Method for Solving Unbalanced Transportation Problems Using Trapezoidal Fuzzy Numbers

Kadhirvel. K, Balamurugan. K.
Assistant professor in Mathematics, T.K.Govt.Arts Colleghe, Vriddhachalam – 606 001.
Associate professor in mathematics, Govt.Arts College – Thiruvannamalai.

Abstract

The fuzzy set theory has been applied in many fields such as operation research, management science and control theory etc. The fuzzy numbers and fuzzy values are widely used in engineering applications because of their suitability for representing uncertain information. In standard fuzzy arithmetic operations we have some problem in subtraction and multiplication operation. In this paper we investigate fuzzy unbalanced transportation problem with the aid of trapezoidal fuzzy numbers and fuzzy U-V distribution methods is proposed to find the optimal solution in terms of fuzzy numbers. A new relevant numerical example is also included.

Key words: Fuzzy numbers, trapezoidal fuzzy numbers, fuzzy vogels approximation method, fuzzy U-V distribution method, ranking function.

I. Introduction

The unbalanced transportation problem refers to a special class of linear programming problems. In a typical problem a product is to transported from 'm' source to n designation and there capacities are $a_1, a_2, \dots \dots \dots \dots a_m$ and $b_1, b_2, \dots \dots \dots \dots b_n$ respectively. In addition there is a penalty C_{ij} associated with transporting unit of product from source i to destination j. This penalty may be cost or delivery time or safety of delivery etc. A variable x_{ij} represents the unknown quantity to be shipped from some i to destination j.

Iserman (1) introduced algorithm for solving this problem which provides effective solutions. Lai and Hwang (5), Bellman and Zadeh (3) proposed the concept of decision making in fuzzy environment. The Ringuest and Rinks (2) proposed two iterative algorithms for solving linear, multicriteria transportation problem. Similar solution in Bit A.K. (8). In works by S.Chunas and D.Kuchta (10) the approach introduced in work (2) and resource management techniques by P.R.Vittal (12).

In this paper the fuzzy unbalanced transfortation problems using trapezoidal fuzzy numbers are discussed here after, we have to propose the methods of fuzzy U-V distribution methods to be finding out the optimal solution for the total fuzzy transportation minimum cost.

II. Fuzzy concepts

L.A.Zadeh advanced the fuzzy theory in 1965. The theory propose a mathematical technique for dealing with imprecise concepts and problems that have many possible solutions.

The concept of fuzzy mathematical programming on a general level was first proposed by Tanaka et.al (1974) in the frame work of the fuzzy decision of Bellman and Zadeh (3) now, we present some necessary definitions

2.1. Definitions

A real fuzzy number $\tilde{a}$ is a fuzzy subset of the real number R with membership function $\mu_{\tilde{a}}$ satisfying the following conditions.

1. $\mu_{\tilde{a}}$ is continuous from R to the closed interval [0,1]
2. $\mu_{\tilde{a}}$ is strictly increasing and continuous on $[a_1, a_2]$
3. $\mu_{\tilde{a}}$ is strictly decreaing and continuous on $[a_2, a_3]$

Where a_1, a_2 *and* a_3 are real numbers and the fuzzy number, denoted by $\tilde{a} = [a_1, a_2, a_3]$ is called a trapezoidal numbers.

2.2.Definition

The fuzzy number $\tilde{a} = [a_1, a_2, a_3]$ is a trapezoidal numbers, denoted by $[a_1, a_2, a_3]$ its membership function $\mu_{\tilde{a}}$ is given below the figure.

