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Fuzzy *C* -Semi-Boundary in IFT Spaces

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Abstract

In this paper we introduce a new concept, Intuitionistic fuzzy ℭ Semi -boundary using the arbitrary complement function namely ℭ: [0,1]→ [0,1] which is introduced by klir[10]. Further we investigate some of the properties of this boundary.

Keywords and phrases: Intuitionistic fuzzy set, Intuitionistic fuzzy topological spaces, Fuzzy boundary, Fuzzy semi-pre boundary.

Mathematics Subject Classification: 54A40

I. Introduction

Athar and Ahmad[2] defined the notion of fuzzy boundary in FTS and studied the properties of fuzzy semi boundary. K.Bageerathi and T.Thangavelu were introduced fuzzy ℭ-semi closed sets in fuzzy topological spaces, where ℭ : [0,1] →[0,1] is an arbitrary complement function which satisfies the monotonic and involutive condition.

After the introduction of fuzzy sets by Zadeh [13], there has been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets was introduced by Atanassov [1] Using this idea Coker[9] introduced the concept of intuitionistic fuzzy topological spaces. For the past few years many concepts in fuzzy topology were extended to IFTS.

In this paper, we are extending the concept of intuitionistic fuzzy semi boundary by using the complement function ℭ instead of the usual complement function. Further we discuss about the basic properties and some of the characterizations of this intuitionistic fuzzy ℭ-semi boundary.

II. Preliminaries

Throughout this paper (X, τ) denotes a intuitionistic topological space in which no separation axioms are assumed unless otherwise mentioned.

Definition 2.1[1]

Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS, for short), A is an object having the form

A = {<x, $\mu_A(x), \gamma_A(x)$> : x∈ X}

Where the mapping $\mu_A : X \to I$ and $\gamma_A : X \to I$ denotes respectively the degree of membership (namely $\mu_A(x)$) and the non-membership (namely$\gamma_A(x)$) of each element x ∈X to a set A, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each x∈X.

A = {<x, $\mu_A(x), \gamma_A(x)$> : x ∈ X

Definition 2.2[1]

Let X be a non-empty set and let the IFS's A and B in the form

A = A = {<x, $\mu_A(x), \gamma_A(x)$> : x ∈ X} ; B = A = {<x, $\mu_B(x), \gamma_B(x)$> : x ∈ X}.

Let $\{A_j : j \in J\}$ be an arbitray family of IFS's in (X, τ). Then ,

(i) $A \leq B$ if and only if $\forall x \in X$, $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$

(ii) $\bar{A} = \{< x,\ \gamma_A(x),\ \mu_A(x) >: x \in X\}$

(iii) $\cap\ A_j = \{< x,\ \wedge \mu_{A_j}(x) ,\vee\ \gamma_{A_j}(x) >: x \in X\}$

(iv) $\cup\ A_j = \{< x,\ \vee \mu_{A_j}(x)\ ,\ \wedge\ \gamma_{A_j}(x) >: x \in X\} >\}$

(v) $\tilde{1} = \{< x, 1, 0 >: x \in X\}$ and $\tilde{0} = \{< x, 0, 1 >: x \in X\}$

(vi) $\bar{\bar{A}} = A$, $\bar{\tilde{1}} = \tilde{0}$ and $\bar{\tilde{0}} = \tilde{1}$.

Definition 2.3[9]

Let X and Y be two non-empty sets and $f : (X, \tau) \to (X, \sigma)$ be a mapping. If $B =$ {< y, $\mu_B(y), \gamma_B(y)$ > : y ∈ Y} is an IFS in Y, then the pre-image of B under f is denoted and defined by $f^{-1}(B) = \{< x, f^{-1}(\mu_B(x)), f^{-1}(\gamma_B(x)) >: x \in$ X } since μ_B, γ_B are fuzzy sets, we explain that $f^{-1}(\ \mu_B)(x) =\ \mu_B(f(x))$

Definition 2.4[9]

An instuitionistic fuzzy topology (IFT, for short) on a non-empty set X is a family τ of IFS's in X satisfying the following axioms:

(i) $\tilde{1}, \tilde{0} \in \tau$

(ii) $A_1 \cap A_2 \in \tau$ for some $A_1, A_2 \in \tau$

(iii) $\cup\ A_j \in \tau$ for any $\{A_j : j \in J\} \in \tau$

In this case, the ordered pair (X, τ) is called intuitionistic fuzzy topological space(IFTS, for short) and each IFS in τ is known as intuitionistic fuzzy open set(IFOS, for short) in X. The complement of an intuitionistic fuzzy open set is called intuitionistic fuzzy closed set(IFCS, for short).

