Exact Value of Pi (Π) = $17 - 8\sqrt{3}$

Mr. Laxman S. Gogawale
Fulora co-operative society,Dhankawadi, Pune-43 (India)

Abstract

In this paper, I show that exact value of pi (π) is $17 - 8\sqrt{3} = 17 - \sqrt{192}$.
My findings are based on geometrical constructions, arithmetic calculation and algebraic formula & proofs.

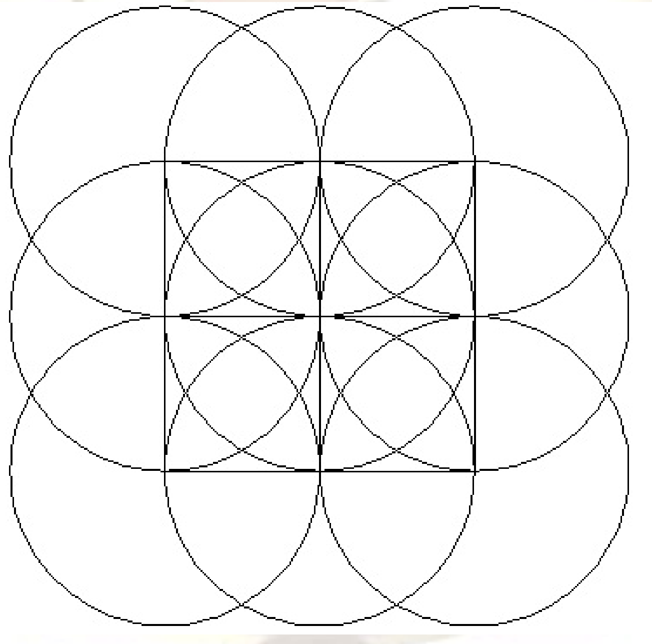
I. Introduction

Increasing the depth of the research published, I additionally found two new methods i.e. "HEXAGON METHOD '' & "DODECAGON METHOD''. Interesting fact is that I obtained the same value of pi from these two new methods.

reference
Complete thesis of my research titled as " Exact value of pi'' is being published in IOSR journal of mathematics in May-June 2012.
One can download it from internet by making Google search as " Exact value of pi ----iosr''.

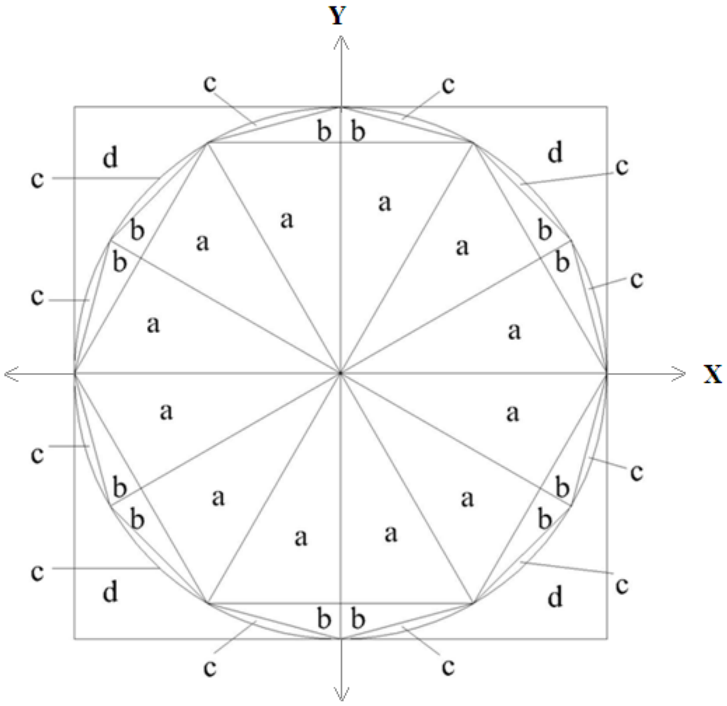
Construction

Basic Figure 1



Modified Figure 1

Draw the square such that circle is totally inside in the square.



Note: - 1) Let a, b, c, d each part shows area.

2) Every Figure in the proof is symmetric about X & Y axis.

From above figure we get ,

12a +12b	= Area of Dodecagon (12 sides Polygon)	------- (1)
12a + 12b+12c	= Area of circle = πr^2	------- (2)
12a + 12b+12c + 4d	= Area of Square = $4r^2$	------- (3)
12c + 4d	= Outside area of the Dodecagon	------- (4)

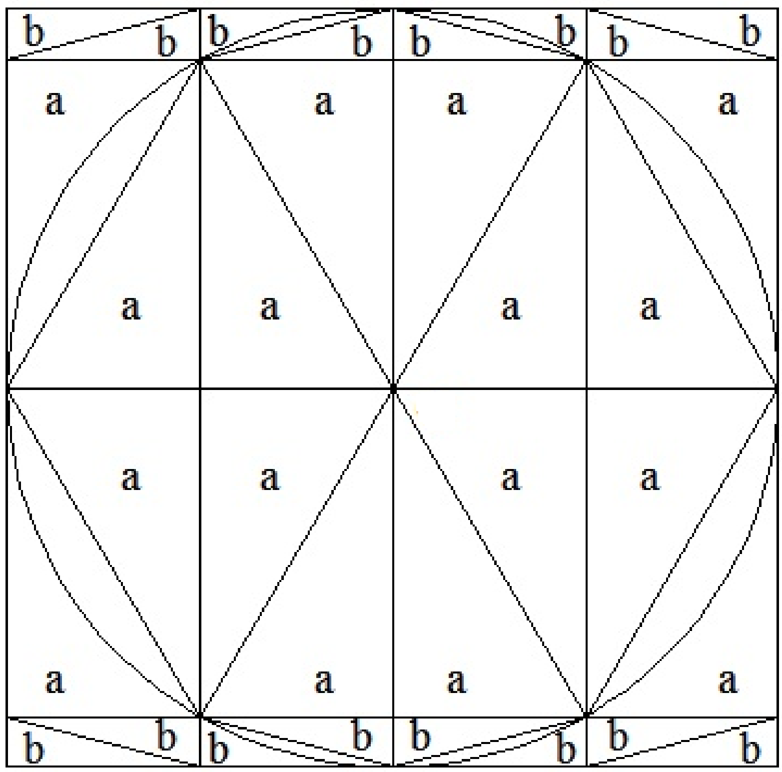


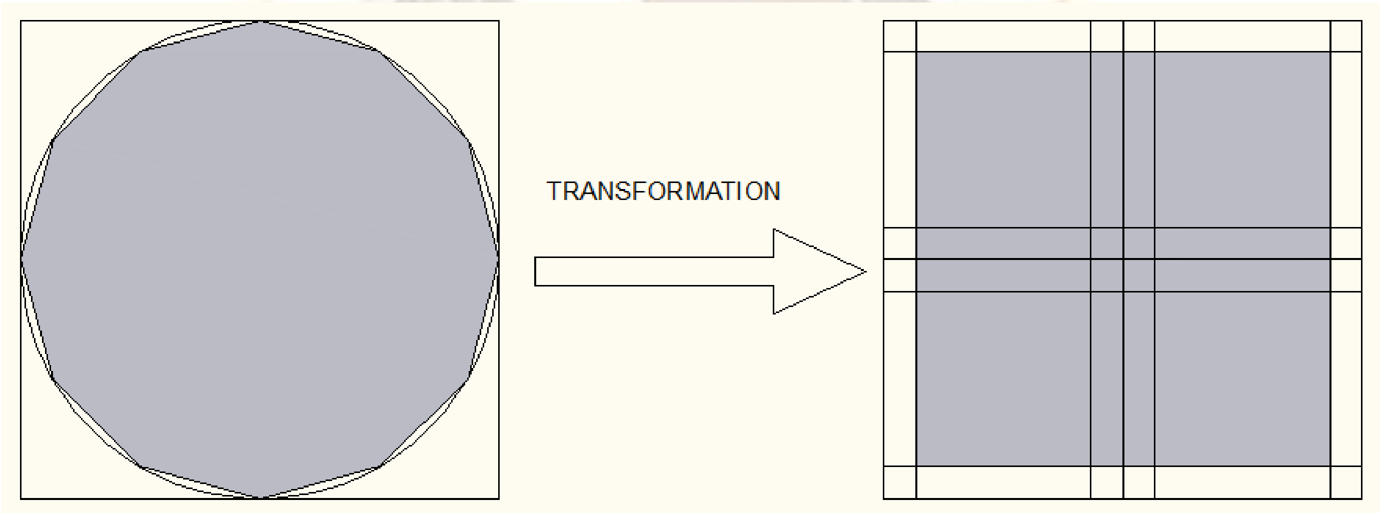
Figure Number – 2

From Figure – 3 ,
$16a + 16b$ = Area of Square = $4r^2$ ---------- (5)
Divide by 4 to above equation we get,
$4a + 4b = r^2$ ---------- (6)
From equation (3) & (5)
$12a + 12b + 12c + 4d = 16a + 16b$
$12a + 12b + 12c + 4d - 16a - 16b = 0$
$-4a - 4b + 12c + 4d = 0$
i.e. $4a + 4b = 12c + 4d$
But by using equation (6)
$4a + 4b = 12c + 4d = r^2$ ---------- (7)
Divided by 4 to above equation we get,

$a + b = 3c + d = \frac{r^2}{4}$ ---------- (8)

Area of Dodecagon (12 sides Polygon) = Inside square area = 12 a +12 b -------(9)

Symmetric aera of part d inside & outside the circle



Area of Dodecagon (12 sides Polygon) Inside square area

Figure Number – 3

Measure part is to find the area of 12c .