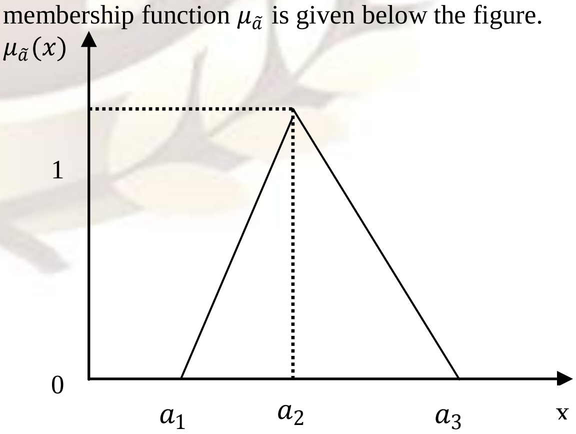


Fig : Membership function of a fuzzy number $\tilde{a}$

1.3. Definition

We define a ranking function R: F(R) ⟶ R, Which maps each fuzzy number into the real line, F(R) represents the set all trapezoidal fuzzy numbers It R be any ranking function , then ,

$$R(\tilde{a}) = (a_1 + a_2 + a_3)/\,3$$

1.4. Arithmetic Operations

Let $(\tilde{a}) == [a_1 a_2 a_3]$ and $(\tilde{b}) = [b_1 b_2 b_3]$ two trapezoidal fuzzy numbers then the arithmetic operations on $(\tilde{a})$ and $(\tilde{b})$ as follows.

Addition: $(\tilde{a}) + (\tilde{b}) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

Subtraction:

$$(\tilde{a}) - (\tilde{b}) = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$$

Multiplication:

$$(\tilde{a}).(\tilde{b}) = \left[\frac{a_1}{3}(b_1 + b_2 + b_3), \frac{a_2}{3}(b_1 + b_2 + b_3), \frac{a_3}{3}(b_1 + b_2 + b_3)\right] \; if \; R(\tilde{a}) > 0$$

$$(\tilde{a}).(\tilde{b}) = \left[\frac{a_3}{3}(b_1 + b_2 + b_3), \frac{a_2}{3}(b_1 + b_2 + b_3), \frac{a_1}{3}(b_1 + b_2 + b_3)\right] \; if \; R(\tilde{a}) < 0$$

III. Fuzzy unbalanced Transportation problem

Consider transportation with m fuzzy origin s (rows) and n fuzzy destinations (Columns) Let $C_{ij} = [\,C_{ij}^{(1)}, C_{ij}^{(2)}, C_{ij}^{(3)}]$ be the cost of transporting one unit of the product from i[th] fuzzy origin to j[th] fuzzy destination $a_i = [a_i^{(1)}, a_i^{(2)}, a_i^{(3)}]$ be the quantity of commodity available at fuzzy origin i $b_j = [b_j^{(1)}, b_j^{(2)}, b_j^{(3)}]$ be the quantity of commodity requirement at fuzzy destination j.

$X_{ij} = [\,X_{ij}^{(1)}, X_{ij}^{(2)}, X_{ij}^{(3)}]$ is quantity transported from i[th] fuzzy origin to j[th] fuzzy destination. The above fuzzy unbalanced transportation problem can be started in the below tabular form.

	FD_1	FD_2	……………………	FD_n	Fuzzy Available a_i
Fo_1	x_{11} C_{11}	x_{12} C_{12}	……………………	x_{1n} C_{1n}	a_1
Fo_2	x_{21} C_{21}	x_{22} C_{22}	……………………	x_{2n} C_{2n}	a_2
	⋮	⋮	……………………	⋮	⋮
fo_m	x_{m1} C_{m1}	x_{m2} C_{m2}		x_{mn} C_{mn}	a_m
Fuzzy Requirement b_j	b_1	b_2		b_n	$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$

Where

$C_{ij} = [\,C_{ij}^{(1)}, C_{ij}^{(2)}, C_{ij}^{(3)}], X_{ij} = [\,X_{ij}^{(1)}, X_{ij}^{(2)}, X_{ij}^{(3)}]$

$a_i = [a_i^{(1)}, a_i^{(2)}, a_i^{(3)}]$ and $b_i = [b_j^{(1)}, b_j^{(2)}, b_j^{(3)}]$

