An Improved Variable Step-Size Incremental Lmsadaptive Algorithm over Distributed Networks for Dealing with Changing Noise

Ehsan Mostafapour[1], Afshin Arjhangmehr[2], Alirza Alizadeh[3]
[1](Department of electrical and computer Engineering, Urmia University Urmia-Iran,
[2](Department of nuclear engineering, Tehran University, Tehran-Iran
[3](Department of technical and engineering, Ahar Azad University Ahar, Iran,

Abstract
In this paper we will study the performance of improved variable step-size LMS algorithm over distributed networks when dealing with jump systems and changing noise environments. In jump systems the energy and variance of noise changes suddenly and then gets back to normal afterwards. The conventional VSSLMS algorithm cannot keep pace with these variations, and the performance degrades dramatically. So we propose an improved adaptive variable stepsize LMS algorithm to mitigate these performance deficiencies over distributed networks that face with jumping and changing noise frequently.

***Keywords**: variable step-size, least mean square, jump noise systems, distributed networks, incremental.*

I. Introduction

Sensor networks are emerging as a key technology for a variety of applications [1]. In all of the supposed applications, each node in the network could collect noisy observations related to a certain parameter or phenomenon of interest. It is necessary to exploit spatial dimension alongside the temporal dimension in order to enhance the robustness of the processing tasks [2]. In general, there are two different strategies to process the collected data from the sensors including, I) centralized processing and II) distributed processing. In a centralized solution, the collected noisy observations are sent to a central location for processing known as fusion center. The central processor would then perform the definite process on the data and broadcast the result back to the network. On the other hand, in distributed processing the estimation task is divided between sensors and fusion center.

In some distributed estimation schemes, the fusion center is perfectly removed. In these schemes each sensor interacts with its neighbors in a certain way, as dictated by the network topology, in order to obtain a suitable estimate of the desired parameter [3-7]. In comparison, centralized solution requires a powerful processor in the central node, in addition to extensive amount of communication between the nodes. Distributed adaptive estimation algorithms are proposed to enhance the performance of distributed estimation algorithms over sensor networks [3-7]. In these algorithms, a network of nodes is equipped in order to function as an adaptive entity. In [3, 4] a distributed adaptive algorithm is developed using incremental optimization technique. The resulting algorithm, known as IDLMS, is distributed, cooperative, and able to respond in real time to changes in the environment. In IDLMS algorithm, each node is allowed to communicate with its immediate neighbor in order to exploit the spatial dimension while limiting the communications burden at the same time. When more communication resources are available the diffusion approach is possible. In this approach each node communicates with all its neighbors as dictated by the network topology. The algorithms given in [6] and [7] are based on diffusion implementation. In general, diffusion based algorithms have better performance but more complexity than incremental-based ones [7].

One of the characteristics of the environments that we employ a wireless sensor network is that the conditions of noise will change in time randomly and the network must be compatible to these abrupt changes. The problem of different noise condition for every sensor has been dealt with in [8]. Here we will overcome the problem of changing noise for all the sensors at the same time. For this reason we proposed a distributed variable step size LMS algorithm that is more adaptable to these sudden changes.In order to do this, first we will study the already proposed variable step size algorithms and then we will choose the most efficient one both in the sense of performance and complexity. Next we will apply our improvement to the chosen algorithm and analyze the steady state and convergence conditions. Finally we will run numerous simulations in different noise conditions to show that how effective is our algorithm. The simulation results are found to corroborate the theoretical findings very well.

The remainder of this paper is as follows: in section IIwe willhave an over view of estimation problem in a sensor network and adaptive distributed algorithms to deal with it. In section III we will overview the incremental variable step size LMS

(VSSIDLMS) algorithm and the existing VSS algorithms that are applied to it. Then we will propose our new VSS algorithm and apply it to IDLMS algorithm. In section IV we will have the performance evaluation and experimental results for different scenarios. Finally, section V will be our concluding remarks.

II. Estimation Problem and the Adaptive Distributed Solution

A. Notation

A list of the symbols used through the paper, for ease of reference, is shown in Table I.

TABLE I
LIST OF THE MAIN USED SYMBOLS

symbol	*description*
w_i	*Weight vector estimate at iteration i*
$e(i)$	*Output estimation error at iteration i*
$d(i)$	*Value of scalar variable d at iteration i*
u_i	*Value of vector variable u at iteration i*

B. Problem Statement

Here the goal is to estimate the unknown vector w^ofrommultiple spatially independent but possibly time-correlated measurements collected at N nodes in a network [3] (See Fig. 1).