Area of Circle Transformation

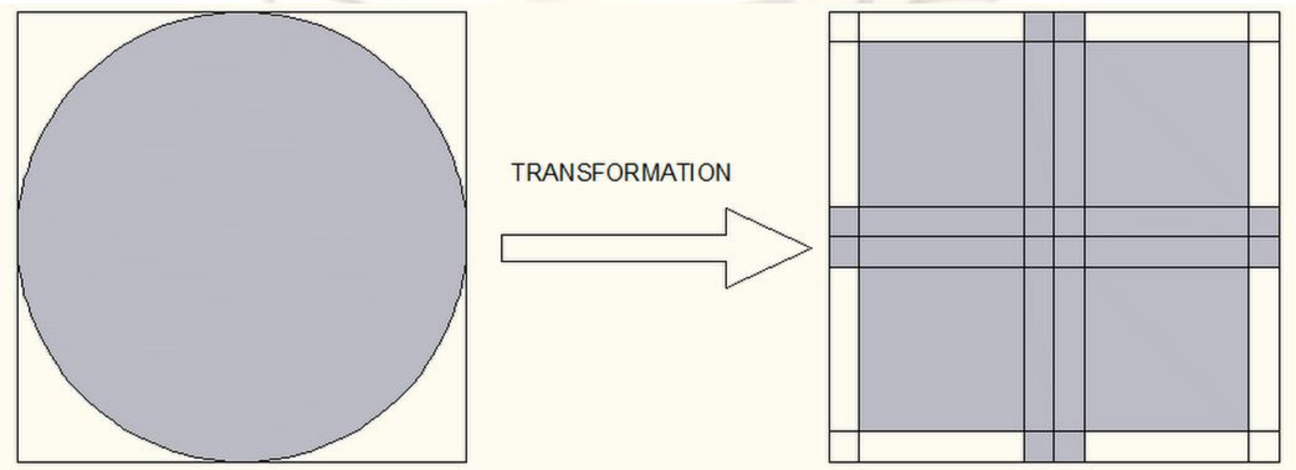


Figure Number – 4

Important Formulae :-
1) Area of Square = 12a +12b +12c +4d = 16a + 16b
2) Area of Circle = 12a +12b +12c = 4a + 68b
3) Area of part 3c = 14b – 2a
4) Area of part d = 3a - 13b

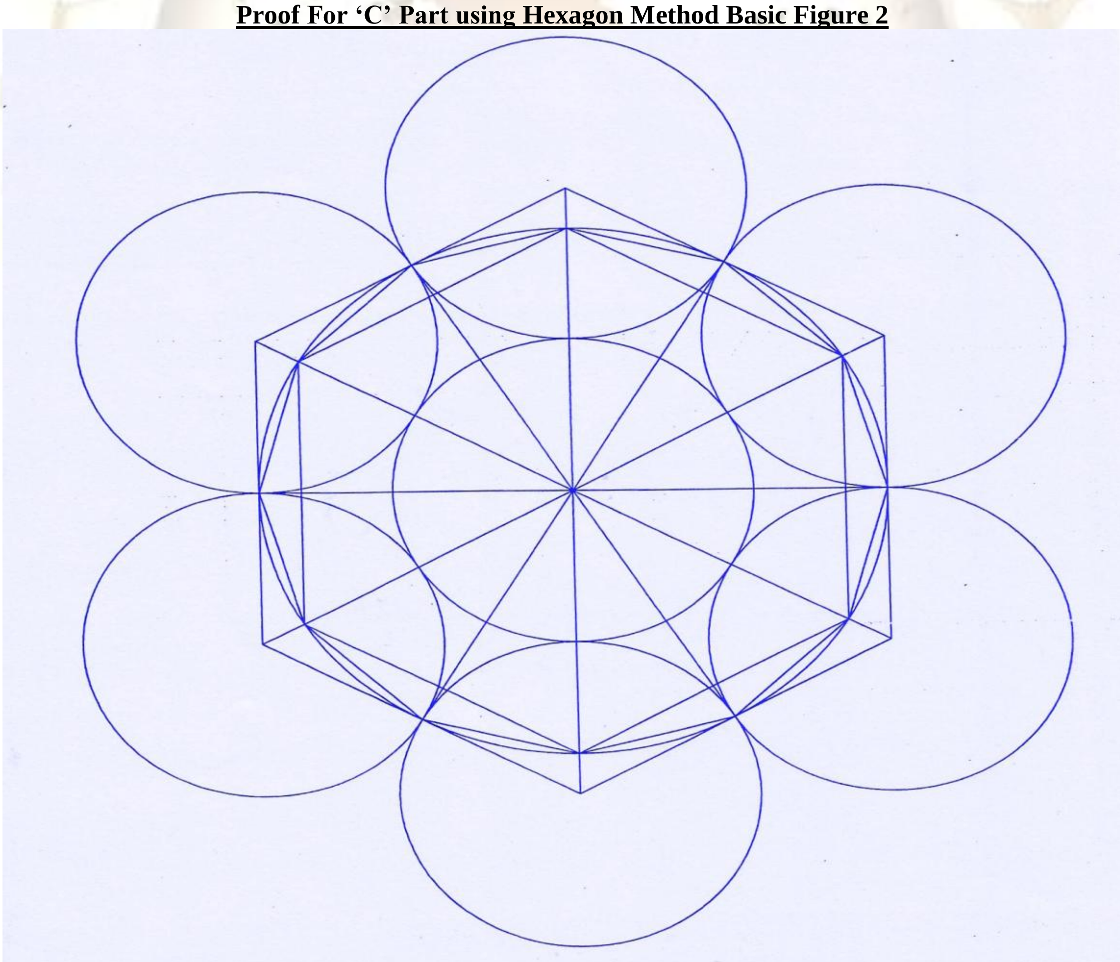
Formulae :-

1) Area of part a = $[0.125(\sqrt{3})]\, r^2$
2) Area of part b = $[0.25 – 0.125(\sqrt{3})]\, r^2$
3) Area of part 3c = $[3.5 - 2\sqrt{3}]\, r^2$
4) Area of Part 12c = $[14 – 8\sqrt{3}]\, r^2$ ---------------(10)
5) Area of part d = $[2\sqrt{3} - 3.25]\, r^2$

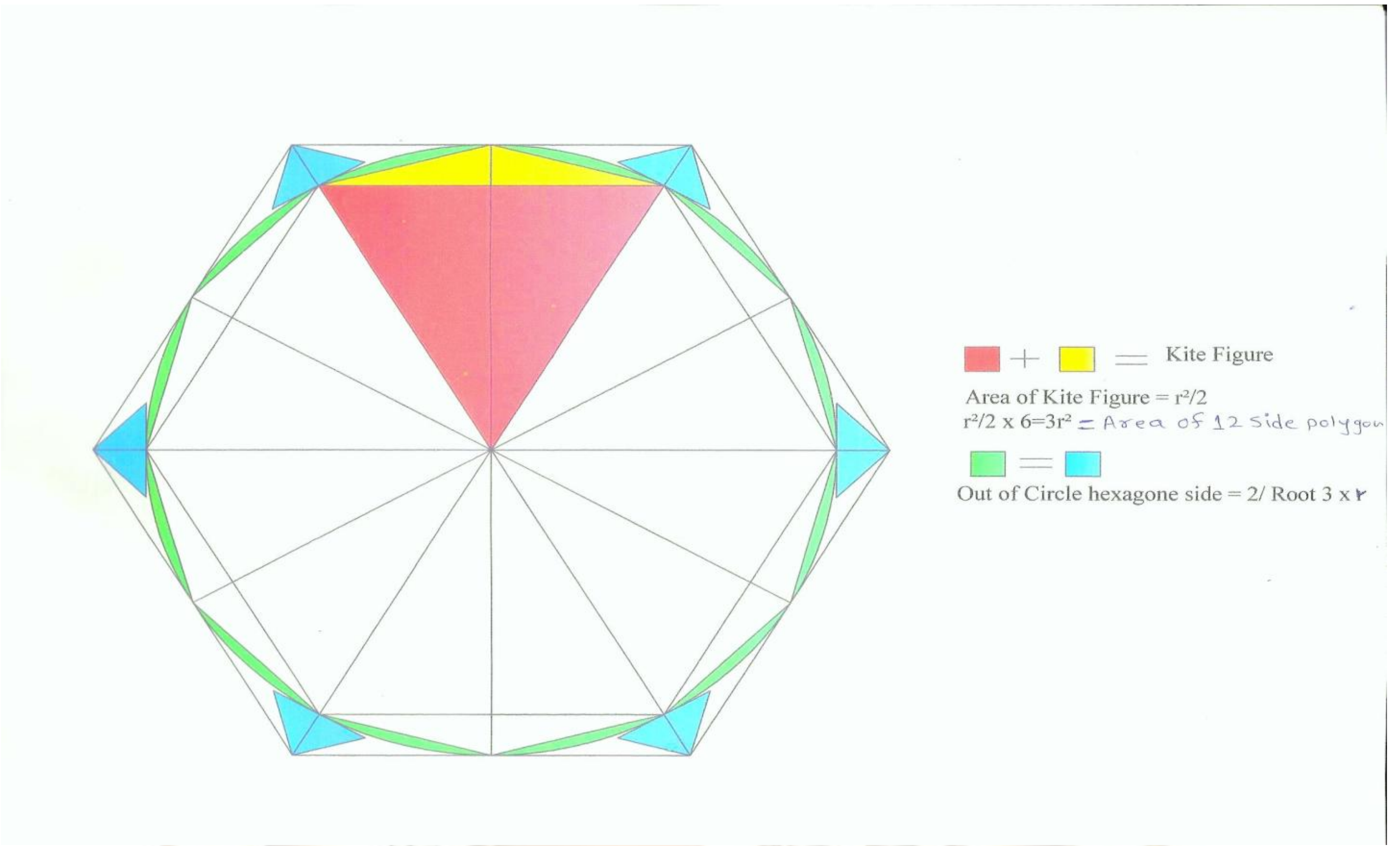
If we put this a, b, c, d values in any above arithmetic equation of area of circle We always get the constant value $\pi = 17 - 8\sqrt{3}$ or $\pi = 17 - \sqrt{192}$.

∴ Finaly I conclude that value of $\pi = 17 - 8\sqrt{3}$ or $\pi = 17 - \sqrt{192}$ or 3.14359........... .