The Linear programming model representing the fuzzy transportation is given by

Minimize,

$$Z = \sum_{i=1}^{m}\sum_{j=1}^{n}[\,C_{ij}^{(1)}, C_{ij}^{(2)}, C_{ij}^{(3)}]\;[\,X_{ij}^{(1)}, X_{ij}^{(2)}, X_{ij}^{(3)}]$$

Subject to the constraints

$$\sum_{j=1}^{n}[\,X_{ij}^{(1)}, X_{ij}^{(2)}, X_{ij}^{(3)}] = [a_i^{(1)}, a_i^{(2)}, a_i^{(3)}]$$

for i= 1,2,…m (Sum Row)

$$\sum_{i=1}^{m}[\,X_{ij}^{(1)}, X_{ij}^{(2)}, X_{ij}^{(3)}] = [b_j^{(1)}, b_j^{(2)}, b_j^{(3)}]$$

for j= 1,2,…n (Column Sum)

$$[\,X_{ij}^{(1)}, X_{ij}^{(2)}, X_{ij}^{(3)}] \geq 0$$

The given fuzzy transportation problem is said to be unbalanced if

$$\sum_{i=1}^{m}[a_i^{(1)},a_i^{(2)},a_i^{(3)}] \neq \sum_{j=1}^{n}[b_j^{(1)},b_j^{(2)},b_j^{(3)}]$$

ie if the total fuzzy available is not equal to the total fuzzy requirement.

3.1.Unbalance Transportation problem to change into balanced transportation problems as follows:

An unbalanced transportation problem is converted into a balanced transportation problem by introducing a dummy origin or a dummy destinations which will provide for the excess availability or the requirement the cost of transporting a unit from this dummy origin (or dummy destination) to any place is taken to be zero. After converting the unbalanced problem into a balanced problem, we adopt the usual procedure for solving a balanced transportation problem.

IV. The computational procedure for fuzzy U-V distribution method

4.1.Fuzzy Vogel's Approximation method

There are numerous method for finding the fuzzy initial basic feasible solution of a transportation problem. In this paper we are to follow fuzzy Vogel's approximation method.

Step 1: Find the fuzzy penalty cost, namely the fuzzy difference between the smallest and next smallest fuzzy costs in each row and column.

Step 2: Among the fuzzy penalties as found in step 1 choose the fuzzy maximum penalty, by ranking method, If this maximum penalty is more than one, choose any one arbitrarily.

Step 3: In the selected row or Column as by step 2, find out the all having the least fuzzy cost. Allocate to this cell as much as possible depending on the fuzzy available and fuzzy requirements.

Step 4: Delete the row or column which is fully exhausted again compute column and row fuzzy penalties for the reduce fuzzy transportation table and then go to step 2, repeat the procedure until all the rim demands are satisfied.

Once the fuzzy initial fuzzy feasible solution can be computed, the next step in the problem is to determine whether the solution obtained is fuzzy optimal or not.

Fuzzy optimality test can be conducted to any fuzzy initial basic feasible solution of a fuzzy transportation provided such allocations has exactly m+n-1 non negative allocations where m is the number of fuzzy origins and n is number of fuzzy distributions. Also these allocations must be independent positions, which fuzzy optimality finding procedure is given below.

4.2. Fuzzy U-V Distribution method

This proposed methods is used for finding the optimal basic feasible solution in fuzzy transportation problem and the following step by step procedure is utilized to find out the same.

Step 1: Find out a set of numbers $[u_i^{(1)},u_i^{(2)},u_i^{(3)}]$ and $[v_j^{(1)},v_j^{(2)},v_j^{(3)}]$ for each row and column satisfying $[u_i^{(1)},u_i^{(2)},u_i^{(3)}] + [v_j^{(1)},v_j^{(2)},v_j^{(3)}] = [C_{ij}^{(1)},C_{ij}^{(2)},C_{ij}^{(3)}]$ for each occupied cell. To start with the assign a fuzzy zero to any row or column having maximum number of allocations, If this maximum number of allocation is more than one. Choose any one arbitrary.