Definition 2.5[9]

Let (X, τ) be an IFTS and let A = {<x, $\mu_A(x), \gamma_A(x)$> : x ∈ X} be oa IFS in X. Then intuitionistic fuzzy interior and intuitionistic fuzzy closure of A are defined by

$$int\ A = \cup \{G : G\ is\ an\ IFOS\ in\ X\ and\ G \subseteq A\}$$

$cl\ A\ = \cap \{K : K\ is\ an\ IFCS\ in\ X\ and\ A\ \subseteq K\}$

Remark 2.6
For any IFS A in (X, τ), we have $cl(\bar{A}) = \overline{int\ A}$ and $int\ (\bar{A}) = \overline{cl\ A}$.

Definition 2.7[5]
Let $\mathfrak{C} : [0,1] \to [0,1]$ be a complement function. If A is a fuzzy subset of (X, τ) then the complement $\mathfrak{C}A$ of a fuzzy subset A is defined by $\mathfrak{C}A(x) = \mathfrak{C}(A(x))$ for all $x \in X$. A complement function $\mathfrak{C}$ is said to satisfy

(i) The boundary condition if $\mathfrak{C}(0) = 1$ and $\mathfrak{C}(1) = 0$,
(ii) Monotonic condition if $x \le y \Rightarrow \mathfrak{C}(x) \ge \mathfrak{C}(y)$ for all x, y $\in$[0,1]
(iii) Involutive condition if $\mathfrak{C}(\mathfrak{C}(x)) = x$ for all x $\in$ [0,1].

Lemma 2.8[5]
Let $\mathfrak{C}: [0,1] \to [0,1]$ be a complement function that satisfies the monotonic and involutive condition. Then for any family $\{\lambda_\alpha : \alpha \in \Delta\}$ of fuzzy subsets of X, we have

(i) $\mathfrak{C}(\sup\{\lambda_\alpha(x) : \alpha \in \Delta\}) = \inf\{\mathfrak{C}(\lambda_\alpha(x)) : \alpha \in \Delta\} = \inf\{\mathfrak{C}\lambda_\alpha x : \alpha \in \Delta\}$ and
(ii) $\mathfrak{C}(\inf\{\lambda_\alpha(x) : \alpha \in \Delta\}) = \sup\{\mathfrak{C}(\lambda_\alpha(x)) : \alpha \in \Delta\} = \sup\{\mathfrak{C}\lambda_\alpha x : \alpha \in \Delta\}$ for x $\in$ X.

Definition 2.9[5]
A fuzzy subset A of X is fuzzy $\mathfrak{C}$-closed in (X, τ) if $\mathfrak{C}$A is fuzzy open in (X, τ). The fuzzy $\mathfrak{C}$-closure of A is defined as the intersction of all fuzzy $\mathfrak{C}$-closed sets B containing A. The fuzzy $\mathfrak{C}$-closure of A is denoted by $cl_{\mathfrak{C}}(A)$ that is equal to $\wedge\{B : B \ge A, \mathfrak{C}B \in \tau\}$.

Lemma 2.10[5]
If the complement function $\mathfrak{C}$satisfies the monotonic and involutive conditions, then for any fuzzy subset A of X, $\mathfrak{C}(int\ A) = cl_{\mathfrak{C}}(\mathfrak{C}A)$ and $\mathfrak{C}(cl_{\mathfrak{C}}A) = int\ (\mathfrak{C}A)$.

Lemma 2.11[5]
Let (X, τ) be a fuzzy topological space. Let $\mathfrak{C}$ be a complement function that satisfies that satisfies the boundary, monotonic and involutive conditions. Then for any family $\{A_\alpha : \alpha \in \Delta\}$ of fuzzy subsets of X. We have $\mathfrak{C}(\vee\{A_\alpha : \alpha \in \Delta\}) = \wedge\{\mathfrak{C}A_\alpha : \alpha \in \Delta\}$ and $\mathfrak{C}(\wedge\{A_\alpha : \alpha \in \Delta\}) = \vee\{\mathfrak{C}A_\alpha : \alpha \in \Delta\}$.

Definition 2.12[5]
Let (X, τ) be a fuzzy topological space and $\mathfrak{C}$ be a complement function. Then for a fuzzy subset A of X, the fuzzy $\mathfrak{C}$-semi interior of A(briefly ${}_s int_{\mathfrak{C}}\ A$), is the union of all fuzzy $\mathfrak{C}$-semi open sets of X contained in A.
That is ${}_s int_{\mathfrak{C}}\ A = \vee\{A : B \le A$, B is fuzzy $\mathfrak{C}$-semi open$\}$.