Proof For 'C' Part using Hexagon Method Basic Figure 2

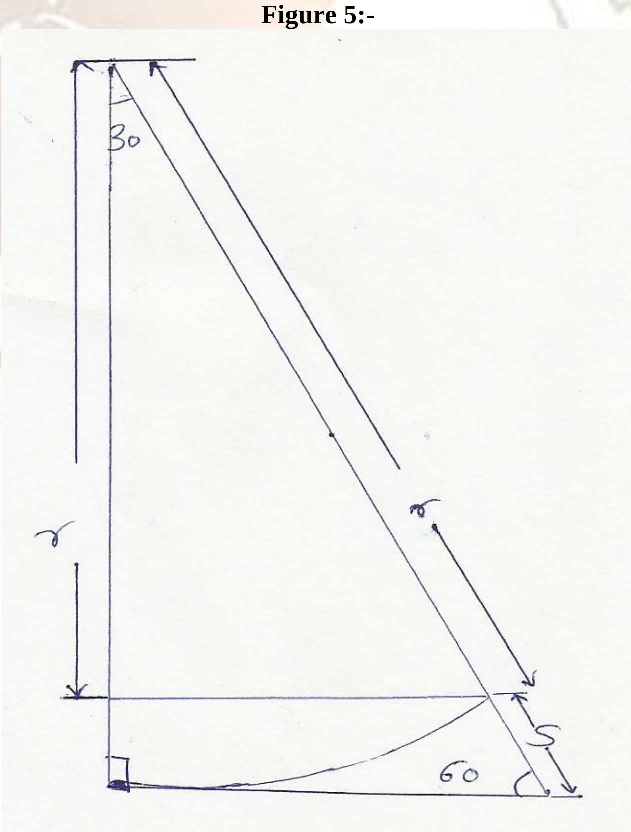


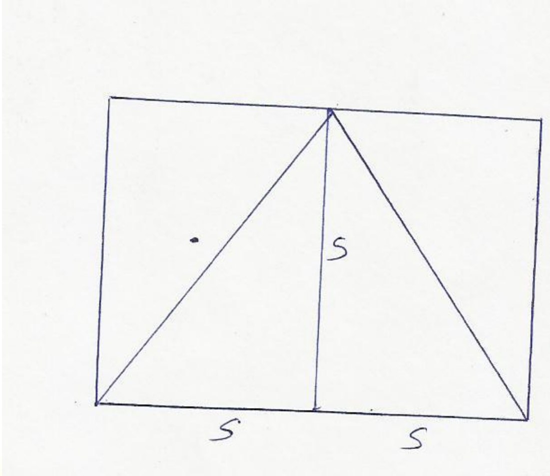
Modified Figure A



$(2\sqrt{3}) * r^2$ =Area of outside hexagon =16a
$(1.5\sqrt{3}) * r^2$=Area of inside hexagon=12a

Figure 5:-





Area of out of circle hexagon = 16a = 16[0.125($\sqrt{3}$)] r^2 = [2$\sqrt{3}$] r^2

Out of circle Hexagon side $= \left(\frac{2}{\sqrt{3}} r\right)$

Area of out of circle Hexagon = 6 (area of Triangle whose base is hexagon side)

$$= 6 \left\{ \frac{\left(\frac{2}{\sqrt{3}} r^2\right)}{2} \right\} = [2\sqrt{3}]\, r^2$$

Note: - out of circle Hexagon side - r = S

$$S = \left(\frac{2}{\sqrt{3}} r\right) - r$$

$$= \left(\frac{2}{\sqrt{3}} - 1\right) r$$

$$S = \left(\frac{2 - \sqrt{3}}{\sqrt{3}}\right) r$$

Where s^2 is Area of Square whose side is s .

$\frac{1}{2}s^2$ is a area of triangle .

$$s^2 = r^2\left(\frac{2-\sqrt{3}}{\sqrt{3}}\right)^2$$

$$s^2 = r^2\left(\frac{7-4\sqrt{3}}{3}\right)$$

Multiply by 6

$$6s^2 = 6\left\{r^2\left(\frac{7-4\sqrt{3}}{3}\right)\right\}$$

$$6s^2 = r^2(14-8\sqrt{3})$$

From equation (10)

$$6s^2 = (12c)$$

$$s^2 = (2c)$$

$$\boxed{c = \frac{s^2}{2}}$$

Area of C part $= \frac{Area\ of\ sqaure\ (\ side\ s)}{2}$

Area of Circle = 12a + 12b + Area of 12C parts

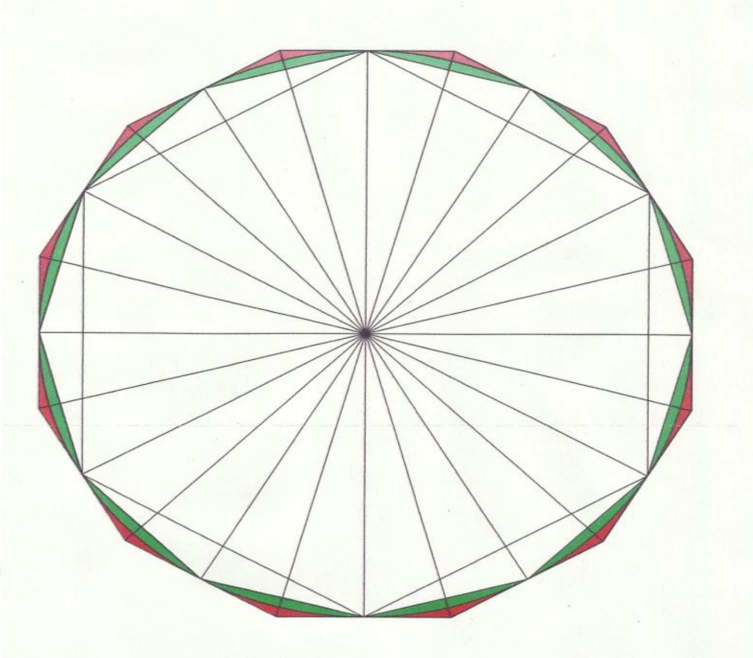
$= 3r^2 + [14 - 8\sqrt{3}\ ]\ r^2$

Area of Circle $= [17 - 8\sqrt{3}\ ]\ r^2$

$\Pi r^2 = [17 - 8\sqrt{3}\ ]\ r^2$

Implies $\pi = 17 - 8\sqrt{3}$

Proof For 'C' Part using Dodecagon Method Modified Figure B



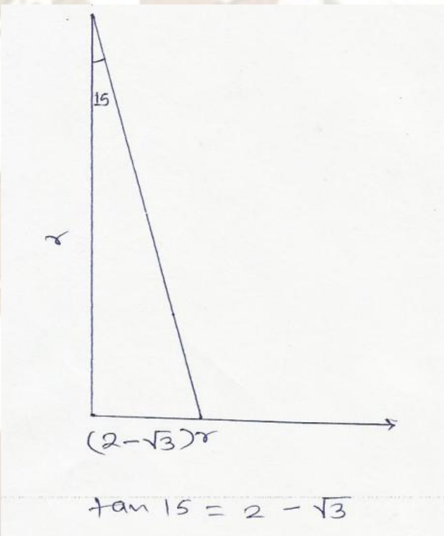


Figure - 6

$\text{Tan}(15^0) = 2 - \sqrt{3}$

Out of circle Dodecagon area = $12(2 - \sqrt{3})r^2 = (24 - 12\sqrt{3})\, r^2 = 96b$ -------- (*)

Area bounded between outside & Inside circle Dodecagon = $12(2 - \sqrt{3})r^2 - 3r^2$

$= (24 - 12\sqrt{3})\, r^2 - 3\, r^2$

Area bounded between outside & Inside circle Dodecagon = $(21 - 12\sqrt{3})\, r^2$

Note :-

Area bounded between outside & inside circle Dodecagon = Area of 12 Green Parts + Area of 12 Red Parts

Area of 12 Green Parts + Area of 12 Red Parts $= (21 - 12\sqrt{3})\, r^2$

$= (14 - 8\sqrt{3})\, r^2 + (7 - 4\sqrt{3})\, r^2$

= Area of 12 Green Parts + $\frac{1}{2}$ (Area of 12 Green Parts)

Implies Area of 12 Red Parts = $\frac{1}{2}$ (Area of 12 Green Parts)

Implies Area of 1 Red Part = $\frac{1}{2}$ (Area of 1 Green Part)

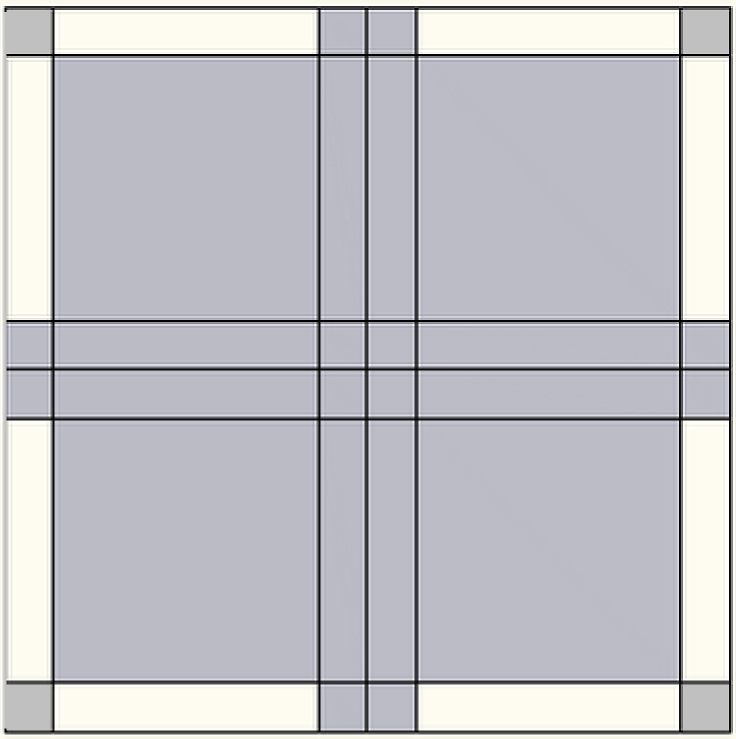
Area of Circle = Area of Out of circle Dodecagon - 12 Red Parts

$= (24 - 12\sqrt{3})\, r^2 - (7 - 4\sqrt{3})\, r^2$

$\Pi r^2 = (17 - 8\sqrt{3})\, r^2$

Implies $\pi = 17 - 8\sqrt{3}$

Supportive work 1 :-



= 96b

Out of circle Dodecagon area = 96b
96b = 12a +12b + 18c
96 b = (12a + 12b +12c) + 6c
96b = Area of circle +6c
Area of circle = 96b – 6c