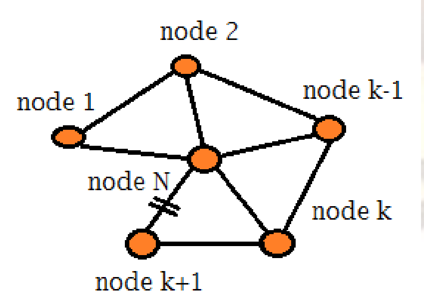

Fig. 1.A distributed network with (N) active nodes accessing space-time data.

Each node k has access to time-realizations $\{d_k(i), u_{k,i}\}$of zero-mean spatial data $\{d_k, u_k\}$where each $d_k$is a scalarmeasurement and each $u_k$is a $1\times M$ row regression vector. Weassume that the unknown vector w^orelates to the $\{d_k(i), u_{k,i}\}$by:

$$d_k(i) = u_{k,i}w^o + v_k(i) \quad (1)$$

Where $v_k(i)$ is some white noise sequence (also known as*observation noise*) with variance$\sigma_{v,k}^2$ and is independent of$\{d_k(i), u_{k,i}\}$. By collecting regression and measurement data into global matrices results:

$$\boldsymbol{U} \triangleq col\{\boldsymbol{u}_1, \boldsymbol{u}_2, \dots, \boldsymbol{u}_N\} \qquad (N \times M) \quad (2)$$

$$\boldsymbol{d} \triangleq col\{\boldsymbol{d}_1, \boldsymbol{d}_2, \dots, \boldsymbol{d}_N\} \qquad (N \times 1) \quad (3)$$

Where the notation $col\{.\}$denotes a column vector (or matrix)with the specified entries stacked on top of each other. The objective is to estimate the vector that solves:

$$J(w) \text{ Where } J(w) = E\|\boldsymbol{d} - \boldsymbol{U}w\|^2 \quad (4)$$

The optimal solution satisfies the normal equations [8]:

$$R_{du} = R_u w^o \quad (5)$$

Where

$$R_u = E\boldsymbol{U}^*\boldsymbol{U}(M \times M), \quad R_{du} = E\boldsymbol{U}^*\boldsymbol{d}(M \times 1) \quad (6)$$

Where the symbol $*$ denotes the Hermitian transform.Note that in order to use (5) to compute w^oeach node must have access to the global statistical information $\{R_u, R_{du}\}$which in turn requires more communications between nodes and computational resources [3, 4]. In another approach we can compute w^ocentrally and send it through all nodes which still require a large amount of communication between nodes and the center. To obtain a distributed algorithm for computing(w^o),we begin by steepest descent method as follows:

C. Incremental Distributed LMS Solution

The standard gradient-descent implementation to solve thenormal equation (5) is as:

$$w_i = w_{i-1} - \mu[\nabla J(w_{i-1})]^*, \quad w_{-1} = \textit{initial condition} \quad (7)$$

Where μ is a suitably chosen step-size parameter, $w_i$is theestimate of optimum weight w^oin ith iteration and $\nabla J(.)$ denotes the gradient vector of $J(w)$evaluated at (w_{i-1}). If $\mu$is sufficiently small then $w_i \rightarrow w^o$as $i \rightarrow \infty$ [3-6]. In order to obtain a distributed version of (8), first the cost function $J(w)$is decomposed as:

$$J(w) = \sum_{k=1}^{N} J_k(w) \quad (8)$$

Where

$$J_k(w) \triangleq E\{|\boldsymbol{d}_k - \boldsymbol{u}_k w|^2\} \quad (9)$$

Using (9) and (10) the standard gradient-descent implementationof (8) can be rewritten as [3-6]:

$$w_i = w_{i-1} - \mu \sum_{k=1}^{N} [\nabla J_k(w_{i-1})]^* \quad (10)$$

By defining the $\psi_k^{(i)}$as the local estimate of the (w^o) at node k and time i, then $w_i$can be evaluated as [3]:

$$\psi_k^{(i)} = \psi_{k-1}^{(i)} - \mu_k[\nabla J_k(w_{i-1})]^*, \quad k = 1, \dots, N \quad (11)$$

This scheme still requires all nodes to share global information (w_{i-1}). The fully distributed solution can be achieved by replacing the local estimate $\psi_k^{(i)}$at each node k with w_{i-1} we have:

$$\psi_k^{(i)} = \psi_{k-1}^{(i)} - \mu_k[\nabla J_k(\psi_{k-1}^{(i)})]^*, \quad k = 1, \dots, \quad (12)$$