Step 2: For each empty (un occupied) cell, we find fuzzy sum$[u_i^{(1)},u_i^{(2)},u_i^{(3)}]$ and $[v_j^{(1)},v_j^{(2)},v_j^{(3)}]$

Step 3 : Find out for each empty cell the net evaluation Value $[Z_{ij}^{(1)},Z_{ij}^{(2)},Z_{ij}^{(3)}] = [C_{ij}^{(1)},C_{ij}^{(2)},C_{ij}^{(3)}] - \{[u_i^{(1)},u_i^{(2)},u_i^{(3)}] + [v_j^{(1)},v_j^{(2)},v_j^{(3)}]\}$ this step gives optimality conclusion.

Case (i) If all R $[Z_{ij}^{(1)},Z_{ij}^{(2)},Z_{ij}^{(3)}] > 0$, then the solution is fuzzy optimal, and a unique solution exists.

Case (ii) If all R $[Z_{ij}^{(1)},Z_{ij}^{(2)},Z_{ij}^{(3)}] \geq 0$, then the solution is fuzzy optimal, but an alternative solution exists.

Case (iii) If R$[Z_{ij}^{(1)},Z_{ij}^{(2)},Z_{ij}^{(3)}] < 0$ Then the solution is not fuzzy optimal. In this case we go to next step, to improve the total fuzzy transportation minimum cost.

Step 4: Select the empty cell having the most negative value of R$[Z_{ij}^{(1)},Z_{ij}^{(2)},Z_{ij}^{(3)}]$ from this cell we draw a closed path drawing horizontal and vertical lines with Cerner cell occupied. Assign sign + and – alternately and find the fuzzy minimum allocation from the cell having negative sign. This allocation should be added to the allocation having negative sign.

Step 5: the above step yield a better solution by making one (or more) occupied cell as empty and one empty cell as occupied for this new set of fuzzy basic feasible allocation repeat from the step 1, till a fuzzy optimal basic feasible solution is obtained.

5. Example: To solve the following fuzzy unbalanced transportation problem staring with the fuzzy initial fuzzy basic feasible solution obtained by fuzzy Vogel's approximation method.

	FD_1	FD_2	FD_3	FD_4	Fuzzy Available
$F0_1$	(-2,3,8)	(-2,3,8)	(-2,3,8)	(-1,1,3)	(0,2,5)
$F0_2$	(4,9,16)	(4,8,12)	(2,5,8)	(1,4,7)	(1,6,11)
FO_3	(2,7,12)	(0,5,10)	(0,5,10)	(4,8,12)	(1,4,8)
Fuzzy Requirement	(1,4,7)	(0,3,5)	(1,4,7)	(2,4,8)	

$$\sum_{i=1}^{m} a_i = (2,12,24) and \sum_{j=1}^{n} b_j = (4,15,27)$$

$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$$

This problem is unbalanced fuzzy transportation problem then the problem convert to balanced fuzzy transportation problem defined as follows.

	FD_1	FD_2	FD_3	FD_4	Fuzzy Available
$F0_1$	(-2,3,8)	(-2,3,8)	(-2,3,8)	(-1,1,3)	(0,2,5)
$F0_2$	(4,9,16)	(4,8,12)	(2,5,8)	(1,4,7)	(1,6,11)
FO_3	(2,7,12)	(0,5,10)	(0,5,10)	(4,8,12)	(1,4,8)
FO_4	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(2,3,3)
Fuzzy Requirement	(1,4,7)	(0,3,5)	(1,4,7)	(2,4,8)	(4,15,27)

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = (4,15,27)$$

The problem is balanced fuzzy transportation problem then there exists a fuzzy initial basic feasible solution by using fuzzy Vogel's approximation method we get.