$= 96[0.25 – 0.125(\sqrt{3})]\, r^2 - 2\,[3.5 - 2\sqrt{3}\,]\, r^2$

$= [24 – 12\sqrt{3}\,]\, r^2 - (7 - 4\sqrt{3})\, r^2$

$\Pi r^2 = (17 - 8\sqrt{3})\, r^2$

Implies $\pi = 17 - 8\sqrt{3}$

Supportive work 2 :- Verification of d value

Area of square = 16a +16b
Area of dodacagon = 96b

Area bounded by square & Dodacagon = 16a +16b – 96b = 16a -80b

$$4d - 6c = 16a - 80b$$

$$4d = 16a - 80b + 6c$$

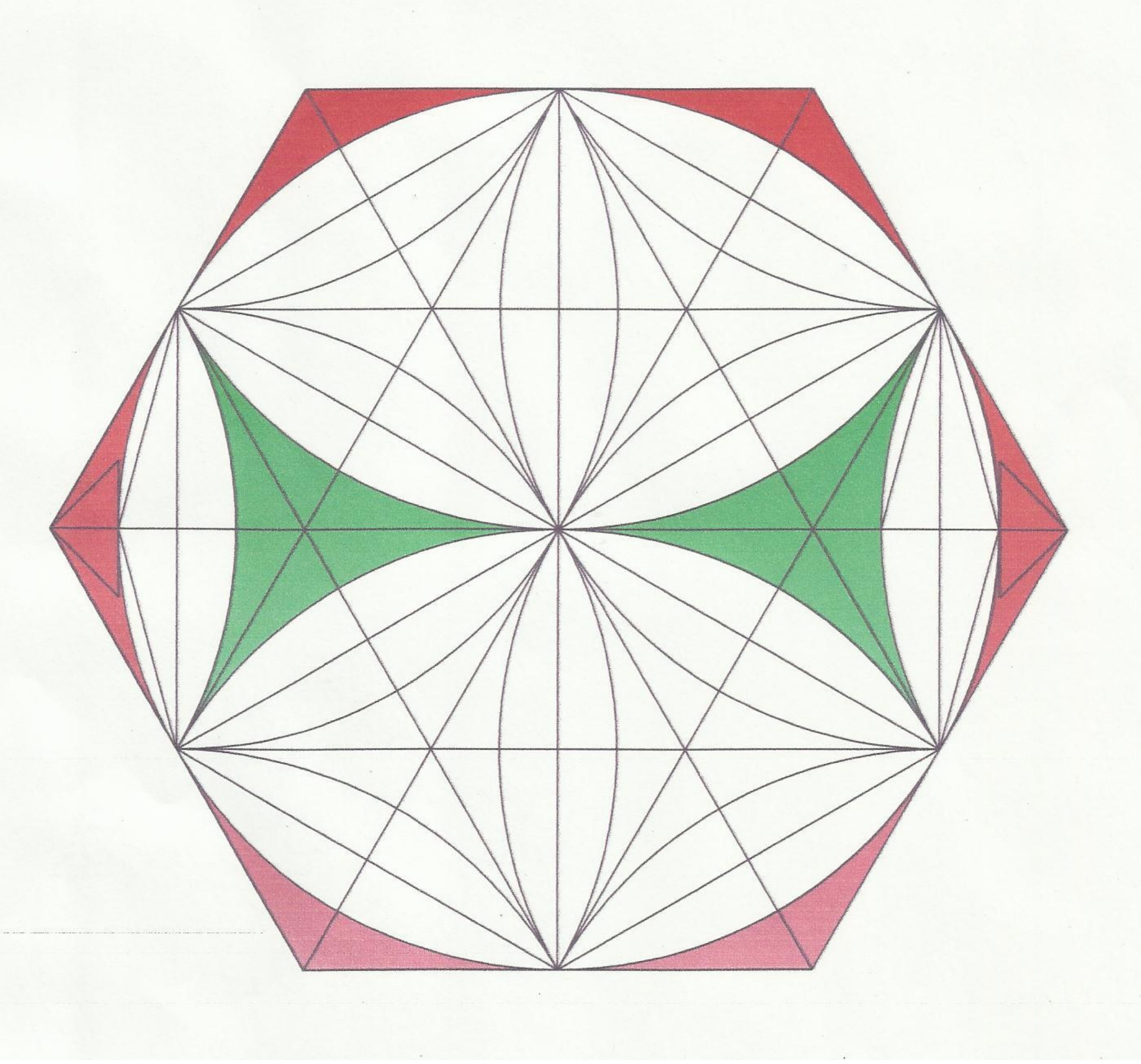
$$4d = 16\,[0.125(\sqrt{3})]\,r^2 - 80[\,0.25 - 0.125(\sqrt{3})\,]\,r^2 + 2\,[\,3.5 - 2\sqrt{3}\,]\,r^2$$

$$4d = [2\sqrt{3} - 20 + 10\sqrt{3} + 7 - 4\sqrt{3}\,]\,r^2$$

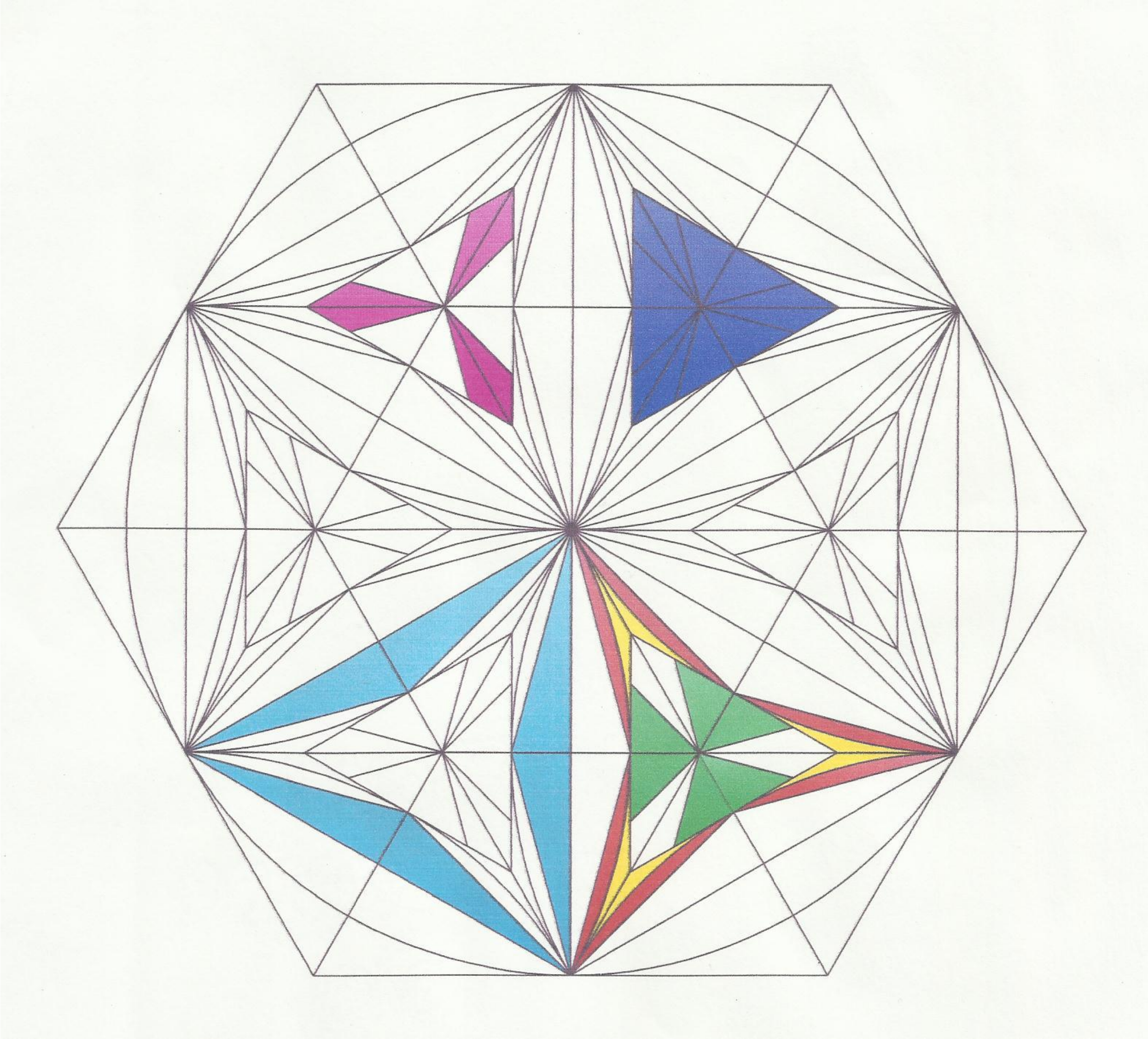
$$4d = [8\sqrt{3} - 13]\,r^2$$

Area of part d $= [\,2\sqrt{3} - 3.25\,]\,r^2$

Extra work :



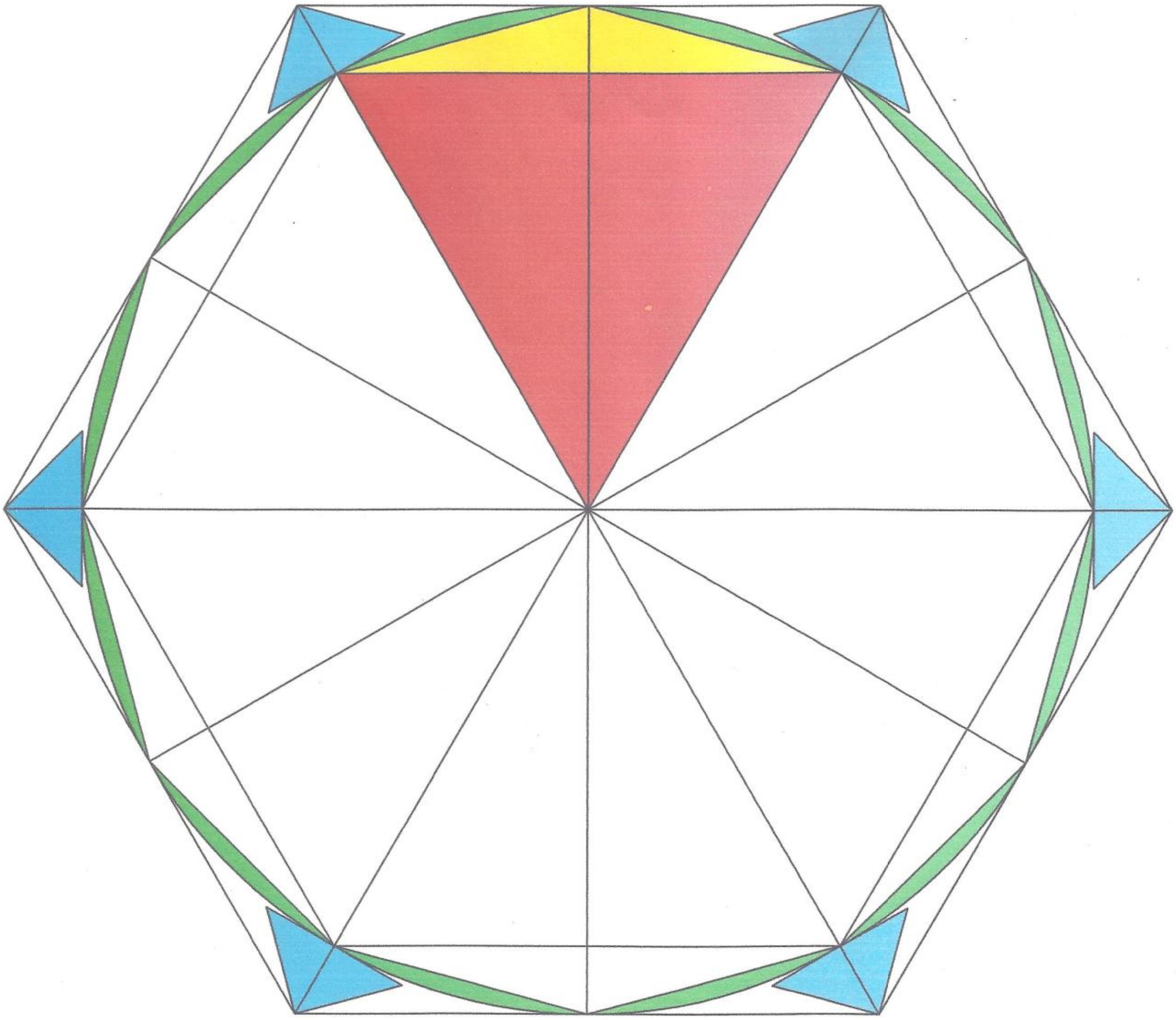
Area of red part = area of green part = 12a-68b



