -\frac{1+\omega_b^2T_a^2}{T_a(1+\sigma^2\omega_b^2)} & \frac{1}{T_a} & \frac{(T_a^2-\sigma^2)K_b\omega_b^2}{T_a(1+\sigma^2\omega_b^2)} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_a^2 & -2\zeta_a\omega_a \end{bmatrix}$$

$$B=\begin{bmatrix} 0 \\ 0 \\ 0 \\ K_q\omega_a^2 \end{bmatrix}$$

$$C=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

State variables of the above state space model are as follows,

$x_1 = \dot{\gamma}$ (Flight path rate demand)
$x_2 = q$ (Body rate in pitch)
$x_3 = \eta$ (Elevator deflection)
$x_4 = \dot{\eta}$ (Rate of change of elevator deflection)

The following numerical data have been taken for the class of missile considered here.

T_a=2.85 sec; $\boldsymbol{\sigma}^2$ =0.00142 sec^2 ; $\boldsymbol{\omega}_b$ =5.6 rad/sec; ζ_a =0.6; K_b = -0.1437 per sec; v = 3000 m/sec; K_p = 28.99; K_q= -1.40; $\boldsymbol{\omega}_a$ =180 rad/sec;

V. DISCUSSION

Original states with and without unknown inputs and the corresponding estimated states of the missile autopilot system under consideration are plotted using MATLAB and shown in fig 1-4 where the red lines indicate the system states with unknown input(x_i),while the black dotted lines indicate the estimated states (x_i hat).Black lines represent system states without unknown input (x_i')

System's initial condition is taken as

x_0 = [1; 0.25; 1.89; 50];

The unknown input (w) has been taken as w=50*(exp(-2*t))*sin(100*t) and the corresponding coefficient matrix is chosen as

E_d = [1; 0; 1; 0] ;

H is any arbitrary matrix of proper dimension.
We have considered

H= [-10 2 ; 3 4; -7 8; -1 2];

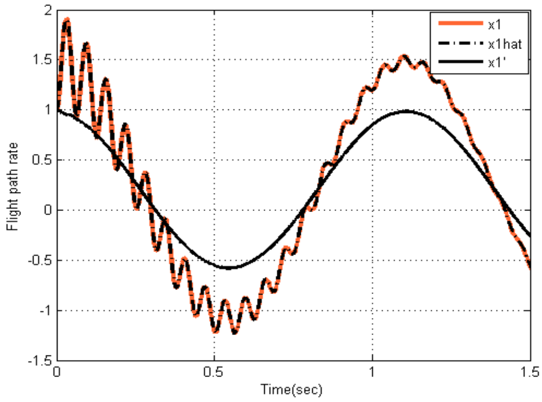

Fig.1: Flight path rate

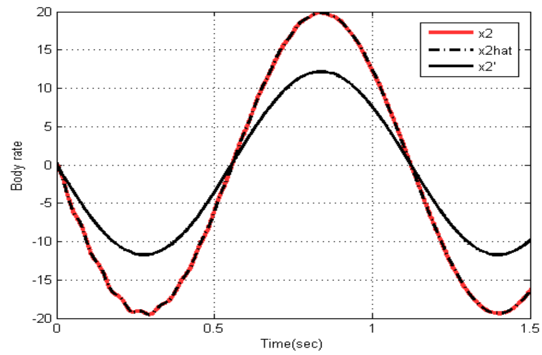

Fig.2: Body rate

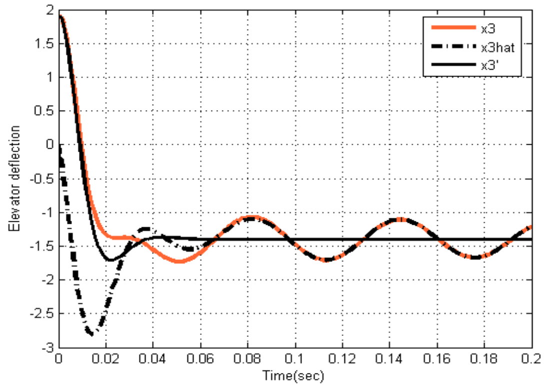

Fig.3: Elevator deflection

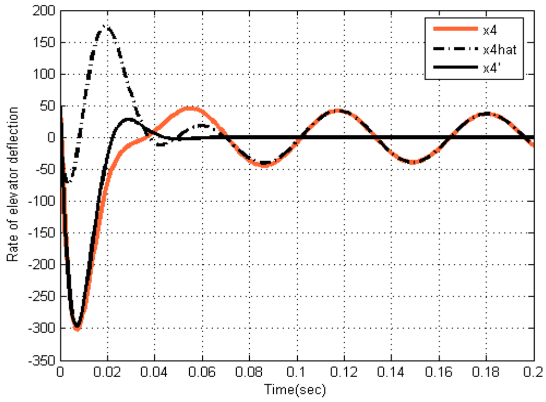

Fig.4: Rate of Elevator deflection

It has been clearly found from the simulation results that the observed states are converging to the system states within very short time span .It can successfully estimate the states of the system in presence of unknown input.

VI. CONCLUSION

In this paper an unknown input full order observer has been directly constructed using generalized matrix inverse. The existence condition for such unknown input observer is presented. The construction method does not presuppose the observer structure. This method also has no restriction on the output distribution matrix and requires no coordinate transformation of the output matrix. Simulations results are presented to justify the proposed method.

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