Definition 2.13[5]
Let (X, τ) be a fuzzy topological space and $\mathfrak{C}$ be a complement function. Then for a fuzzy subset A of X, the fuzzy $\mathfrak{C}$-semi closure of A(briefly ${}_s cl_{\mathfrak{C}}\ A$), is the union of all fuzzy $\mathfrak{C}$-semi open sets of X contained in A.
That is ${}_s cl_{\mathfrak{C}}\ A = \vee\{B : B \ge A$, B is fuzzy $\mathfrak{C}$-semi closed$\}$.

Lemma 2.14[2]
Let (X, τ) and (Y, σ) be $\mathfrak{C}$-product related fuzzy topological spaces. Then A is called fuzzy $\mathfrak{C}$-semi open if there exists a $\mathfrak{C}$-open set B such that $B \le A \le cl_{\mathfrak{C}} B$.

Lemma 2.15[4]
If A_1, A_2, A_3, A_4 are the fuzzy subsets of X then $(A_1 \wedge A_2) X (A_3 \wedge A_4) = (A_1 X A_4) \wedge (A_2 X A_3)$.

Lemma 2.16[4]
Suppose f is a function from X to Y. Then $f^{-1}(\mathfrak{C}B) = \mathfrak{C}(f^{-1}(B))$ for any fuzzy subset B of Y.

Definition 2.17[12]
If A is a fuzzy subset of X and B is a fuzzy subset of Y, then AxB is a fuzzy subset of XxY defined by (AxB)(x,y) = min {A(x), B(y)} for each (x,y) $\in$ XxY.

Lemma 2.18[4]
Let $f : X \to Y$ be a function. If $\{A_\alpha\}$ a family of fuzzy subsets of Y, then

(i) $f^{-1}(\vee A_\alpha) = \vee f^{-1}(A_\alpha)$
(ii) $f^{-1}(\wedge A_\alpha) = \wedge f^{-1}(A_\alpha)$

Lemma 2.19[4]
If A is an IFS of X and B is a IFS of Y, then $\mathfrak{C}$(A x B) = $\mathfrak{C}$A x 1 $\vee$ 1 x $\mathfrak{C}$B.

III. Intuitionistic fuzzy $\mathfrak{C}$-semi boundary

In this section intuitionistic fuzzy $\mathfrak{C}$-semi closure and interior were introduced. Using these two concepts we develop the idea of intuitionistic fuzzy $\mathfrak{C}$-semi boundary.

Definition 3.1
Let (X, τ) be a IFTS. Let A be a IFS and let $\mathfrak{C}$ be the complement function. Then intuitionistic fuzzy $\mathfrak{C}$-semi boundary is defined as
${}_{is}Bd_{\mathfrak{C}}(A) = {}_{is}cl_{\mathfrak{C}}A \wedge {}_{is}cl_{\mathfrak{C}}(\mathfrak{C}A)$, where semi closure and semi interior were defined as
${}_{is}int_{\mathfrak{C}}\ A = \vee\{A : B \le A$, B is intuitionistic fuzzy $\mathfrak{C}$-semi open$\}$.
${}_{is}cl_{\mathfrak{C}}\ A = \vee\{B : B \ge A$, B is intuitionistic fuzzy $\mathfrak{C}$-semi closed$\}$.

Proposition 3.2

Let (X, τ) be a IFTS and $\mathfrak{C}$ b ${}_{is}Bd_{\mathfrak{C}}(A)$ e a complement function that satisfies the monotonic and involutive condition. Then for any IFS A of X, ${}_{is}Bd_{\mathfrak{C}}(A)= {}_{is}Bd_{\mathfrak{C}}(\mathfrak{C}A)$.

Proof

By the definition 3.1, ${}_{is}Bd_{\mathfrak{C}}(A) = {}_{is}cl_{\mathfrak{C}}A \wedge {}_{is}cl_{\mathfrak{C}}(\mathfrak{C}A)$. Since $\mathfrak{C}$ satisfies the involutive condition $\mathfrak{C}(\mathfrak{C}A) = A$, that implies ${}_{is}Bd_{\mathfrak{C}}(A) = {}_{is}cl_{\mathfrak{C}}(\mathfrak{C}A) \wedge {}_{is}cl_{\mathfrak{C}}\mathfrak{C}(\mathfrak{C}A)$. That is ${}_{is}Bd_{\mathfrak{C}}(A)= {}_{is}Bd_{\mathfrak{C}}(\mathfrak{C}A)$.

Proposition 3.3

Let (X, τ) be a IFTS and $\mathfrak{C}$ be a complement function that satisfies the monotonic and involutive conditions. If A is intuitionistic fuzzy $\mathfrak{C}$-semi closed, then ${}_{is}Bd_{\mathfrak{C}}(A) \leq A$.

Proof

Let A be intuitionistic fuzzy $\mathfrak{C}$-semi closed, and $\mathfrak{C}$ satisfies the involutive and monotonic conditions we have ${}_{is}Bd_{\mathfrak{C}}(A) \leq {}_{is}cl_{\mathfrak{C}} A = A$.