Area of (red part + yellow part +green part) = area of 15c .
Area of dark blue triangle = 8a-48b.

$2\sqrt{3}\,r^2$ = area of outside hexagon = 16 a

$1.5\sqrt{3}\,r^2$ = area of inside hexagon = 12 a



+ = Kite Figure

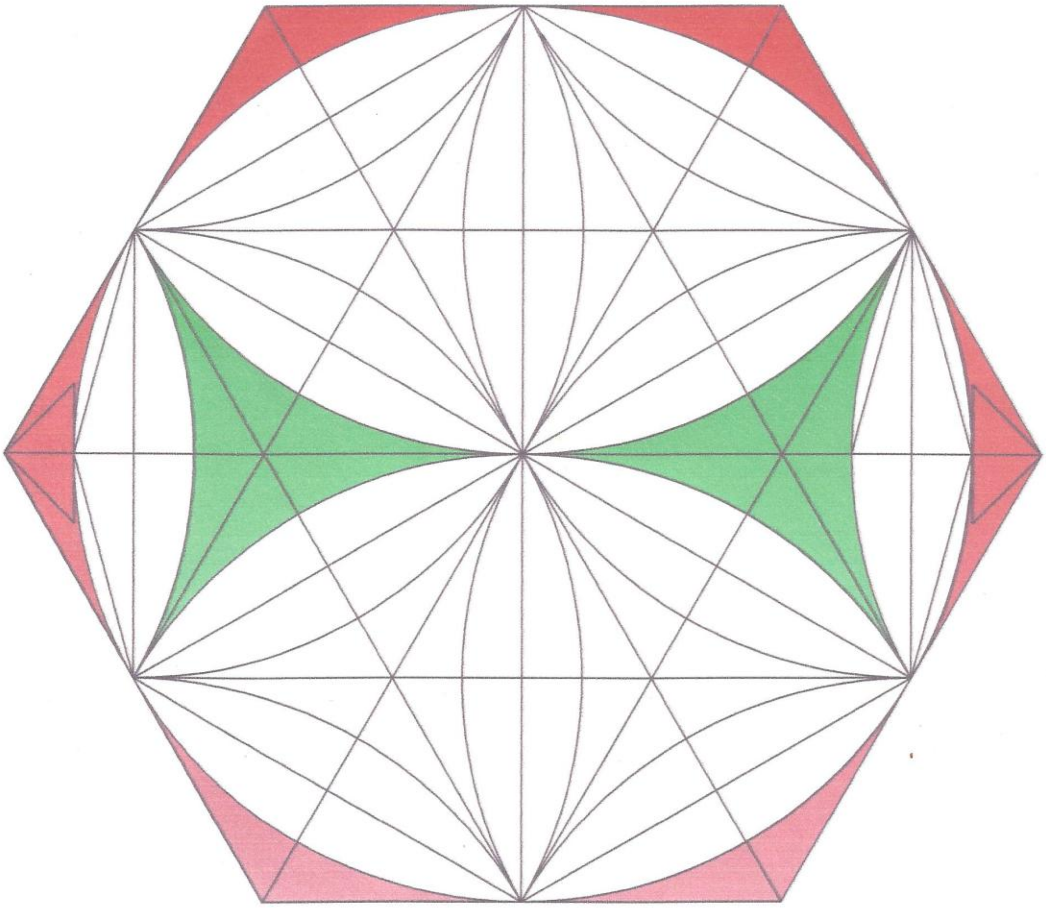
Area of Kite Figure = $r^2/2$

$r^2/2 \times 6 = 3r^2$

=

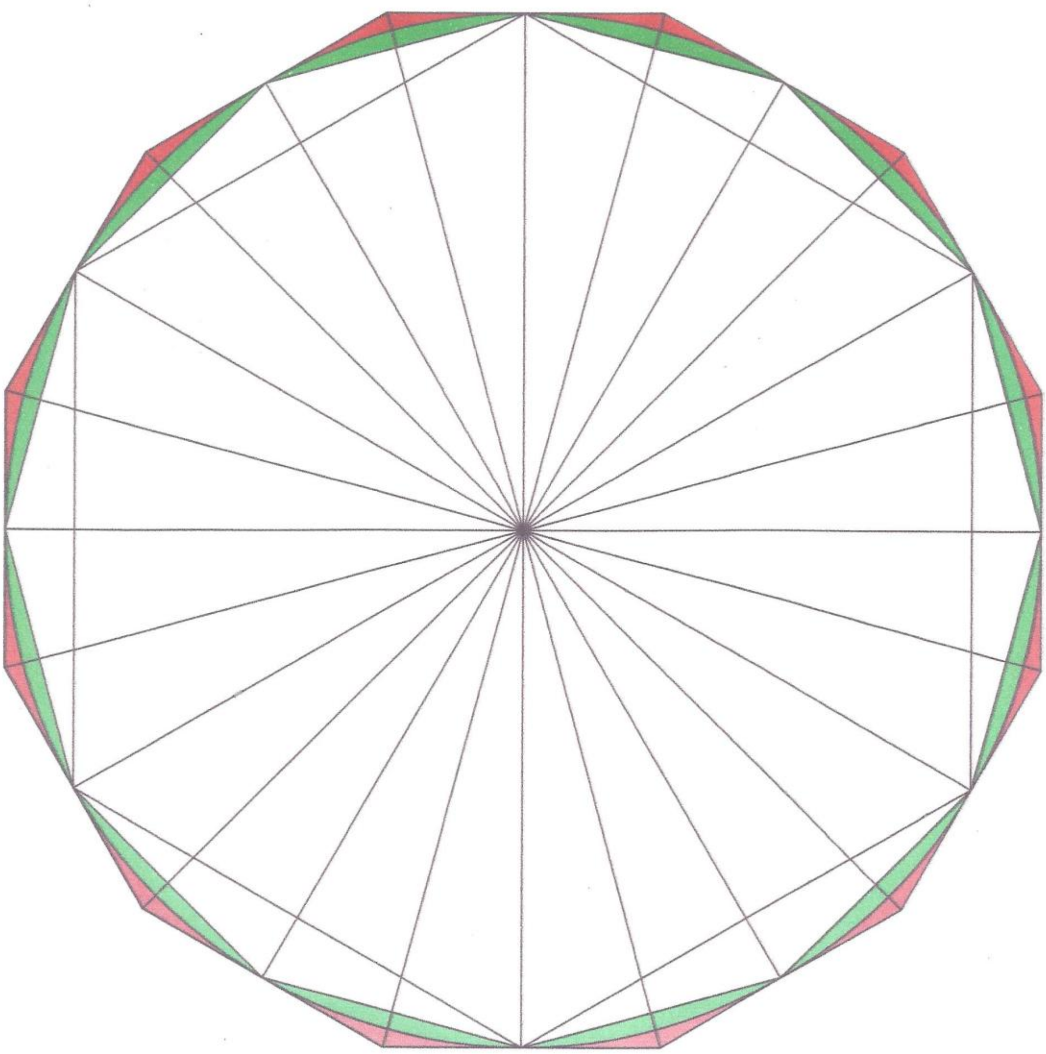
Out of Circle hexagone side = 2/ Root 3 x r