Now, we need to determine the gradient of $J$and replaceit in (13). To do this, the following LMS approximations are used:

$$R_{du,k} \approx d_k(i)u_{k,i}^* \quad (13)$$

$$R_{u,k} \approx u_{k,i}^* u_{k,i} \quad (14)$$

The resulting IDLMS algorithm is as follows [5]:

$$\begin{cases} \psi_0^{(i)} = w_{i-1} \\ \psi_k^{(i)} = \psi_{k-1}^{(i)} - \mu_k u_{k,i}^*(d_k(i) - u_{k,i}\psi_{k-1}^{(i)}), \quad k = 1, \dots, N \\ w_i = \psi_n^{(i)} \end{cases} \quad (15)$$

III. Variable Step Size incremental distributed LMS

A. overview

In previous sections we assumed that the step size parameter μ is fixed. Although this assumption leads to a less complex algorithm, much more improvements can be achieved by taking it variable.In [9], a variable stepsize LMS (VSSLMS) algorithm was introducedin which the step-size is adapted at each iteration, using the instantaneous power of the error. Since then various VSS strategies have been proposed. However, the authors of [10] proved that an optimally designed VSSLMS algorithm of [9] outperformed the other VSS variations. Therefore, in this paper we will use the VSS algorithm of [11].

B. variable step size IDLMS algorithm

The variable step size parameter is given as follows by considering the algorithm of [5]:

$$\mu_k(i) = \alpha\mu_k(i-1) + \gamma e_k^2(i) \quad (16)$$

Where α and γ are controlling parameters and $e_k(i)$ is given as follows:

$$e_k(i) = d_k(i) - u_{k,i}\psi_{k-1,i} \quad (17)$$

It can be shown that best convergence results yield for α parameter close to 1 and γ parameter close to zero. The performance of this algorithm degrades as α gets larger, and it improves when γ increases [9]. Using (16), (17) the variable step size IDLMS algorithm can be written by following recursion:

$$\begin{cases} \psi_0^{(i)} = w_{i-1} \\ \psi_k^{(i)} = \psi_{k-1}^{(i)} - \mu_k(i) u_{k,i}^*(d_k(i) - u_{k,i}\psi_{k-1}^{(i)}), \quad k = 1, \dots, N \\ w_i = \psi_n^{(i)} \end{cases} \quad (18)$$

c.Improved variable step size IDLMS algorithm

For more compatibility to sudden changes, as stated in introduction section, an improved method is proposed [9] that replaces $e_k^2(i)$ in $\mu_k(i)$ equation with $e_k(i)e_k(i-1)$ which we will apply to the IDLMS algorithm to achieve another variable step size algorithm. Equation (16) will change as follows:

$$\mu_k(i) = \alpha\mu_k(i-1) + \gamma e_k(i)e_k(i-1) \quad (19)$$

Our improved algorithm is achieved by applying (16) into equation (18) that is less sensitive to sudden variations.Although applying this algorithm to the adaptive sensor network makes it resistant in front of sudden changes, it is important to state it requires a little more communication between nodes, and that is the need for error in recent iteration which is calculated by previous node.

IV. Experimental Results

A. Performance Evaluation

Here our tools for performance evaluation of proposed algorithm are described. Evaluating the performance of an adaptive distributed algorithm over a network means that how close does each $\psi_k^{(i)}$ (local estimate at node) gets to the desired solution w^o as time evolves. We will use Mean Square Deviation (MSD) and Mean Square Error (MSE) to measure this amount. The MSE is given by [1]:

$$MSE \triangleq E|e_k(i)|^2 \quad (20)$$

And the MSD is given by [1]:

$$MSD(n) \triangleq E\left\|w^o - \psi_{k-1}^{(i)}\right\|^2 \quad (21)$$

B. Simulation

To compare the performance of proposed algorithm with the already existing VSS algorithm of [9] over a distributed network we run several simulations for different noise conditions. As we will see in situations the proposed algorithm outperforms the already existing algorithm.

Case 1: The MSD of VSSLMS and proposed algorithm is compared in this section. For now we assume that the noise is zero mean Gaussian and variance of 0.01with slight variation over the time of experiment. The number of iterations is 600 and results are averaged for 50 experiments. The controlling parameters α, γ are taken 0.997 and 2×10^{-4} respectively [9]. The initial value of step size is an important factor for all VSS algorithms because taking a small initial value will lead to a slow convergence rate and by taking it large we will threaten the convergence itself. For this we take the initial value of 0.005 for (μ). The results are given in Fig. 2.