Proposition 3.4

Let (X, τ) be a IFTS and $\mathfrak{C}$ be a complement function that satisfies the monotonic and involutive conditions. If A is intuitionistic fuzzy $\mathfrak{C}$-semi open, then ${}_{is}Bd_{\mathfrak{C}}(A) \leq \mathfrak{C}A$.

Proof

Let A be intuitionistic fuzzy $\mathfrak{C}$-semi open. Since $\mathfrak{C}$ satisfies the involutive condition, we get, ${}_{is}Bd_{\mathfrak{C}}(A) \leq \mathfrak{C}A$.

Proposition 3.5

Let (X, τ) be a IFTS. Let $\mathfrak{C}$ be a complement function satisfies the monotonic and involutive conditions. Then for any IFS A of X, we have $\mathfrak{C}\, {}_{is}Bd_{\mathfrak{C}}(A)= {}_{is}int_{\mathfrak{C}}(A) \vee {}_{is}int_{\mathfrak{C}}(\mathfrak{C}A)$.

Proof

Using Definition 3.1, ${}_{is}Bd_{\mathfrak{C}}(A) = {}_{is}cl_{\mathfrak{C}}A \wedge {}_{is}cl_{\mathfrak{C}}(\mathfrak{C}A)$. Taking complement on both sides, we get $\mathfrak{C}\,[{}_{is}Bd_{\mathfrak{C}}(A)] = \mathfrak{C}\,[{}_{is}cl_{\mathfrak{C}}A \wedge {}_{is}cl_{\mathfrak{C}}(\mathfrak{C}A)]$. Since $\mathfrak{C}$ the monotonic and involutive conditions, $\mathfrak{C}\,[{}_{is}Bd_{\mathfrak{C}}(A)] = \mathfrak{C}[{}_{is}cl_{\mathfrak{C}}A] \vee \mathfrak{C}[{}_{is}cl_{\mathfrak{C}}(\mathfrak{C}A)]$. Using proposition 5.6 in [6], $\mathfrak{C}\, {}_{is}Bd_{\mathfrak{C}}(A)= {}_{is}int_{\mathfrak{C}}(A) \vee {}_{is}int_{\mathfrak{C}}(\mathfrak{C}A)$.

Proposition 3.6

Let (X, τ) be a IFTS. Let $\mathfrak{C}$ be a complement function that satisfies the monotonic and involutive conditions. Then for any IFS A of X, we have ${}_{is}Bd_{\mathfrak{C}}(A) = {}_{is}cl_{\mathfrak{C}}A \wedge {}_{is}int_{\mathfrak{C}}(A)$.

Proof

By the Definition 3.1, we have ${}_{is}Bd_{\mathfrak{C}}(A) = {}_{is}cl_{\mathfrak{C}}A \wedge {}_{is}cl_{\mathfrak{C}}(\mathfrak{C}A)$. Since $\mathfrak{C}$ satisfies the monotonic and involutive conditions, we have ${}_{is}Bd_{\mathfrak{C}}(A) = {}_{is}cl_{\mathfrak{C}}A \wedge {}_{is}int_{\mathfrak{C}}(A)$.

Proposition 3.7

Let (X, τ) be a IFTS. Let $\mathfrak{C}$ be a complement function that satisfies the monotonic and involutive conditions. Then for any IFS A of X, ${}_{is}Bd_{\mathfrak{C}}[{}_{\beta}int_{\mathfrak{C}}(A)] \leq {}_{is}Bd_{\mathfrak{C}}(A)$.

Proof

Since $\mathfrak{C}$ satisfies the monotonic and involutive conditions, by using proposition 3.6, we have ${}_{is}Bd_{\mathfrak{C}}[{}_{is}int_{\mathfrak{C}}(A)]= {}_{is}cl_{\mathfrak{C}}(A)[{}_{is}int_{\mathfrak{C}}(A)] \wedge {}_{is}int_{\mathfrak{C}}[{}_{is}int_{\mathfrak{C}}(A)]$. Since ${}_{is}int_{\mathfrak{C}}(A)$ is intuitionistic fuzzy $\mathfrak{C}$-semi- open, ${}_{is}Bd_{\mathfrak{C}}[{}_{is}int_{\mathfrak{C}}(A)]= {}_{is}cl_{\mathfrak{C}}(A)[{}_{is}int_{\mathfrak{C}}(A)] \wedge \mathfrak{C}\,[{}_{is}int_{\mathfrak{C}}(A)]$. Since ${}_{is}int_{\mathfrak{C}}(A) \leq A$, by using proposition 5.6(ii) in [6], and proposition 5.5 in [6],
${}_{is}Bd_{\mathfrak{C}}[{}_{is}int_{\mathfrak{C}}(A)] \leq {}_{is}cl_{\mathfrak{C}}(A) \wedge {}_{is}cl_{\mathfrak{C}}[\mathfrak{C}A]$. By using the definition 3.1, we have ${}_{is}Bd_{\mathfrak{C}}[{}_{is}int_{\mathfrak{C}}(A)] \leq {}_{is}Bd_{\mathfrak{C}}(A)$.