	FD_1	FD_2	FD_3	FD_4	Fuzzy Available
$F0_1$	(-1,1,4) (-2,3,8)	(1,1,1) (-2,3,8)	(-2,3,8)	(-1,1,3)	
$F0_2$	(4,9,16)	(4,8,12)	(-1,2,3) (2,5,8)	(2,4,8) (1,4,7)	
FO_3	(2,7,12)	(-1,2,4) (0,5,10)	(2,2,4) (0,5,10)	(4,8,12)	
FO_4	(2,3,3) (0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	
Fuzzy Requirement					

Since the number of occupied cell having m+n-1 = 7 and are also independent, there exists a non – degenerate fuzzy basic feasible solution.

Therefore the initial fuzzy transportation minimum cost is

$$[Z^{(1)}, Z^{(2)}, Z^{(3)}] = (-1,1,4)(-2,3,8) + (1,1,1)(-2,3,8) + (-1,2,3)(2,5,8) +$$

$$(2,4,8)(1,4,7) + (-1,2,4)(0,5,10) + (2,2,4)(0,5,10) + (2,3,3)(0,0,0)$$

Now

(-1,1,4)(-2,3,8) $=[\frac{-1}{3}(-2+3+8), \frac{1}{3}(-2+3+$ *8,43(−2+3+8)]*

$= [\frac{-1}{3}(9), \frac{1}{3}(9), \frac{4}{3}(9)]$

= (-3,3,12)

Using this way we get

$[Z^{(1)}, Z^{(2)}, Z^{(3)}]$ =(-3,3,12)+(3,3,3)+(5,10,15)+(8,16,32)+(5,10,20)+(10,10,20)+(0,0,0)

$[Z^{(1)}, Z^{(2)}, Z^{(3)}]$ = (28,52,102)

To find the optimal solution

Applying the fuzzy U-V distribution method, we determine a set of number $U_i = [u_i^{(1)}, u_i^{(2)}, u_i^{(3)}]$ and $V_j = [v_j^{(1)}, v_j^{(2)}, v_j^{(3)}]$ each row and column such that $[C_{ij}^{(1)}, C_{ij}^{(2)}, C_{ij}^{(3)}] = [u_i^{(1)}, u_i^{(2)}, u_i^{(3)}] + [v_j^{(1)}, v_j^{(2)}, v_j^{(3)}]$ for each occupied cell since first, second and third row has two allocations, we choose any one of this row now we choose the first row, we give fuzzy number $[u_1^{(1)}, u_1^{(2)}, u_1^{(3)}]$ = [0,0,0], the remaining numbers can be obtained as given below

$[C_{11}^{(1)}, C_{11}^{(2)}, C_{11}^{(3)}] = [u_1^{(1)}, u_1^{(2)}, u_1^{(3)}] + [v_1^{(1)}, v_1^{(2)}, v_1^{(3)}]$

$[v_1^{(1)}, v_1^{(2)}, v_1^{(3)}] = (-2,3,8)$

Using this way we set

$[v_2^{(1)}, v_2^{(2)}, v_2^{(3)}] = (-2,3,8)$

$[u_3^{(1)}, u_3^{(2)}, u_3^{(3)}]$ = (2,2,2)

$[v_3^{(1)}, v_3^{(2)}, v_3^{(3)}] = (-2,3,8)$

$[u_2^{(1)}, u_2^{(2)}, u_2^{(3)}]$ = (4,2,0)

$[v_4^{(1)}, v_4^{(2)}, v_4^{(3)}]$ = (-2,3,7)

$[u_4^{(1)}, u_4^{(2)}, u_4^{(3)}]$ = (2,-3,-8)

We find, for each empty cell of the sum $[u_i^{(1)}, u_i^{(2)}, u_i^{(3)}]$ and $[v_j^{(1)}, v_j^{(2)}, v_j^{(3)}]$ Next we find net evolution $[z_{ij}^{(1)}, z_{ij}^{(2)}, z_{ij}^{(3)}]$ is given by

$* [z_{ij}^{(1)}, z_{ij}^{(2)}, z_{ij}^{(3)}] = [C_{ij}^{(1)}, C_{ij}^{(2)}, C_{ij}^{(3)}] - \{[u_i^{(1)}, u_i^{(2)}, u_i^{(3)}] + [v_j^{(1)}, v_j^{(2)}, v_j^{(3)}]\}$

