

## Determination of Spectrum of $\psi$ Mesons with Interaction Potential and Spin and Isospin Effects

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### Abstract

As we know  $j/\psi$  meson consists of a quark C and anti-quark C. Here this meson is considered as a bi particle system and Schrödinger equation is solved by interaction Potential. Then potentials of Spin-Spin, Isospin-Isospin and Spin-Isospin interactions are entered in a perturbation way and the mass (energy) of spectrum of  $j/\psi$  mesons can be determined. The obtained results can be compared with the results of other potentials and real results.

**Keywords:** Quark; perturbation; spin; energy.

### I. Introduction

Hadrons are such particles which have powerful interaction on each other. If we consider protons as an exception, all Hadrons are unstable. Hadrons include Baryons, Fermions which their lighter members are Nucleons and Mesons which are Bosons that their lighter members are Pions. Each meson includes one quark and an anti-quark. The interaction between two quarks or a quark and anti-quark is done by the exchange of Gluons. For the system of high mass quarks, their spin is non-relative and their interaction is described logically by a potential. Each meson has a spectrum that shows a spectrum of states and each one corresponds with different relative spins of quarks and anti-quarks of its constituents. Potential of quark-anti-quark can be considered in different ways. In some practical uses, a harmonic oscillator potential, as it is determined in QCD theory, is used. This model of harmonic oscillator has simple mathematics and most of the time is used. Otherwise the colony is not enough by itself, because allows the free quarks exist. One of the most practical potentials is shown as below:

$$\frac{d^2 \phi}{dx^2} + \frac{2\mu}{\hbar^2} \left[ E - V(x) - \frac{l(l+1)\hbar^2}{2\mu x^2} \right] \phi(x) = 0$$

$$r_{ij} = x_{ij} = r_i - r_j$$

In this potential, there is not only term of colon but also linear term.

Here we consider the potential between quark and anti-quark in  $\psi$  meson as below:

$$\begin{cases} f_v(x) = \prod_1^v (x - \alpha_i^v) & v = 1, 2, 3, \dots \\ f_0(x) = 1 & v = 0 \end{cases}$$

$ax^2 + bx$  potential not only has the role of confinement potential, but also it shows the oscillation of one quark to the other one. In fact, this phrase is a confining potential. In accordance with br term, separating quarks from hadrons is not possible and leads producing pairs of new quarks and anti-quarks. If coefficient b is near to 16 tons; It is mentioned that quark and anti-quark, notwithstanding the space between them, magnet each other with the energy of at least 16 tons. Then maybe understanding the concept of this subject would be easier that nobody can take out a quark from hadrons. To solve Schrödinger equation accurately, we use a binary system as shown below:

$$E_{0l} = \sqrt{\frac{a}{2\mu}} (2l + 3) - \frac{b^2}{4a}$$

Schrödinger equation is represented as regards to motion of one particle to another one.

$\mu = \frac{m_q m_{\bar{q}}}{m_q + m_{\bar{q}}}$  is the decreasing mass of quark and

anti-quark;  $l$  is the number of node and  $l$  is the orbital quantum number of motion of one particle to another one.

By choosing  $\psi_x = \frac{\phi(x)}{x}$ , we have:

$$V = \sum \left( \frac{1}{2} k r_{ij}^2 + \frac{a_s}{r_{ij}} + b r_{ij} \right) + C$$

If  $\hbar = C = 1$  and  $\phi(x) = f(x)e^{g(x)}$ , we can compute  $\phi(x)$  functions. Functions of  $f(x)$  and  $g(x)$  are as below:

$$V(r) = ax^2 + bx - \frac{c}{x}$$

$f(x)$  function is similar to Hermit Polynomial and  $\alpha_i^v$  coefficient is obtained from potential coefficients. And  $g(x)$  function is presented as below:

$$g(x) = -\frac{1}{2}\alpha x^2 + \beta x + \delta \ln x$$

By placing these relations in Schrödinger equation equalizing the coefficients of equal exponents (power), the exponents of relations between passive coefficients can be obtained as below:

$$\alpha = \sqrt{2\mu a}, \beta = -\frac{2\mu b}{2\sqrt{2\mu a}}$$

$$\omega = \sqrt{\frac{2a}{\mu}}$$

Particular amounts of energy for the condition of  $v = 0$  and the size of motion  $l$  can be obtained as below:

$$-\frac{\hbar^2}{2\mu} \frac{1}{x^2} \frac{\partial}{\partial x} \left( x^2 \frac{\partial \psi_{v,l}(x)}{\partial x} \right) + \left( V(x) - E_{v,l} + \frac{l(l+1)\hbar^2}{2\mu x^2} \right) \psi_{v,l}(x) = 0$$

$$\psi_{0l} = N_0 x^{l+1} \exp\left(-\frac{\mu\omega x^2}{2} - \frac{bx}{\omega}\right)$$

Particular functions of higher conditions can be obtained. At this time spin-spin, isospin-isospin and spin-isospin potentials can be considered as below and then  $j/\psi$  meson spectrum can be compared.

$$H_{T_1 T_2}^{(x)} = A_T \left( \frac{1}{\sqrt{\pi\sigma_T}} \right)^3 e^{-\frac{x^2}{\sigma_T^2}} (T_1 \cdot T_2)$$

$$H_{S_1 S_2}^{(x)} = A_S \left( \frac{1}{\sqrt{\pi\sigma_S}} \right)^3 e^{-\frac{x^2}{\sigma_S^2}} (S_1 \cdot S_2)$$

$$H_{S_T}^{(x)} = A_{ST} \left( \frac{1}{\sqrt{\pi\sigma_{ST}}} \right)^3 e^{-\frac{x^2}{\sigma_{ST}^2}} (S_1 \cdot S_2)(T_1 \cdot T_2)$$

In which fixed amounts are represented as below:

$$A_S = 122.75 \text{ fm}, \sigma_S = 0.8 \text{ fm}$$

$$A_T = 51.7 \text{ fm}, \sigma_T = 3.45 \text{ fm}$$

$$A_{ST} = -106.2 \text{ fm}, \sigma_{ST} = 2.31 \text{ fm}$$

$$M_{q\bar{q}} = m_q + m_{\bar{q}} + E_{v,l} + \langle H_{in} \rangle$$

### Conclusion

Spectrum of different states of  $\psi$  mesons can be determined by the mass of quark C,  $m_C = 1784 \text{ MeV}$ , and the mass of  $\psi(3097)$ ,  $\psi(4030)$  as pre-assumption amounts for obtaining a and b parameters. Obtained results are shown in Table 1. These results can be compared with other potentials and experimental results.

Table 1. Mass of  $\psi$  family mesons with parameters of potential.

	$\psi$ family	$v$	$l$	Theory (MeV)	Exp(eV)
1S	$\psi(3097)$	0	0	3097	3097
2S	$\psi(3686)$	0	1	3563	3686
3S	$\psi(4030)$	0	2	4029	4030
4S	$\psi(4415)$	0	3	4496	4415

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