Proposition 3.8

Let (X, τ) be a IFTS. Let $\mathfrak{C}$ be a complement function that satisfies the monotonic and involutive conditions. Then for any IFS A of X, ${}_{is}Bd_{\mathfrak{C}}[{}_{is}cl_{\mathfrak{C}}(A)] \leq {}_{is}Bd_{\mathfrak{C}}(A)$.

Proof

Since $\mathfrak{C}$ be a complement function that satisfies the monotonic and involutive conditions, by using proposition 3.11, ${}_{is}Bd_{\mathfrak{C}}[{}_{is}cl_{\mathfrak{C}}(A)]= {}_{is}cl_{\mathfrak{C}}(A)[{}_{is}cl_{\mathfrak{C}}(A)] \wedge {}_{is}int_{\mathfrak{C}}[{}_{is}cl_{\mathfrak{C}}(A)]$. By using proposition 5.6(iii) in [6], we have ${}_{is}cl_{\mathfrak{C}}[{}_{is}cl_{\mathfrak{C}}(A)] = {}_{is}cl_{\mathfrak{C}}(A)$, that implies
${}_{is}Bd_{\mathfrak{C}}[{}_{is}cl_{\mathfrak{C}}(A)] = {}_{is}cl_{\mathfrak{C}}A \wedge {}_{is}int_{\mathfrak{C}}[{}_{is}cl_{\mathfrak{C}}(A)]$. Since $A \leq [{}_{is}cl_{\mathfrak{C}}(A)]$, that implies ${}_{is}int_{\mathfrak{C}}(A) \leq {}_{is}int_{\mathfrak{C}}[{}_{is}cl_{\mathfrak{C}}(A)]$. Therefore ${}_{is}Bd_{\mathfrak{C}}[{}_{is}cl_{\mathfrak{C}}(A)] \leq {}_{is}cl_{\mathfrak{C}}A \wedge {}_{is}int_{\mathfrak{C}}(A)$. By using proposition 5.5(ii) in [6], and using the definition 3.1, we get ${}_{is}Bd_{\mathfrak{C}}[{}_{is}cl_{\mathfrak{C}}(A)] \leq {}_{is}Bd_{\mathfrak{C}}(A)$.

Theorem 3.9

Let (X, τ) be a IFTS. Let $\mathfrak{C}$ be a complement function that satisfies the monotonic and involutive conditions. Then for any IFS A and B of X,

$${}_{is}Bd_{\mathfrak{C}}[A \vee B] \leq {}_{is}Bd_{\mathfrak{C}}(A) \vee {}_{is}Bd_{\mathfrak{C}}(B).$$

Proof

From the definition 3.1, ${}_{is}Bd_{\mathfrak{C}}[A \vee B]= {}_{is}cl_{\mathfrak{C}}[A \vee B] \wedge {}_{is}cl_{\mathfrak{C}}(\mathfrak{C}[A \vee B])$. Since $\mathfrak{C}$ satisfies the monotonic and involutive conditions, by the proposition 5.7(i) in [6], that implies ${}_{is}Bd_{\mathfrak{C}}[A \vee B]= \{{}_{is}cl_{\mathfrak{C}}[A] \vee [{}_{is}cl_{\mathfrak{C}}(B)]\} \wedge {}_{is}cl_{\mathfrak{C}}(\mathfrak{C}[A \vee B])$. By using , proposition 5.7 (ii) in [6], ${}_{is}Bd_{\mathfrak{C}}[A \vee B] \leq \{{}_{is}cl_{\mathfrak{C}}[A] \vee [{}_{is}cl_{\mathfrak{C}}(B)]\} \wedge \{{}_{is}cl_{\mathfrak{C}}(\mathfrak{C}A) \wedge {}_{is}cl_{\mathfrak{C}}(\mathfrak{C}B)\}$. That is ${}_{is}Bd_{\mathfrak{C}}[A \vee B] \leq \{{}_{is}cl_{\mathfrak{C}}[A] \wedge [{}_{is}cl_{\mathfrak{C}}(\mathfrak{C}A)]\} \vee \{{}_{is}cl_{\mathfrak{C}}(A) \wedge {}_{is}cl_{\mathfrak{C}}(\mathfrak{C}B)\}$. Again by using definition 3.1, ${}_{is}Bd_{\mathfrak{C}}[A \vee B] \leq {}_{is}Bd_{\mathfrak{C}}(A) \vee {}_{is}Bd_{\mathfrak{C}}(B)$.