$2\sqrt{3}r^2 = 16a$



(green) = (red) = 12a − 68b

16a − 12a − 68b = 4a + 68b = Area of circle

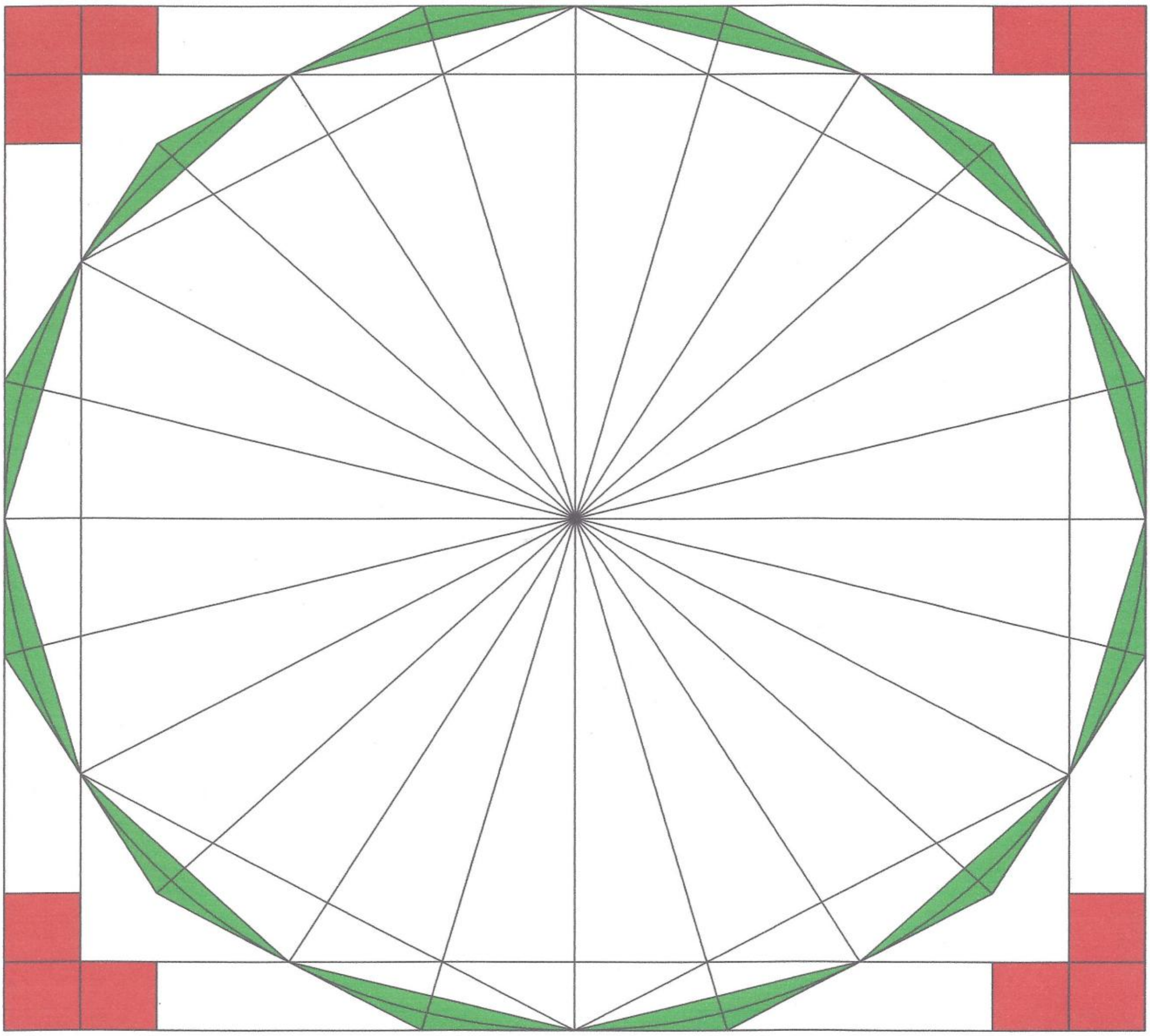


Area of ouside dodecagon $= \left[12 \left(2-\sqrt{3}\right)r^2\right] = 96b$

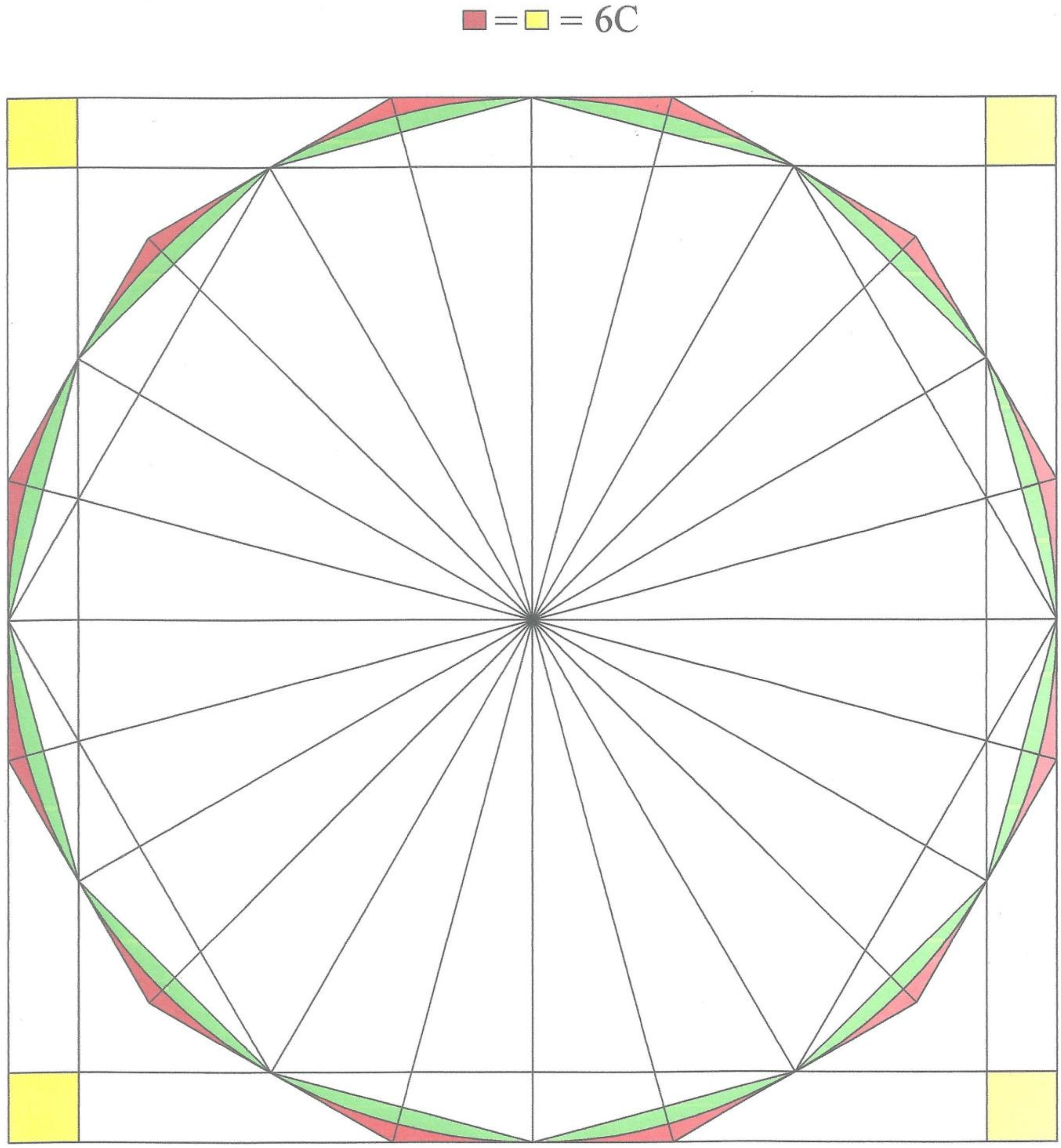
Area of inside dodecagon $= 3r^2$

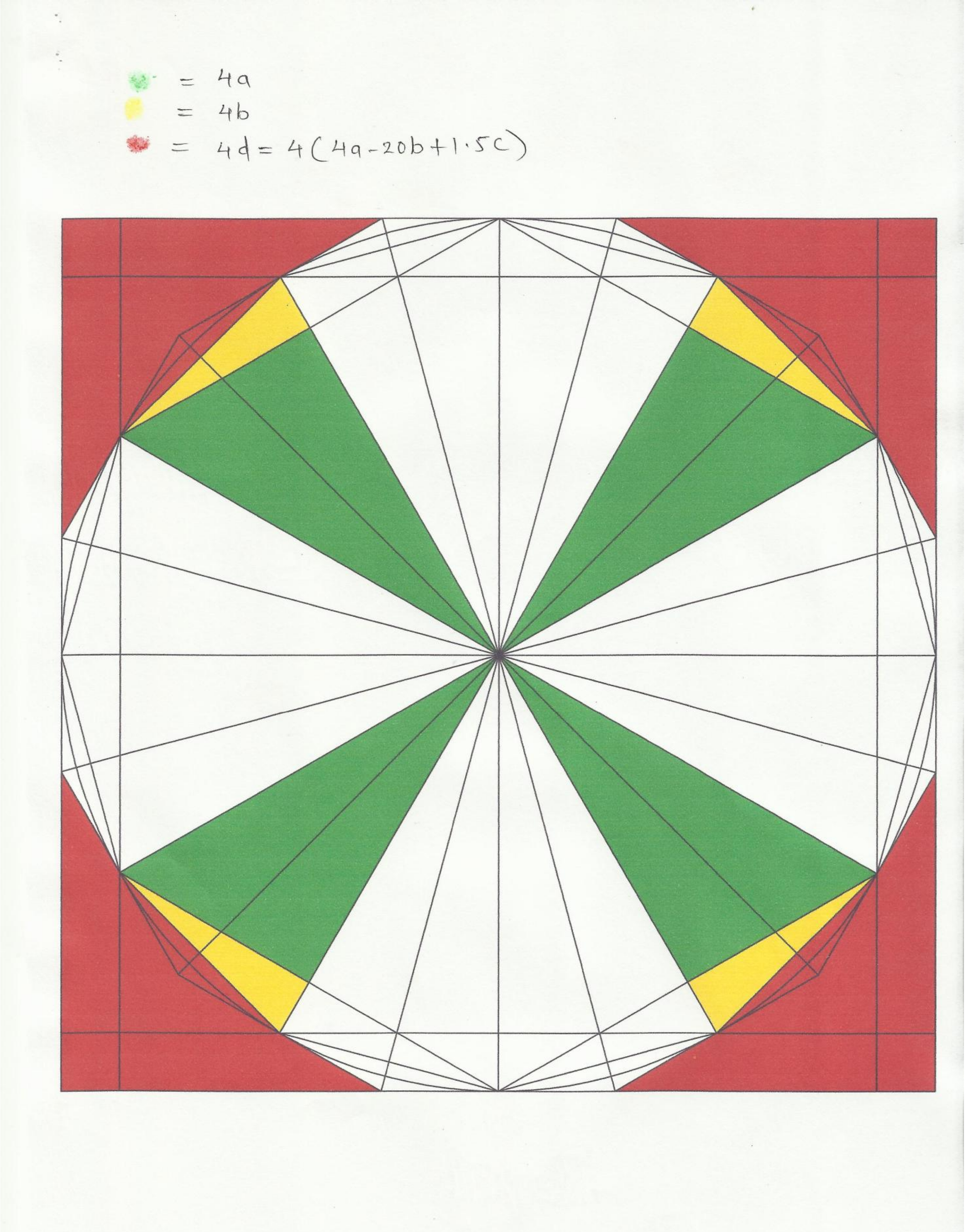
$96b = 12a + 12b + 18c$

$\therefore$ $96b - 6c =$ Area of circle



= = 18C = 84b − 12a

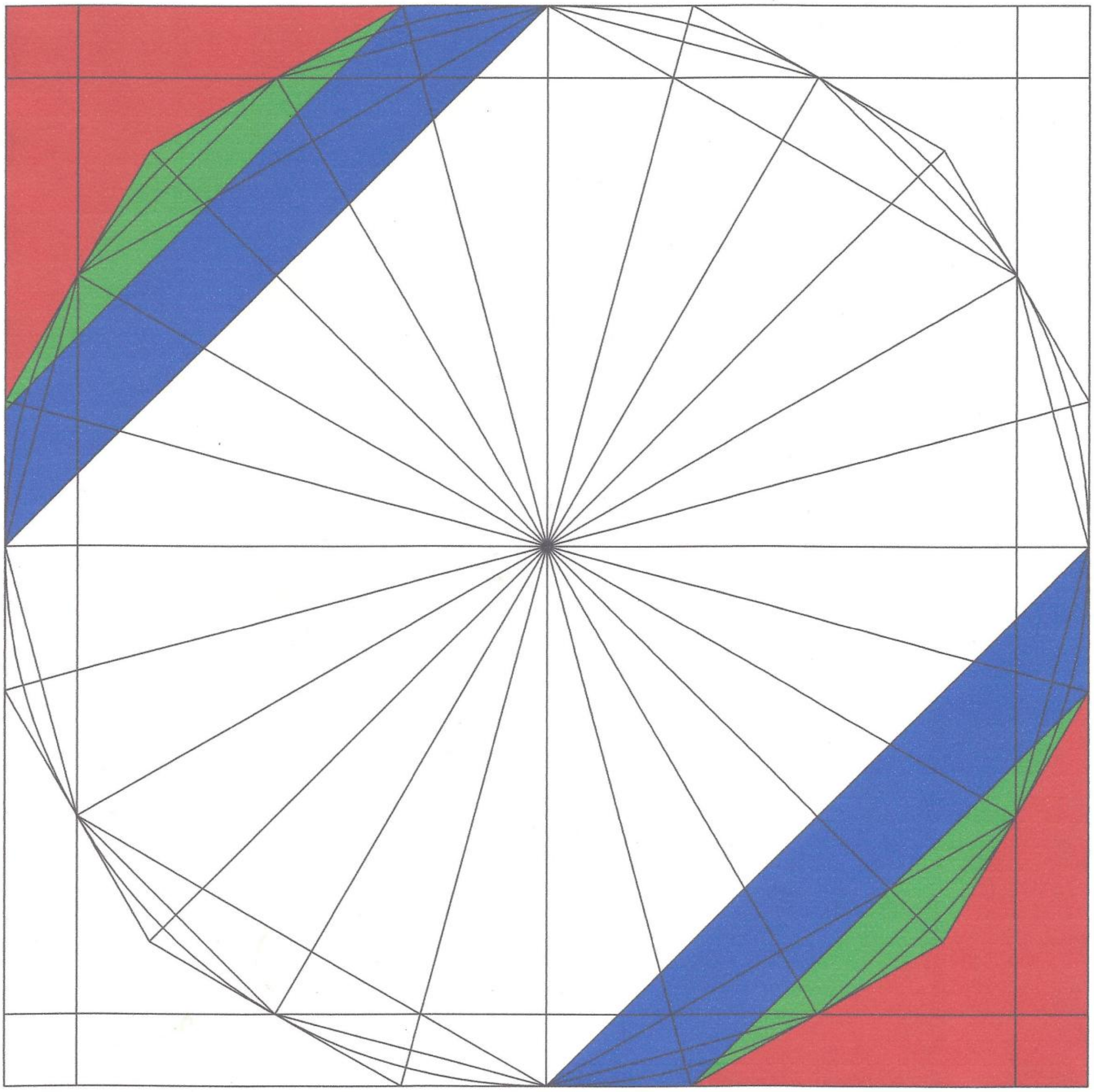

= = 6C


= 4a
= 4b
= 4d = 4(4a-20b+1·5c)

[blue] $= 2a - 6b = d + 1.5c$

[green] $= 6c = 28b - 4a$

[red] $= 4a - 20b = d - 1.5c$



([red] + [green] + [blue]) − ([red] + [blue])

