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Adaptive Control of Angular Position & Angular Velocity for A DC Motor with Full State Measureable

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Abstract: Adaptive control is the control method used by a controller which must adapt to a controlled system with parameters which vary, or are initially uncertain. Adaptive control is different from robust control in that it does not need a priori information about the bounds on these uncertain or time-varying parameters; robust control guarantees that if the changes are within given bounds the control law need not be changed, while adaptive control is concerned with control law changing themselves. In this paper the methods of adaptive control are used and explored on a pilot-scale hardware platform. A computer-aided design procedure is used to achieve the specifications, as part of the overall adaptive systems.

I. INTRODUCTION

Most current techniques for designing control systems are based on a good understanding of the plant under study and its environment [1]. However, in a number of instances, the plant to be controlled is too much complex and the basic physical processes in it are not fully understood. Control design techniques then need to be augmented with an identification technique aimed at obtaining a progressively better understanding of the plant to be controlled. It is thus intuitive to aggregate system identification and control. Often, the two steps will be taken separately. If the system identification is recursive-that is the plant model is periodically updated on the basis of previous estimates and new data identification and control may be performed concurrently. Adaptive control is a direct aggression of a control methodology with some form of recursive system identification.

Among the various types of adaptive system configurations, model reference adaptive systems are important since they lead to relatively easy-to-implement systems with a high speed of adaptation which can be used in a variety of situations [2]. In a model based adaptive system there should be a reference index of performance (IP). To generate this reference index of performance, one uses an auxiliary dynamic system called the reference model, which is excited by the same external inputs as adjustable system. The reference model specifies in terms of input and model states a given index of performance. Model reference adaptive methods might be classified as evolving from three different approaches [3]. (i) Full state access method which assumes that the state variables are measurable. (ii) Input-output method, where adaptive observers are incorporated into the controller to overcome the inability to access the entire vector. (iii) Output feedback method which requires neither full state feedback nor adaptive observers [5], [6]. In full state measurable method, the comparison between the given index of performance and the measured index of performance is obtained directly by comparing the states of the adjustable system and of the reference model using a typical feedback comparator [7], [8]. The difference between the states of the reference model and those of the adjustable system is used by the adaptation mechanism either to modify the parameters of the adjustable system or to generate an auxiliary input signal in order to minimize the difference between the two index of performance expressed as a function of the difference between the states of the adjustable system and those of the model in order to maintain the measured index of performance in the neighbourhood of the reference index of performance [9].

In this paper a DC motor kit is used as the experimental platform and a PC-based data acquisition system with a graphical icon-driven software (NI LabVIEW) is used and MATLAB software package is used for the simulation analysis. Adaptive controller has been designed with a full state measurable case for this DC motor experimental system.

II. CALIBRATION OF D.C. MOTOR SENSORS

D.C. motor is used in this project as the hardware plant model. The D.C. motor apparatus and the nominal dynamic model of the motor are shown in figure 1 and figure 2 respectively.