Theorem 3.10

Let (X, τ) be a IFTS. Let $\mathfrak{C}$ be a complement function that satisfies the monotonic and involutive conditions. Then for any IFS A and B of X,

${}_{is}Bd_{\mathfrak{C}}[A \wedge B] \leq \{{}_{is}Bd_{\mathfrak{C}}[A] \wedge [{}_{is}cl_{\mathfrak{C}}(B)]\} \vee \{{}_{is}Bd_{\mathfrak{C}}(B) \wedge {}_{is}cl_{\mathfrak{C}}(A)\}$.

Proof

By 3.1 ${}_{is}Bd_{\mathfrak{C}}[A \wedge B] = {}_{is}cl_{\mathfrak{C}}[A \wedge B] \wedge {}_{is}cl_{\mathfrak{C}}(\mathfrak{C}[A \wedge B])$. Since $\mathfrak{C}$ satisfies the monotonic and involutive conditions, by using propositions 5.7(i), in [6] and by using lemma 2.11, we get ${}_{is}Bd_{\mathfrak{C}}[A \wedge B] \leq \{{}_{is}cl_{\mathfrak{C}}[A] \wedge [{}_{is}cl_{\mathfrak{C}}(B)]\} \wedge \{{}_{is}cl_{\mathfrak{C}}(\mathfrak{C}A) \vee {}_{is}cl_{\mathfrak{C}}(\mathfrak{C}B)\}$ is equal to ${}_{is}Bd_{\mathfrak{C}}[A \wedge B] \leq \{\{{}_{is}cl_{\mathfrak{C}}[A] \wedge [{}_{is}cl_{\mathfrak{C}}(\mathfrak{C}A)]\} \wedge [{}_{is}cl_{\mathfrak{C}} A]\} \vee \{[{}_{is}cl_{\mathfrak{C}}(B) \wedge {}_{is}cl_{\mathfrak{C}}(\mathfrak{C}B)] \wedge [{}_{is}cl_{\mathfrak{C}} B]\}$. Again by definition 3.1, we get

${}_{is}Bd_{\mathfrak{C}}[A \wedge B] \leq \{{}_{is}Bd_{\mathfrak{C}}[A] \wedge [{}_{is}cl_{\mathfrak{C}}(B)]\} \vee \{{}_{is}Bd_{\mathfrak{C}}(B) \wedge {}_{is}cl_{\mathfrak{C}}(A)\}$.

Proposition 3.11

Let Let (X, τ) be a IFTS. Let $\mathfrak{C}$ be a complement function that satisfies the monotonic and involutive conditions. Then for any IFS A of X,we have

1. ${}_{is}Bd_{\mathfrak{C}}[{}_{is}Bd_{\mathfrak{C}}(A)] \leq {}_{is}Bd_{\mathfrak{C}}(A)$.
2. $[{}_{is}Bd_{\mathfrak{C}}(A)]\{{}_{is}Bd_{\mathfrak{C}}[{}_{is}Bd_{\mathfrak{C}}(A)]\} \leq {}_{is}Bd_{\mathfrak{C}}(A)$.

Proof

From the definition 3.1, ${}_{is}Bd_{\mathfrak{C}}A = {}_{is}Bd_{\mathfrak{C}}A \wedge {}_{is}Bd_{\mathfrak{C}}\mathfrak{C}B$. We have ${}_{is}Bd_{\mathfrak{C}}\, {}_{is}Bd_{\mathfrak{C}}A = {}_{is}cl_{\mathfrak{C}}[{}_{is}Bd_{\mathfrak{C}}A] \wedge {}_{is}cl_{\mathfrak{C}}[{}_{is}Bd_{\mathfrak{C}}A] \leq {}_{is}Bd_{\mathfrak{C}}[A \vee B]\, {}_{is}cl_{\mathfrak{C}}[{}_{is}Bd_{\mathfrak{C}}A]$. Since $\mathfrak{C}$ satisfies the monotonic and involutive conditions, we have $[{}_{is}cl_{\mathfrak{C}}(A)] = A$, where A is A is intuitionistic fuzzy $\mathfrak{C}$-semi closed. Hence the proof.

Proposition 3.12

Let A be a intuitionistic fuzzy $\mathfrak{C}$-semi closed subset of a IFTS X and B be a intuitionistic fuzzy $\mathfrak{C}$-semi closed subset od a IFTS Y, then A x B is a intuitionistic fuzzy $\mathfrak{C}$-seml closed set of the intuitionistic fuzzy product space X x Y where the complement function $\mathfrak{C}$ satisfies the monotonic and involutive conditions.