Where

$U_i = [u_i^{(1)}, u_i^{(2)}, u_i^{(3)}]$ $V_j = [v_j^{(1)}, v_j^{(2)}, v_j^{(3)}]$

Now,

$* [z_{13}^{(1)}, z_{13}^{(2)}, z_{13}^{(3)}] = [C_{13}^{(1)}, C_{13}^{(2)}, C_{13}^{(3)}] - \{[u_1^{(1)}, u_1^{(2)}, u_1^{(3)}] + [v_3^{(1)}, v_3^{(2)}, v_3^{(3)}]$

= (-2,3,8) – [(0,0,0) + (-2,3,8)]

= (-2,3,8)-(-2,3,8)

= (0,0,0)

Using this way we get

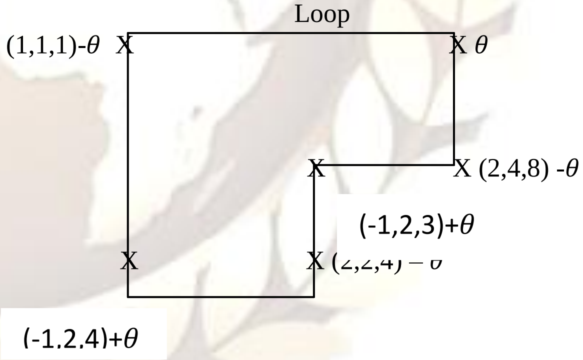
$*[Z_{14}^{(1)}, Z_{14}^{(2)}, Z_{14}^{(3)}]$= (2;,-1,-3) $*[Z_{21}^{(1)}, Z_{22}^{(2)}, Z_{23}^{(3)}]$= (2,4,8) $*[Z_{22}^{(1)}, Z_{22}^{(2)}, Z_{23}^{(3)}]$= (2,3,4) $*[Z3_1^{(1)}, Z_{31}^{(2)}, Z_{31}^{(3)}]$= (2,2,2) $*[Z_{34}^{(1)}, Z_{34}^{(2)}, Z_{34}^{(3)}]$= (3,4,5) $*[Z_{42}^{(1)}, Z_{42}^{(2)}, Z_{42}^{(3)}]$= (0,0,0) $*[Z_{43}^{(1)}, Z_{43}^{(2)}, Z_{43}^{(3)}]$= (0,0,0) $*[Z_{44}^{(1)}, Z_{44}^{(2)}, Z_{44}^{(3)}]$= (1,1,1)

Fuzzy optimal u-v distribution method table

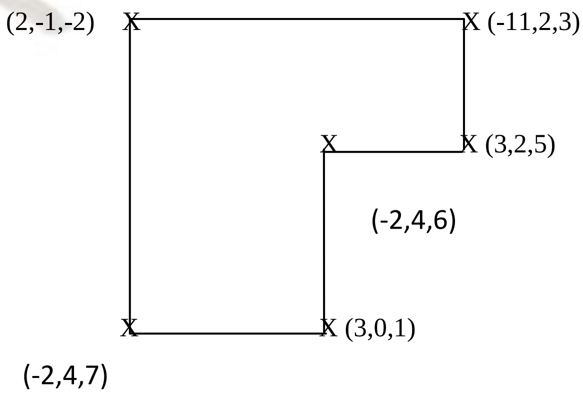
	FD1	FD2	FD3	FD4	Ui
F01	(-1,1,4) (-2,3,8)	(1,1,1) (-2,3,8)	*(0,0,0) (-2,3,8)	*(2,-1,-3) (-1,1,3)	(0.0.0)
F02	*(2,4,8) (4,9,16)	*(2,3,4,) (4,8,12)	(-1,2,3) (2,5,8)	(2,4,8) (1,4,7)	(4,2,0)
F03	(2,2,2) (2,7,12)	(-1,2,4) (0,5,10)	(2,2,4) (0,5,10)	∴∴*(3,4,5) (4,8,12)	(2,2,2)
F04	(2,3,3) (0,0,0)	*(0,0,0) (0,0,0)	*(0,0,0) (0,0,0)	*(1,1,1) (0,0,0)	(2,-3,-8)
Vj	(-2,3,8)	(-2,3,8)	(-2,3,8,)	(-3,2,7)	