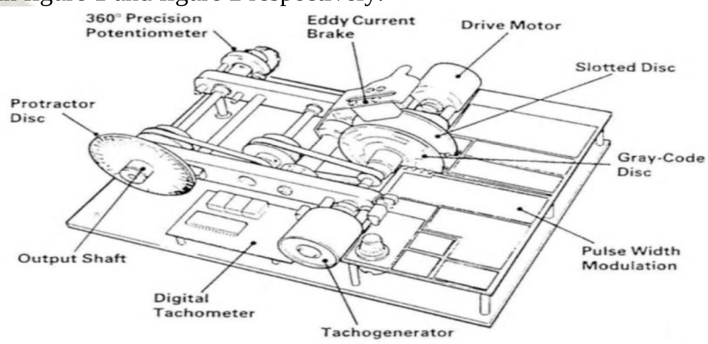

Figure 1: DC Motor Apparatus

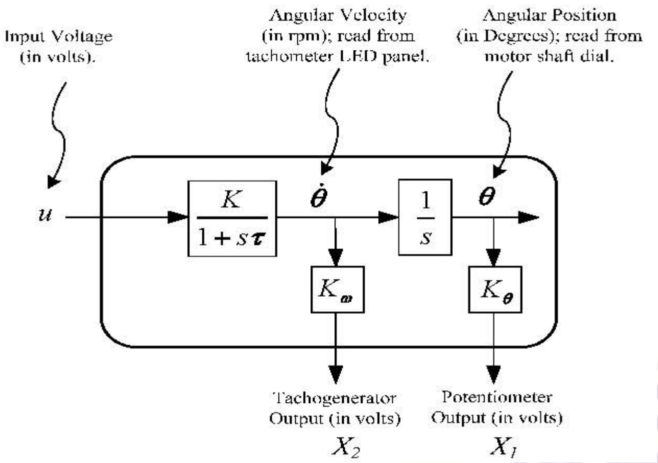

Figure 2: Nominal Dynamic model of D.C. motor

As it can be seen from figure 2 of nominal dynamic model of motor, the plant (D.C. motor) has two states to be measured. So for full state measurable adaptive control, we need two sensors as it can be seen. D.C. motor two types of sensors- (i) Potentiometer for angular displacement measurement (ii) Tachometer for angular velocity measurement. For getting actual angular position and angular velocity measurements, voltage outputs of the sensors need to be calibrated. For calibration purpose, a LabVIEW program was coded as seen from figure 3 and the corresponding front panel view is shown in figure 4 and figure 5. For calibration of potentiometer, the motor protractor disc was rotated by hand by certain amount of angle and corresponding amplitude of position was observed in the front plane. This procedure was repeated for few times to collect few data in order to plot potentiometer output (volts) vs angular position (radian) and therefore to obtain K_θ (slope of the straight line obtained by curvefit) which is the calibrated gain for potentiometer. This procedure is shown in table 1 and figure 6. For calibration of tachometer, a specific voltage signal is applied to the plant (by entering the voltage values in the control signal in the front panel [fig.3]) and corresponding tachometer output was observed from the tachometer display (rpm). This procedure was repeated for few times to collect few data in order to plot tachometer output (volts) vs angular velocity (radian/sec) and therefore to obtain K_w (slope of the straight line obtained by curvefit) which is the calibrated gain for potentiometer. This procedure is shown in table 2 and figure 7. Using table 2, another plot of angular velocity (rad/sec) vs Input (volts) can be obtained [figure 8] and slope of the straight line found to find the static gain , K of the plant. Applying a step input voltage; the corresponding transient response can be obtained. From the transient response of figure 5, time constant τ of the plant can be obtained.

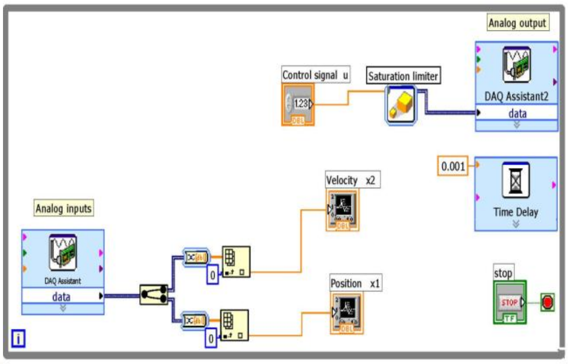

Figure 3: Block diagram for calibration of sensors

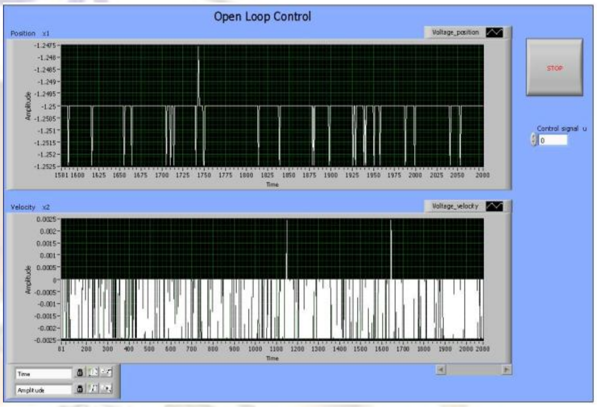

Figure 4: Front panel view for calibration of potentiometer (motor shaft rotated by hand)

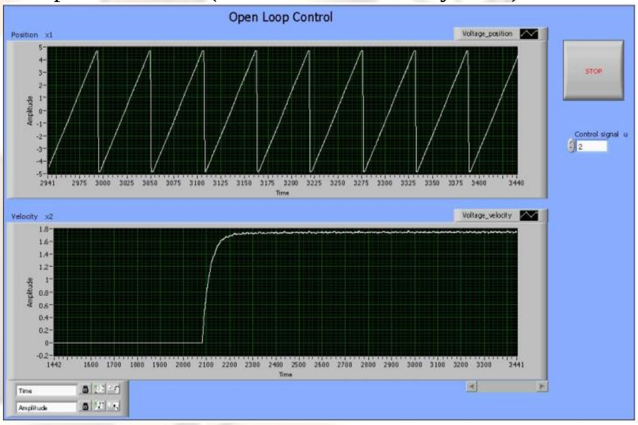

Figure 5: Front panel view for calibration of tachometer (a control voltage is given)

Table 1: Calibration of Position K_θ

Potentiometer Output x_1(volts)	Angular Position (degrees)	Angular Position θ (radians)
-2.1536	0	0
-1.3427	30	0.5236
-0.4906	60	1.0472
0.4404	90	1.5708
1.2255	120	2.0944
2.068	150	2.6180
2.92	180	3.1416
3.762	210	3.6652
4.626	240	4.1888

Table 2: Calibration for velocity K_w

Input Voltage u (volts)	Tachogenerator Output x_2 (volts)	Angular Velocity (rpm)	Angular Velocity $\dot{\theta}$ (rad/sec)
-5	-4.525	-265	-27.7507
-4	-3.678	-210	-21.9911
-3	-2.688	-155	-16.2316
-2	-1.72	-99	-10.3673
-1	-0.755	-45	-4.7124
0	0	0	0
1	0.7725	44	4.6077
2	1.738	100	10.4720
3	2.718	156	16.3363
4	3.692	214	22.41
5	4.5	272	28.4838