$2a + 2b - 2d\ (6a - 26b) = 6c =$

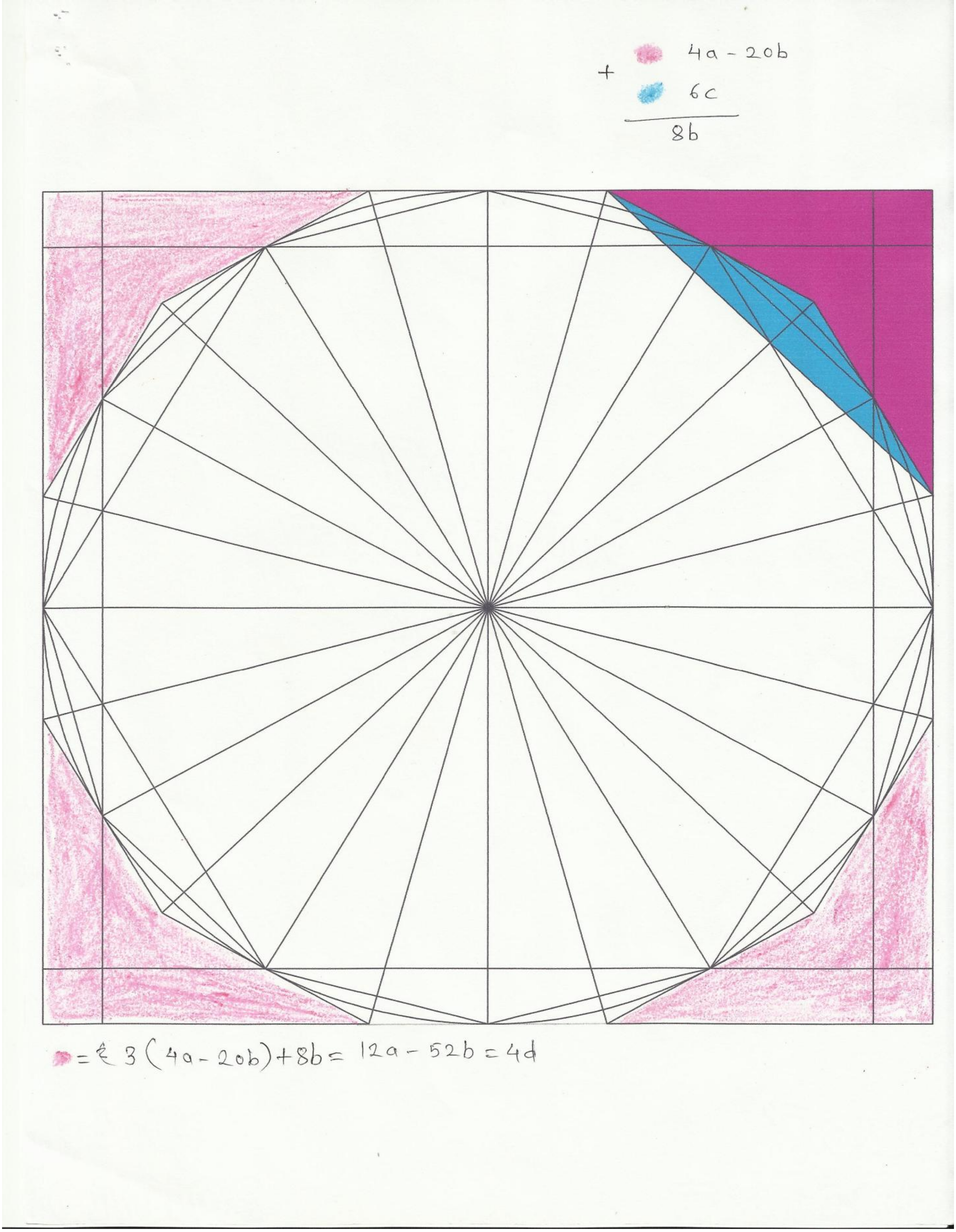
$28b - 4a$

[red] + [green] + [blue] $= 2a + 2b = 2d + 6c$

[red] + [green] $= 8b = d + 4.5c$

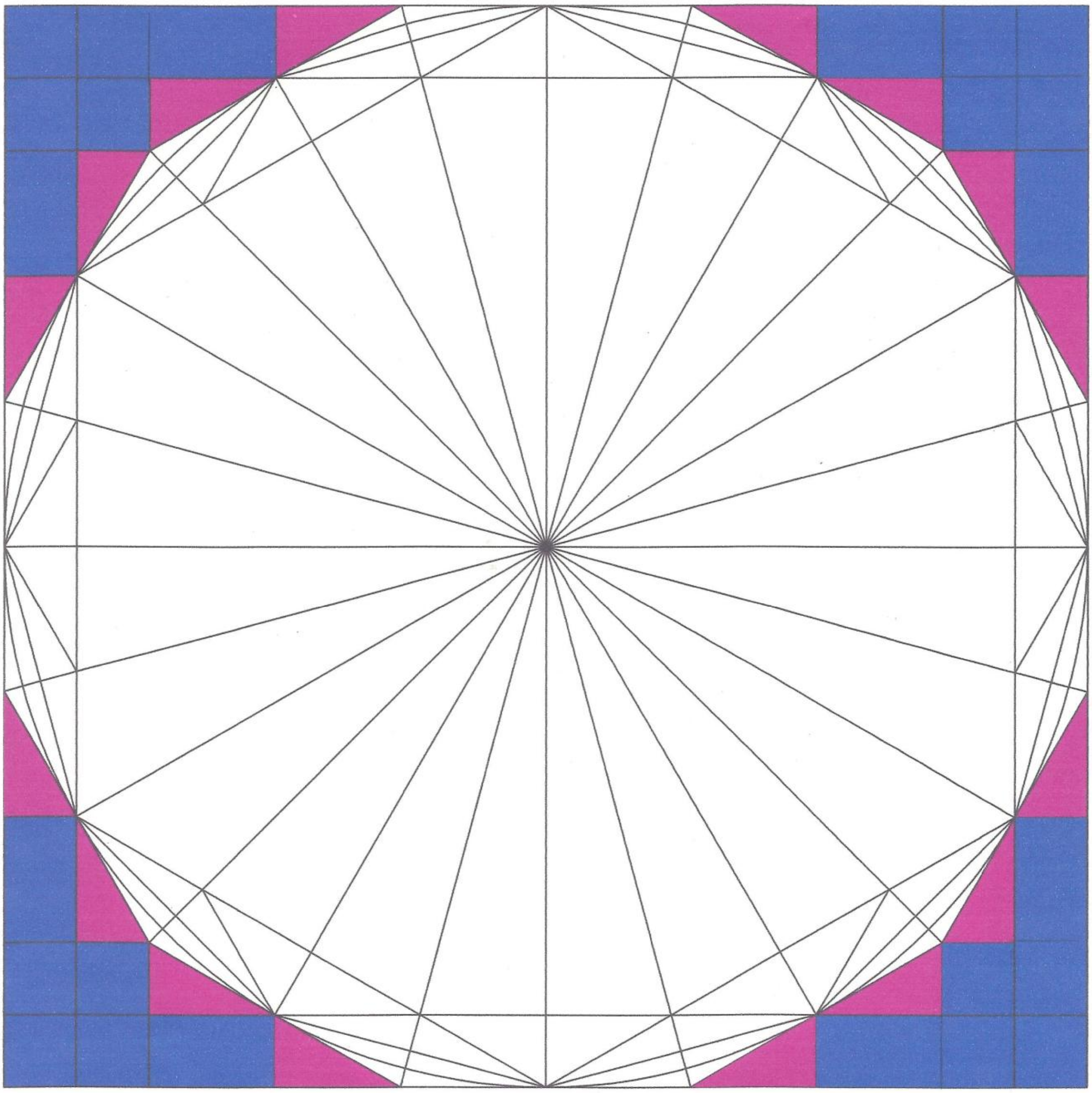
[red] + [blue] $= 2d = 6a - 26b$

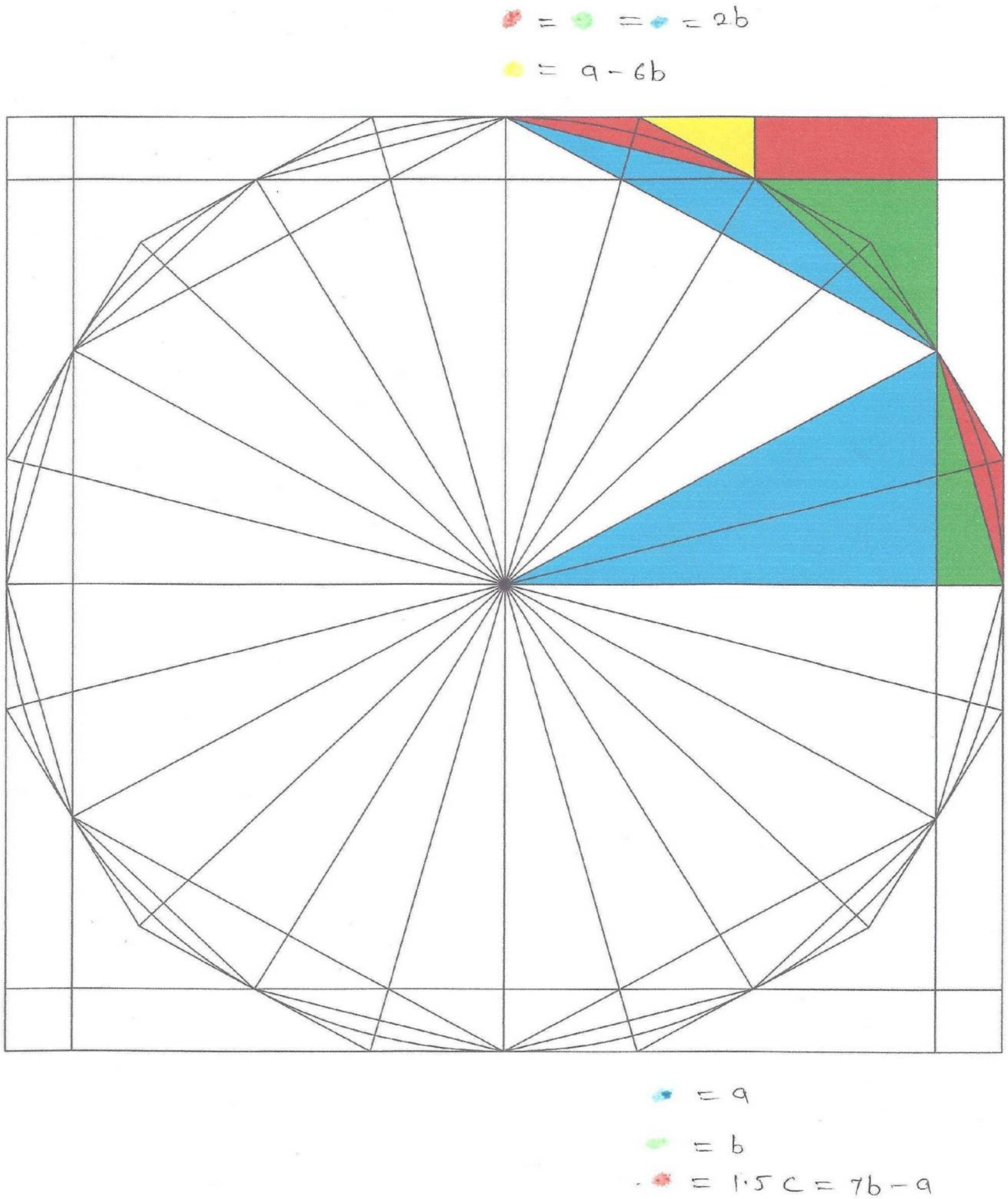
[blue] − [red] $= 3c$

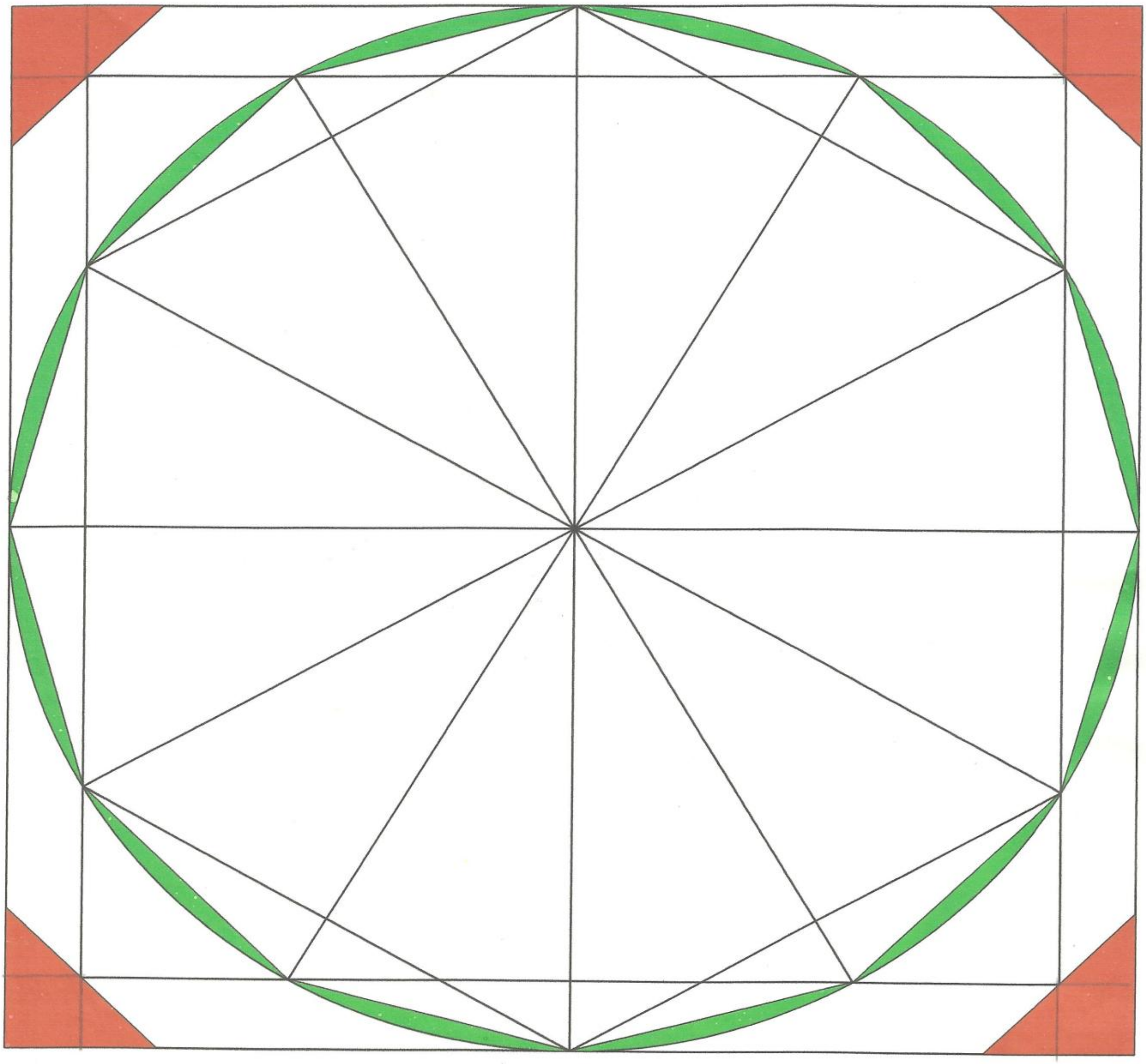

4a - 20b
+
6c
8b
= 3 (4a - 20b) + 8b = 12a - 52b = 4d

= 16b

= 16(a − 6b)




= = = 2b
= a − 6b
= a
= b
= 1.5 C = 7b − a



Green part equal to red part

Conclusions:-

I conclude that we get 12c value is $(14 - 8\sqrt{3})\, r^2$
by using square , Hexagon , Dodecagon method & it is same .
Inside circle Dodecagon area = $3r^2$.
Area of circle = Inside circle Dodecagon area + 12c Area

$$\Pi r^2 = (14 - 8\sqrt{3})\, r^2 + 3r^2$$

Implies $\pi = 17 - 8\sqrt{3}$

i.e $\pi = 3.14359353944...$

References :-

1) Basic algebra & geometry concepts .
2) History of Pi(π) from internet .
3) Exact value of PI(π) , IOSR International Journal .