Proof

Let A be a intuitionistic fuzzy $\mathfrak{C}$-semi closed subset of a IFTS X. Then $\mathfrak{C}A$ is intuitionistic fuzzy semi open in X. Then $\mathfrak{C}A$ x 1 is intuitionistic fuzzy semi open in X x Y. Similarly let B be a intuitionistic fuzzy $\mathfrak{C}$-semi closed subset od a IFTS Y, then 1 x $\mathfrak{C}B$ is intuitionistic fuzzy semi open in X x Y. Since the arbitrary union of intuitionistic fuzzy $\mathfrak{C}$-semi open sets is intuitionistic fuzzy $\mathfrak{C}$-semi open , $\mathfrak{C}A\, x\, 1 \vee 1\, x\, \mathfrak{C}B$ is intuitionistic fuzzy $\mathfrak{C}$-semi open in X x Y. Then by lemma 2.19 $\mathfrak{C}$ (A x B) = $\mathfrak{C}$ A x 1 ∨ 1 x $\mathfrak{C}$ B, hence $\mathfrak{C}$(A x B) is intuitionistic fuzzy $\mathfrak{C}$-semi open. Therefore A x B is intuitionistic fuzzy $\mathfrak{C}$-semi closed of intuitionistic fuzzy product space X x Y.

Theorem 3.13

Let (X, τ) be a IFTS. Let $\mathfrak{C}$ be a complement function that satisfies the monotonic and involutive conditions. Then for any IFS A and B of X,

(i) ${}_{is}cl_{\mathfrak{C}}(A)$ x ${}_{is}cl_{\mathfrak{C}}(B) \geq {}_{is}cl_{\mathfrak{C}}$(A x B)

(ii) ${}_{is}nt_{\mathfrak{C}}(A)$ x ${}_{is}int_{\mathfrak{C}}(B) \leq {}_{is}int_{\mathfrak{C}}$(A x B)

proof

By the definition 2.17,[${}_{is}cl_{\mathfrak{C}}$(A) x ${}_{is}cl_{\mathfrak{C}}(B)$] (x,y)= min { ${}_{is}cl_{\mathfrak{C}}A$(x), ${}_{is}cl_{\mathfrak{C}}B$(y)}≥ min { A(x), B(y)} = (A x B) (x,y). This shows that ${}_{is}cl_{\mathfrak{C}}(A)$ x ${}_{is}cl_{\mathfrak{C}}(B) \geq$ (A x B).
By using proposition 5.6 in [6], ${}_{is}cl_{\mathfrak{C}}$(A x B) ≤ ${}_{is}cl_{\mathfrak{C}}$ (${}_{is}cl_{\mathfrak{C}}(A)$ x ${}_{is}cl_{\mathfrak{C}}(B)$)= ${}_{is}cl_{\mathfrak{C}}(A)$ x ${}_{is}cl_{\mathfrak{C}}(B)$. By using definition 2.17 ,[${}_{is}int_{\mathfrak{C}}$(A) x ${}_{is}int_{\mathfrak{C}}(B)$] (x,y)= min { ${}_{is}int_{\mathfrak{C}}A$(x), ${}_{is}int_{\mathfrak{C}}B$(y)} ≤ min { A(x), B(y)} = (A x B) (x,y). This shows that

${}_{is}int_{\mathfrak{C}}(A)$ x ${}_{is}int_{\mathfrak{C}}(B) \leq {}_{is}int_{\mathfrak{C}}$(A x B).

Theorem 3.14

Let X and Y be $\mathfrak{C}$-product related IFTSs. Then for a IFS A of X and a IFS B of Y,

(i) ${}_{is}cl_{\mathfrak{C}}$(A x B) = ${}_{is}cl_{\mathfrak{C}}(A)$ x ${}_{is}cl_{\mathfrak{C}}(B)$

(ii) ${}_{is}int_{\mathfrak{C}}$(A x B) = ${}_{is}int_{\mathfrak{C}}(A)$ x ${}_{is}int_{\mathfrak{C}}(B)$.