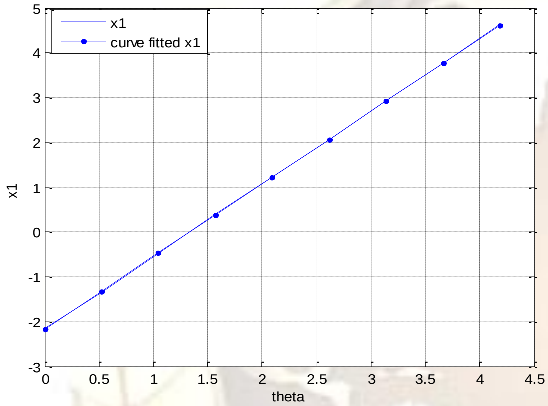

Figure 6: [Potentiometer output (volts)] x_1 vs θ [Angular position (radian)]

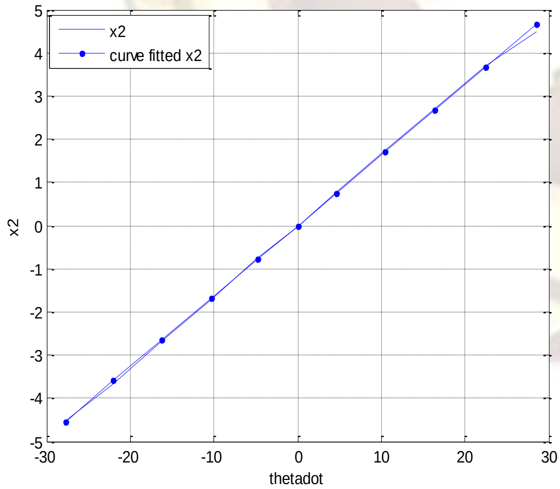

Figure 7: [Tachometer output (volts)] x_2 vs $\dot{\theta}$ [Angular velocity (rad/sec)]

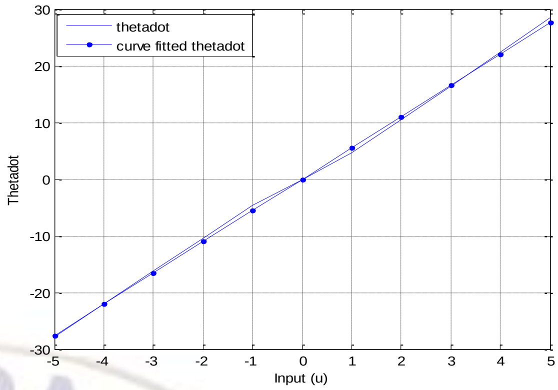

Figure 8: [Angular velocity (rad/sec)] $\dot{\theta}$ vs u [Input (volts)]

Investigating the above four figures 5, 6, 7, 8; the following parameters are obtained

Time constant, τ =0.6 sec

Static state gain, K= 5.5225

K_θ = 1.6188

K_w = 0.1633

III. ADAPTIVE CONTROL OF ANGULAR POSITION

3.1 Adaptive control algorithm for angular displace-ment control

For full state measurable adaptive control design for angular position of D.C motor, the plant model has the following state-space model

$$\dot{x}_p = A_p x_p + gbu \quad (1)$$

With $x_p = [x_1 \; x_2]^T$ measurable , b= $\begin{bmatrix}0\\1\end{bmatrix}$ and sgn(g) is known

Where x_1 = angular position and x_2 = angular velocity

Reference second order model, $\dot{x}_m = A_m x_m + g_m br$ (2)

With $x_m = [x_{m1} \; x_{m2}]^T$, $g_m b = [g_{m1} \;\; g_{m2}]^T$ *and r=* reference input

Where $x_{m1=}$ reference angular position and $x_{m1=}$ reference angular velocity

Control Law:

$$u(t) = \theta_x^T(t)\, x_p(t) + \theta_r(t)\, r(t) = \theta^T(t)\, w(t) \quad (3)$$

where $\theta = [\theta_{x1}(t) \quad \theta_{x2}(t) \quad \theta_r(t)]^T$, $w(t) = [\, x_{p1}(t) \;\; x_{p2}(t) \;\; r(t)]$

Adaptive Law:

Error, $e_1 = x_{p1} - x_{m1}$ *and* $e_2 = x_{p2} - x_{m2}$

$$\begin{bmatrix}\dot{\theta}_{x1}\\ \dot{\theta}_{x2}\\ \dot{\theta}_r\end{bmatrix} = -sgn(g)\,\Gamma \begin{bmatrix}x_{p1}\\ x_{p2}\\ r\end{bmatrix} [e_1 \quad e_2] \begin{bmatrix}P_{11} & P_{12}\\ P_{21} & P_{22}\end{bmatrix} \begin{bmatrix}0\\1\end{bmatrix} = -\Gamma \begin{bmatrix}x_{p1}\\ x_{p2}\\ r\end{bmatrix} [e_1 \quad e_2] \begin{bmatrix}P_{12}\\ P_{22}\end{bmatrix} \quad (6)$$

By applying a unit step signal and observing the output, it can be easily concluded that, sgn(g)=1 and Γ *is a* 3-by-3 positive definite matrix.

The LabVIEW block diagram for angular displacement control is shown in figure 9.