Since the value of $*[Z_{14}^{(1)}, Z_{14}^{(2)}, Z_{14}^{(3)}]$<0 then the solution is not fuzzy optimal solution then form a loop as above table.

Loop



Let θ=(-1,2,3) we get



Fuzzy optimal U-V distribution method table

	FD_1	FD_2	FD_3	FD_4	U_i
F01	(-1,1,4) (-2,3,8)	(2,-1,-2) (-2,3,8)	*(0,0,0) (-2,3,8)	(-1,2,3) (-1,1,3)	(0,0,0)
F02	*(2,4,8) (4,9,16)	*(2,3,4,) (4,8,12)	(-2,4,6) (2,5,8)	(3,2,5) (1,4,7)	(4,2,0)
F03	*(2,2,2) (2,7,12)	(-2,4,7) (0,5,10)	(3,0,1) (0,5,10)	*(3,5,7) (4,8,12)	(2,2,2)
F04	(2,3,3) (0,0,0)	*(0,0,0) (0,0,0)	*(0,0,0) (0,0,0)	*(-1,2,3) (0,0,0)	(2,-3,-8)
V_j					

$\therefore$ all $*\left[z_{ij}^{(1)}, z_{ij}^{(2)}, z_{ij}^{(3)}\right] \geq 0$ then the solution is fuzzy optimal but an alternative solution exists ic the one of the solution fuzzy optimal is given below

The fuzzy optimal solution in terms of trapezoidal fuzzy numbers.

$[x_{11}^{(1)}, x_{11}^{(2)}, x_{11}^{(3)}]$ = (-1,1,4); $[x_{12}^{(1)}, x_{12}^{(2)}, x_{12}^{(3)}]$ = (2,-1,-2)

$[x_{13}^{(1)}, x_{13}^{(2)}, x_{13}^{(3)}]$ = (-1,2,3); $[x_{23}^{(1)}, x_{23}^{(2)}, x_{23}^{(3)}]$ = (-2,4,6)

$[x_{24}^{(1)}, x_{24}^{(2)}, x_{24}^{(3)}]$ = (3,2,5); $[x_{32}^{(1)}, x_{32}^{(2)}, x_{32}^{(3)}]$ = (-2,4,7)

$[x_{33}^{(1)}, x_{33}^{(2)}, x_{33}^{(3)}]$ = (3,0,1); $[x_{41}^{(1)}, x_{42}^{(2)}, x_{41}^{(3)}]$ = (2,3,3)

Hence the total fuzzy transportation minimum cost is

$*\left[z_{ij}^{(1)}, z_{ij}^{(2)}, z_{ij}^{(3)}\right]$ = (-1,1,4) (-2,3,8)+(2,-1,-2) (-2,3,8) + (-1,2,3),(-1,1,4) + (-2,4,6) (2,5,8) + (3,2,5) (1,4,7) + (-2,4,7,) (0,5,10) + (3,0,1) (0,5,10) + (2,3,3) (0,0,0)

$\therefore [Z^{(1)}, Z^{(2)}, Z^{(3)}]$ = [-3,50,111]

V. Conclusion

We have thus obtained an optimal solution for a fuzzy unbalanced transportation problem using trapezoidal fuzzy numbers. A new approach called fuzzy U-V computational procedure to find the optimal solution is also discussed. The new arithmetic operation trapezoidal fuzzy numbers are employed to get the fuzzy optimal solution. The same approach of solving the fuzzy problems may also be utilized in future studies of operational research.

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