Proof

By the theorem 3.13, it is sufficient to show that ${}_{is}cl_{\mathfrak{C}}(A)$ x ${}_{is}cl_{\mathfrak{C}}(B) \leq {}_{is}cl_{\mathfrak{C}}$(A x B). By using the definition in [6], we have ${}_{is}cl_{\mathfrak{C}}$(A x B) = inf { $\mathfrak{C}(A_\alpha$ x $B_\beta)$: $\mathfrak{C}$(A x B) ≥ A x B where A_α and B_β are intuitionistic fuzzy $\mathfrak{C}$-semi open }. By using lemma 2.17, we have,

${}_{is}cl_{\mathfrak{C}}$(A x B) = inf { $\mathfrak{C}(A_\alpha)$ x 1 ∨ 1 x $\mathfrak{C}(B_\beta)$: $\mathfrak{C}(A_\alpha)$ x 1 ∨ 1 x $\mathfrak{C}(B_\beta)$ ≥ A x B}

=inf { $\mathfrak{C}(A_\alpha)$ x 1 ∨ 1 x $\mathfrak{C}(B_\beta)$: $\mathfrak{C}(A_\alpha) \geq$ A or $\mathfrak{C}(B_\beta) \geq$B}

= min (inf { $\mathfrak{C}(A_\alpha)$ x 1 ∨ 1 x $\mathfrak{C}(B_\beta)$: $\mathfrak{C}(A_\alpha) \geq$ A }, inf { $\mathfrak{C}(A_\alpha)$ x 1 ∨ 1 x $\mathfrak{C}(B_\beta)$: $\mathfrak{C}(B_\beta) \geq$B}).

Now inf{ $\mathfrak{C}(A_\alpha)$ x 1 ∨ 1 x $\mathfrak{C}(B_\beta)$: $\mathfrak{C}(A_\alpha) \geq$ A} ≥ inf { $\mathfrak{C}(A_\alpha)$ x 1 : $\mathfrak{C}(A_\alpha) \geq$ A}

= inf { $\mathfrak{C}(A_\alpha)$: $\mathfrak{C}(A_\alpha) \geq$ A} x 1

= [${}_{is}cl_{\mathfrak{C}}(A)$] x 1.

Also inf { $\mathfrak{C}(A_\alpha)$ x 1 ∨ 1 x (B_β) : $\mathfrak{C}(B_\beta)$ ≥B}
≥ inf { 1 x $\mathfrak{C}(B_\beta)$: $\mathfrak{C}(B_\beta)$ ≥B}

= 1 x inf { $\mathfrak{C}(B_\beta)$: $\mathfrak{C}(B_\beta)$ ≥B}

= 1 x [${}_{is}cl_{\mathfrak{C}}(B)$].
Thus we conclude the result as ,
${}_{is}cl_{\mathfrak{C}}$(A x B) ≥ min ([${}_{is}cl_{\mathfrak{C}}A$] x 1 , 1 x [${}_{is}cl_{\mathfrak{C}}B$]
= [${}_{is}cl_{\mathfrak{C}}A$]x [${}_{is}cl_{\mathfrak{C}}B$].

Theorem 3.15

Let X_i, i= 1, 2, 3, ….. n be a family of $\mathfrak{C}$-product related IFTSs. If A_i is a IFS of X_i, and the complement function $\mathfrak{C}$ satisfies the monotonic and involutive conditions, then ${}_{is}Bd_{\mathfrak{C}}[\prod_{i=1}^{n} A_i]$={ ${}_{is}Bd_{\mathfrak{C}}(A_1)$ x ${}_{is}cl_{\mathfrak{C}}(A_2)$ x ……… x ${}_{is}cl_{\mathfrak{C}}(A_n)$} ∨ { ${}_{is}cl_{\mathfrak{C}}(A_1)$ x ${}_{is}Bd_{\mathfrak{C}}(A_2)$ x ……… x ${}_{is}cl_{\mathfrak{C}}(A_n)$} ∨ ………. ∨ { ${}_{is}cl_{\mathfrak{C}}(A_1)$ x ${}_{is}cl_{\mathfrak{C}}(A_2)$ x ……… x ${}_{\beta is}Bd_{\mathfrak{C}}(A_n)$}.

Proof

Using lemma 2.14, lemma 2.19, we get the result.

Theorem 3.16

Let $f: X \rightarrow Y$ be a intutionistic fuzzy continuous function. Suppose the complement function $\mathfrak{C}$ satisfies the monotonic and involutive conditions, then ${}_{is}Bd_{\mathfrak{C}}(f^{-1}(B) \leq f^{-1}({}_{is}Bd_{\mathfrak{C}}$[B]), for any IFS B in Y.

Proof

Let f be a IF continuous function and B be a IFS in Y. By using definition 3.1, we have ${}_{is}Bd_{\mathfrak{C}}(f^{-1}(B))$ = ${}_{is}cl_{\mathfrak{C}}(f^{-1}(B))$ ∧ [${}_{is}cl_{\mathfrak{C}}(\mathfrak{C}\,(f^{-1}(B)))$. Then using the lemma's 2.16, 2.18, finally we get ${}_{is}Bd_{\mathfrak{C}}(f^{-1}(B) \leq f^{-1}({}_{is}Bd_{\mathfrak{C}}$[B]). Hence the result.

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