3.2 Experimental results

Case (a): Here the effect of changing the adaptive gain Γ will be examined. The reference model is chosen as, $\dot{\boldsymbol{x}}_{\boldsymbol{m}} = \boldsymbol{A}_{\boldsymbol{m}} x_m + g_m br$

Where $A_m = \begin{bmatrix} 0 & 1 \\ -4 & -3.6 \end{bmatrix}$ and $g_m b = \begin{bmatrix} g_{m1} \\ g_{m2} \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$ (7)

Choosing Q=$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and after solving Lyapunov equation,

$$P = \begin{bmatrix} 1.144 & 0.1250 \\ 0.1250 & 0.1736 \end{bmatrix}$$

Here the adaptive gain Γ is selected as a 3-by-3 positive diagonal matrix as follows

$$\Gamma_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \Gamma_2 = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$$\Gamma_3 = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 200 \end{bmatrix}$$

Choosing Q to solve the Lyapunov equation ,

$A_m^T P + PA_m = -Q$ (5)

Where Q is a 2-by-2 symmetric positive definite matrix and thereby P will be a 2-by-2

$$\Gamma_4 = \begin{bmatrix} 1000 & 0 & 0 \\ 0 & 1000 & 0 \\ 0 & 0 & 2000 \end{bmatrix}$$

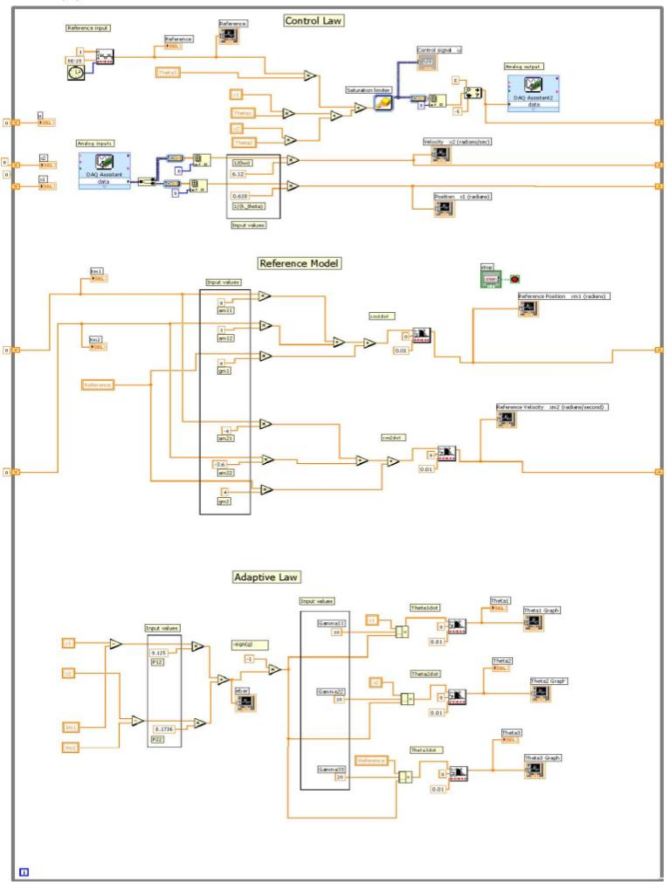

Figure 9: LabVIEW block diagram for angular displacement control

For this case, the corresponding results are shown in figure 10, 11, 12 and 13. So for this Case (a), after observing the 4 figures it can be concluded that when the adaptive gain matrix Γ is being increased, output converges with the reference output faster, but after a certain higher limit, the output signal becomes distorted. Because control signal reaches its saturation at that time. We know

$$\begin{bmatrix} \dot{\theta}_{x1} \\ \dot{\theta}_{x2} \\ \dot{\theta}_r \end{bmatrix} = -\Gamma \begin{bmatrix} x_{p1} \\ x_{p2} \\ r \end{bmatrix} \begin{bmatrix} e_1 & e_2 \end{bmatrix} \begin{bmatrix} P_{12} \\ P_{22} \end{bmatrix}$$

So, when a large Γ is applied, then $\dot{\theta}$ will be higher, that means change of controller gain will larger. So the control signal $u(t) = \theta^T(t)\ w(t)$ suddenly grows larger and the it reaches the saturation. So, once saturation occurs after that whatever the change happens in θ(t) and corresponding to u(t), the output will not change correspondingly and at the same times as output is not changing so it can be said that adaptive law and control law will not affect the plant output.