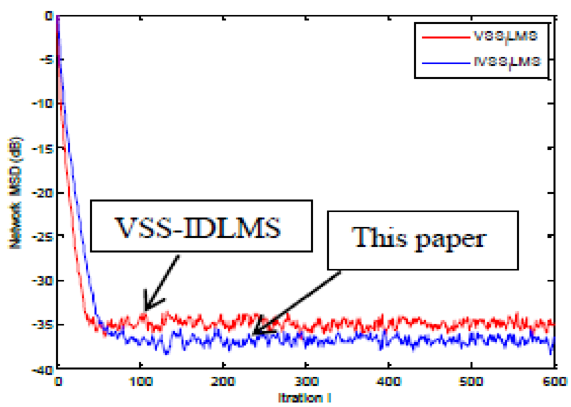

Fig. 2.Comparison between proposed algorithm and already existing VSS algorithm in MSD sense

Case 2: Ability of tracking a jump system

Due to the unsteady property of the network communication environment, it is possible thetime-varying system is disturbed by a random strong signal.Here, a disturbance signal whose intensity is 6 times of theinput signal is applied to the system at 300th sampling time.According to the simulation in Fig. 3, the result of the newalgorithm returns to the steady state after disturbance in afaster rate compared with the original VSS method,which illustrates this method possesses a good ability oftracking a jump system.The results are shown in MSD sense.

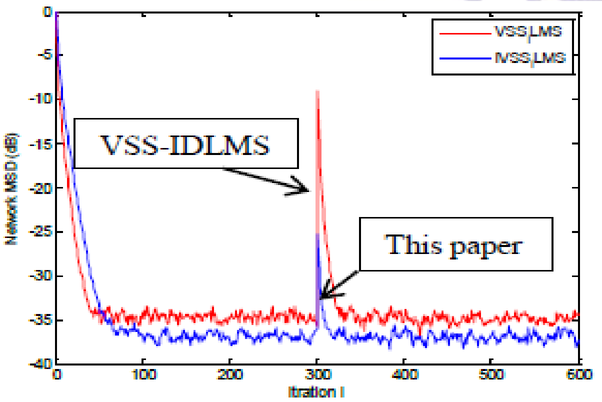

Fig. 3: performance in a jump system

Case 3: Anti-Noise ability

The noise$v_k(i)$ is strengthened 6 times in variance at the sampling point 300. The two simulation results are shown in Fig. 4 and Fig. 5 in MSE and MSD sense correspondingly, where the improved algorithm provides a lower noise level compared with the VSSLMS. It can be concluded that the proposedalgorithm has a better ability in anti-noise than the VSSLMS algorithm. That is to say, the newalgorithm in the paper is more suitable in the low SNR conditions.

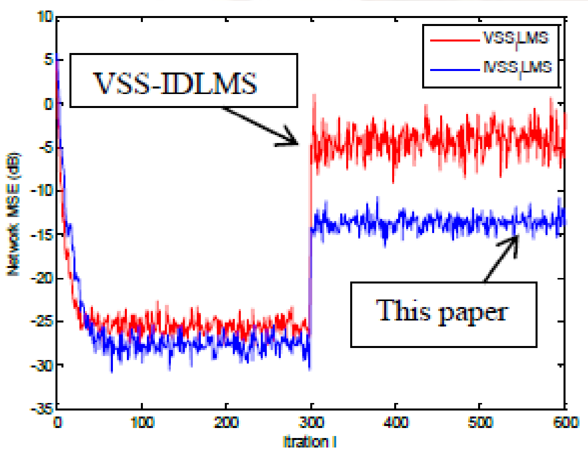

Fig. 4: Anti-Noise performance in MSE sense

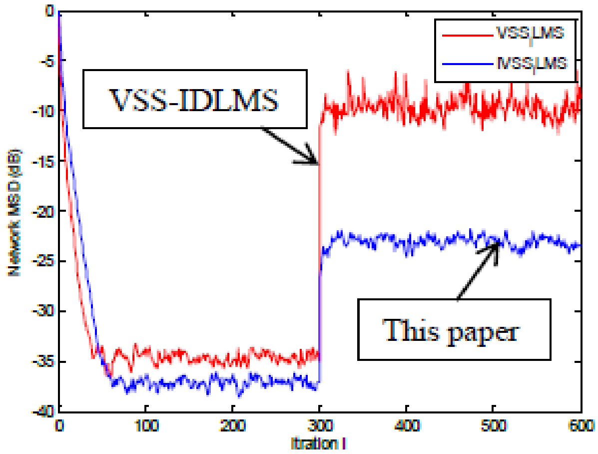

Fig. 5: Anti-Noise performance in the sense of MSD

V. Conclusion

In this paper we studied the problem of estimating a desired value over a sensor network using adaptive distributed incremental LMS algorithm with variable step size. We had a preview about already existing variable step size algorithms and selected the best as the base of our improvement. We stated the deficiencies of this original algorithm when dealing with sudden changes in noise condition and then proposed our new algorithm to overcome this challenge. By running several simulations we showed that our proposed algorithm outperforms the original VSS algorithm both in the sense of MSD and MSE.

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