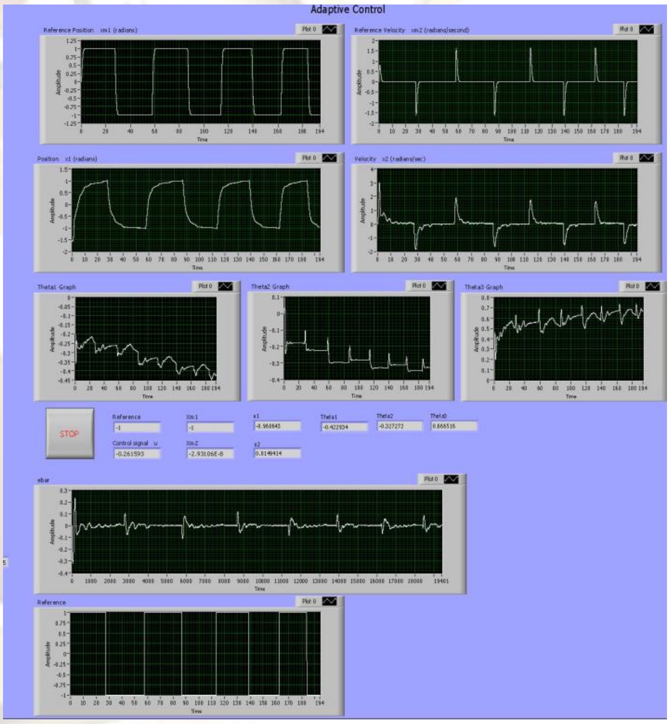

Figure 10: Front panel view for Γ_1

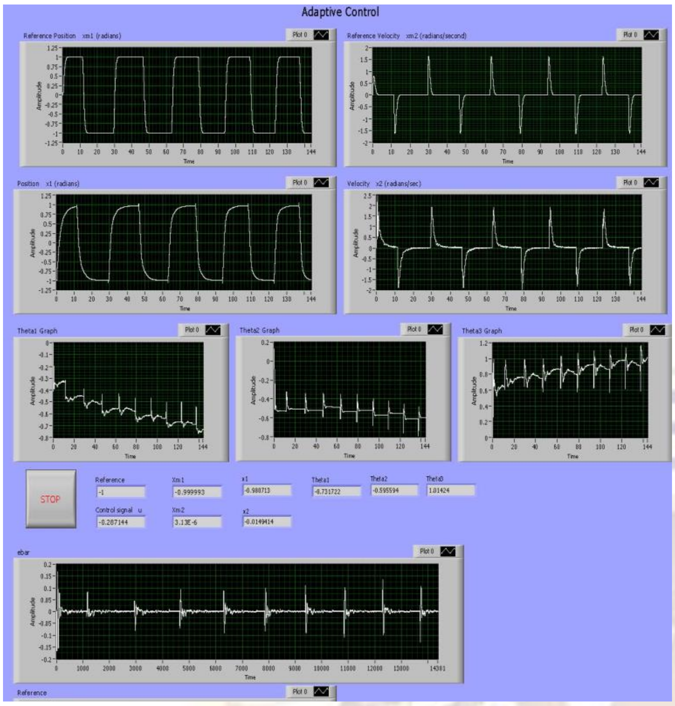

Figure 11: Front panel view for Γ_2

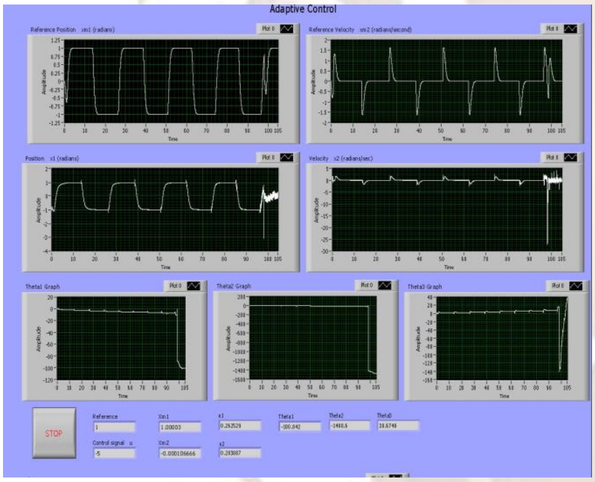

Figure 12: Front panel view for Γ_3

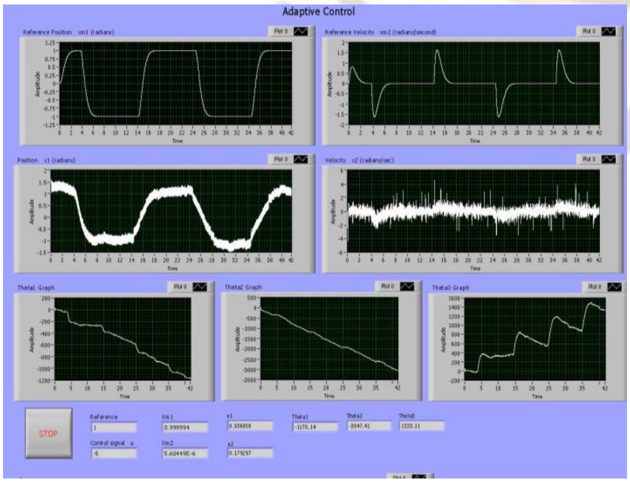

Figure 13: Front panel view for Γ_4

Case (b): Here the effect of changing the Q of Lyapunov equation (5) will be examined. The reference model is chosen as (7). In case (a), Q was chosen as $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, now

Choosing $Q_1=\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ and after solving Lyapunov equation, $P=\begin{bmatrix} 5.722 & 0.6250 \\ 0.6250 & 0.8681 \end{bmatrix}$

Here the adaptive gain Γ is selected as a 3-by-3 positive diagonal matrix as follows

$$\Gamma_1=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\Gamma_2=\begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$$\Gamma_2=\begin{bmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 200 \end{bmatrix}$$

For this case, the corresponding results are shown in figure 14, 15 and 16 respectively.

From the those figures it can be seen by comparing with the case (a). For choosing larger Q it gives larger P, resulting larger adaptive change so convergence will be faster than for the corresponding Γ of case (a).

In case (b), for Γ_3 saturation occurs but in case (a) it happens for Γ_4. Therefore, it results that changing Q to a larger value is equivalent to increasing the adaptive gain .It can be further investigated by choosing $Q_2=\begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix}$ and for this convergence is faster even for Γ_1 and saturation occurs for Γ_2. These results are shown in figure 17 and 18.

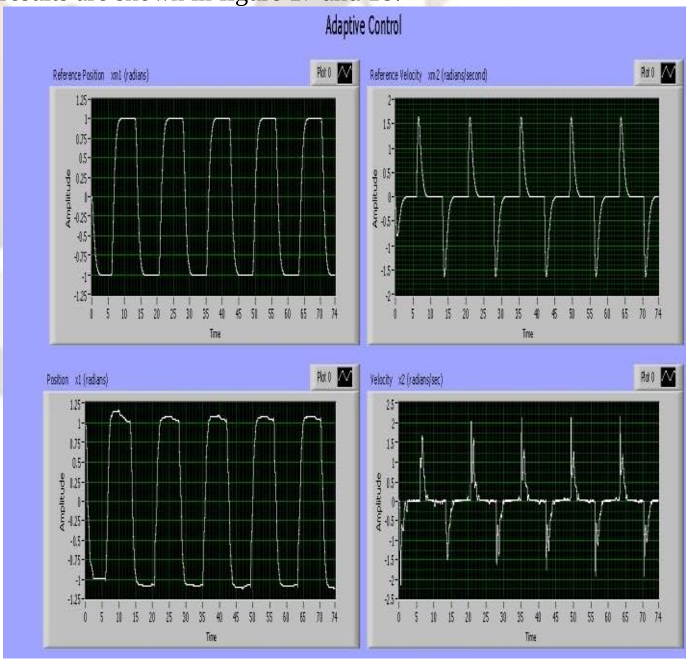

Figure 14 : Front panel view for Γ_1 and Q_1

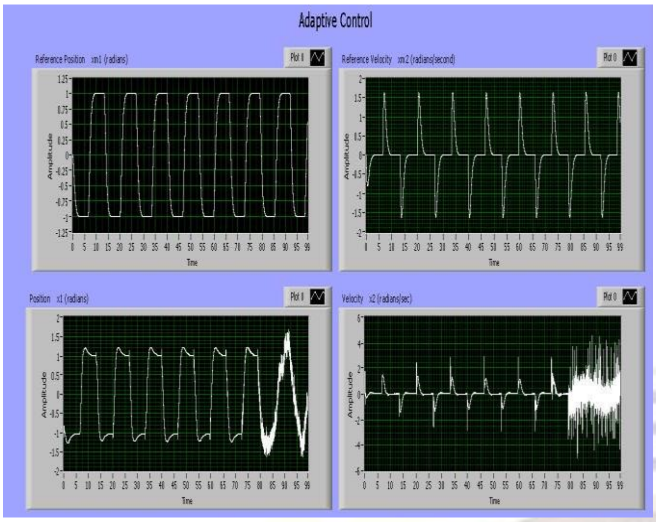

Figure 15 : Front panel view for Γ_2 and Q_1

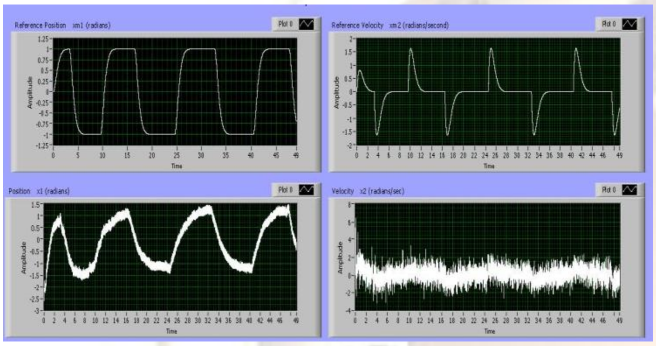

Figure 16 : Front panel view for Γ_3 and Q_1

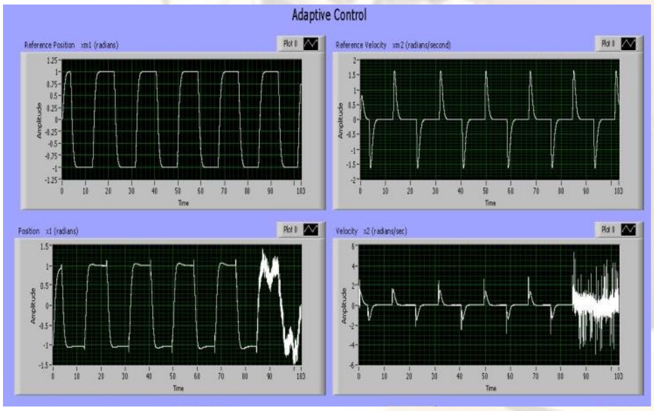

Figure 17: Front panel view for Γ_1 and Q_2

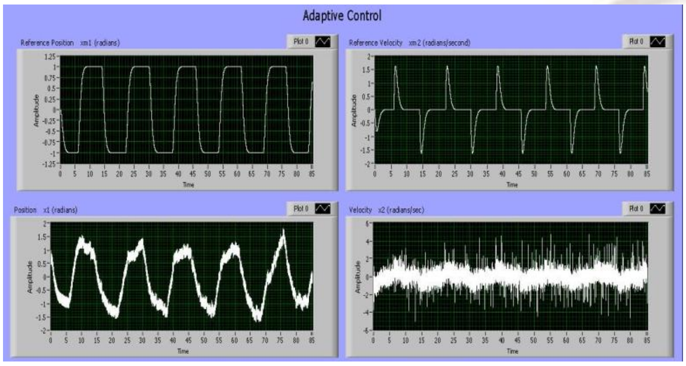

Figure 18: Front panel view for Γ_2 and Q_2

3.3 Effect of Disturbance

There must be some disturbance in any control system. So here, performance of the adaptive control system will be investigated when the disturbance being added. Depending on the disturbance type, the control system behaves differently because there is no addition control mechanism for disturbance rejection. The following figures 19,20 and 21 are showing the response corresponding to square wave disturbance with period less than the reference period, square wave disturbance with period larger than the reference period and unit step disturbance respectively. From the figures it can be concluded that for unit step disturbance as it is quite predictable, so adaptive controller can reject the effect of disturbance and quite correctly follow the reference output. But for square wave disturbance, as it is changing with some time period, so adaptive cannot reject the effect of disturbance and therefore output cannot follow the reference output.

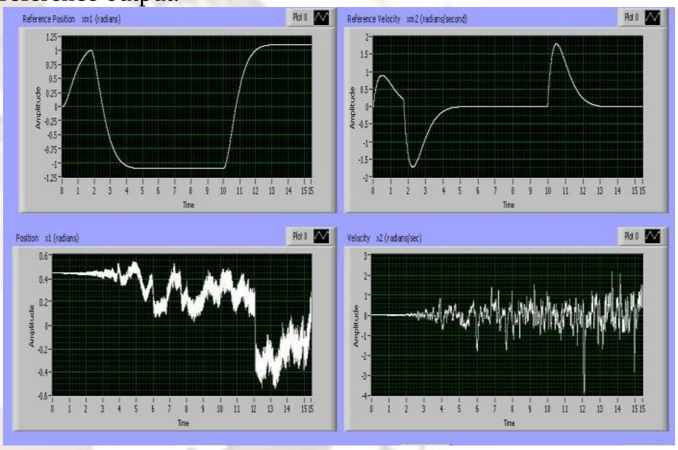

Figure 19: Square wave disturbance with period less than reference period

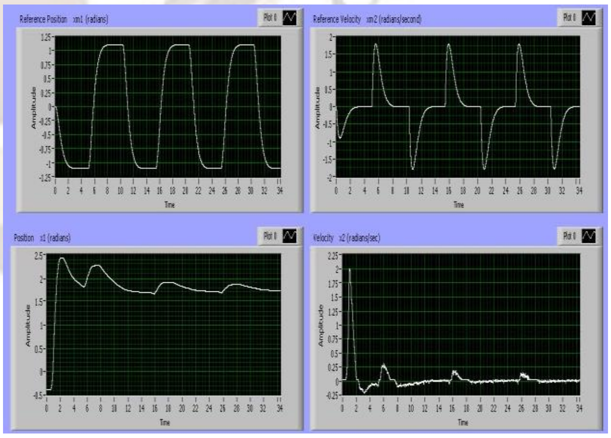

Figure 20: Square wave disturbance with period less than reference period

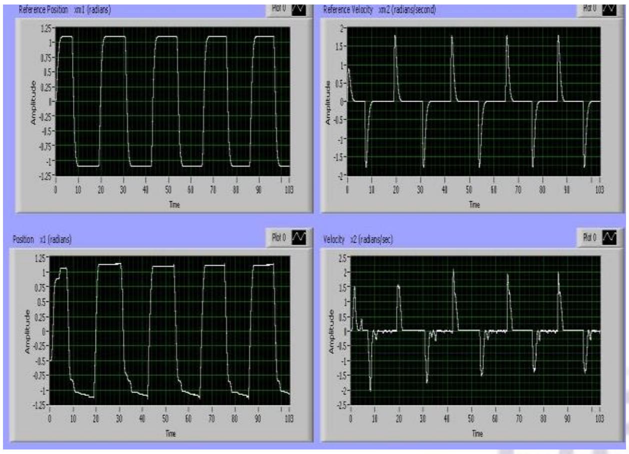

Figure 21: Unit step disturbance

3.4 Comparing Experimental results with Simulation results

For comparing the accuracy of the real time implementation of the adaptive controller, it can be compared with the corresponding simulation results. During the time of calibration we measured the time constant τ and static state gain K of the first order plant for output velocity. By using an integrator the output position/displacement can be obtained the Simulink model for this adaptive control system is shown in figure 22. The same reference model and Q as case (a) are being used for this simulation and Γ_2 is chosen as adaptive gain. The corresponding simulation result for position output is shown in figure 23. By comparing the simulation results with the experimental results, it can be concluded that real-time adaptive control system works quite fine. Although simulation result seems showing better performance, but there may be lot of issues, like external noise, sensor noise etc. which are excluded in the simulation design.

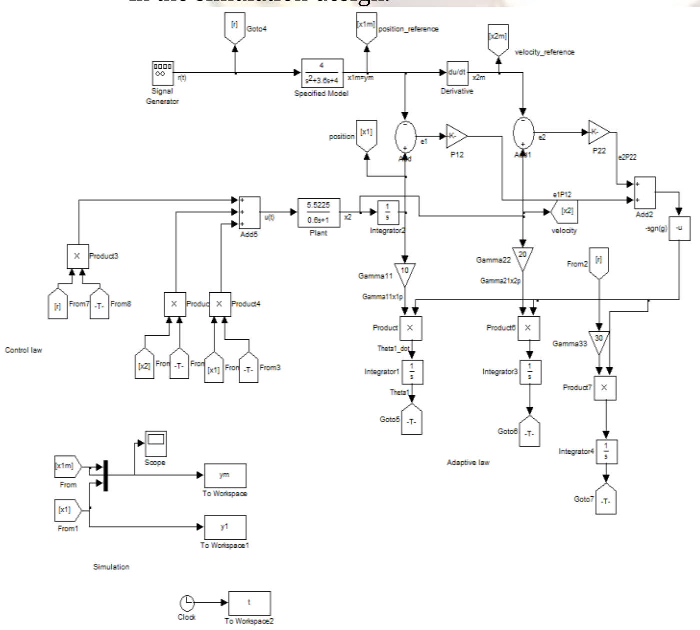

Figure 22: Simulink design for adaptive control system for angular displacement of DC motor

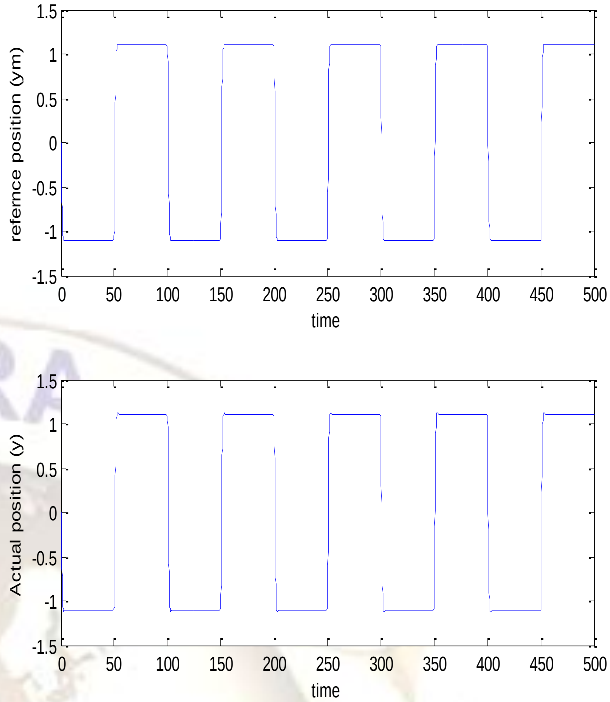

Figure 23: Output response for angular position control

IV. ADAPTIVE CONTROL OF ANGULAR VELOCITY

4.1 Adaptive control algorithm for angular velocity control

For full adaptive control design for angular velocity of D.C motor, the plant model has the following first order differential model

$\dot{y}(t)= a_p y(t) + k_p u(t), \quad a_p < 0$ (8)

With y(t)= angular velocity (plant output) and u(t)= control input

Reference first order model,

$\dot{y}_m(t) = a_m y_m(t) + k_m r(t), \quad a_m < 0$ (9)

With y_m = reference output r(t)=reference input

Control Law:

$u(t)= \theta(t)y(t)+k(t)r(t)$ (10)

Adaptive Law:

$\dot{\theta}(t)=-sgn(k_p)\gamma_1 e(t)y(t)=-\gamma_1 e(t)y(t)$ (11)

$\dot{k}(t)=-sgn(k_p)\gamma_2 e(t)r(t)= -\gamma_2 e(t)r(t)$ (12)

$e(t)=y(t)-y_m(t)$ (13)

with γ_1 *and* γ_2 are adaptive gains and by applying a unit step signal and observing the output, it can be easily concluded that, sgn(g)=1. The LabVIEW block diagram for angular displacement control are shown in figure 24.

4.2 Experimental results

Case (a): Here the effect of changing the adaptive law gains γ_1 *and* γ_2 will be examined. The reference model is chosen as,

$\dot{y}_m = -3.6y_m + 4r(t)$

γ_1 and γ_2 are selected as

$$\begin{bmatrix}\gamma_1\\ \gamma_2\end{bmatrix}=\begin{bmatrix}3\\ 4\end{bmatrix},\ \begin{bmatrix}\gamma_1\\ \gamma_2\end{bmatrix}=\begin{bmatrix}7\\ 10\end{bmatrix}$$

$$\begin{bmatrix}\gamma_1\\ \gamma_2\end{bmatrix}=\begin{bmatrix}20\\ 30\end{bmatrix},\ \begin{bmatrix}\gamma_1\\ \gamma_2\end{bmatrix}=\begin{bmatrix}100\\ 50\end{bmatrix}$$

For this case, the corresponding results are shown in figure 25, 26, 27 and 28 respectively.

From those figures, it can be concluded that when the adaptive law gains γ_1 and γ_2 are being increased, output converges with the reference output faster, but after a certain higher limit the output signal becomes distorted. Because control signal reaches its saturation at that time.

Besides, it can be seen that in comparison with the position output in part 2, the velocity output is quite noisy. It may be because the velocity sensor (tachometer) is more sensitive to noise than the position sensor (potentiometer).

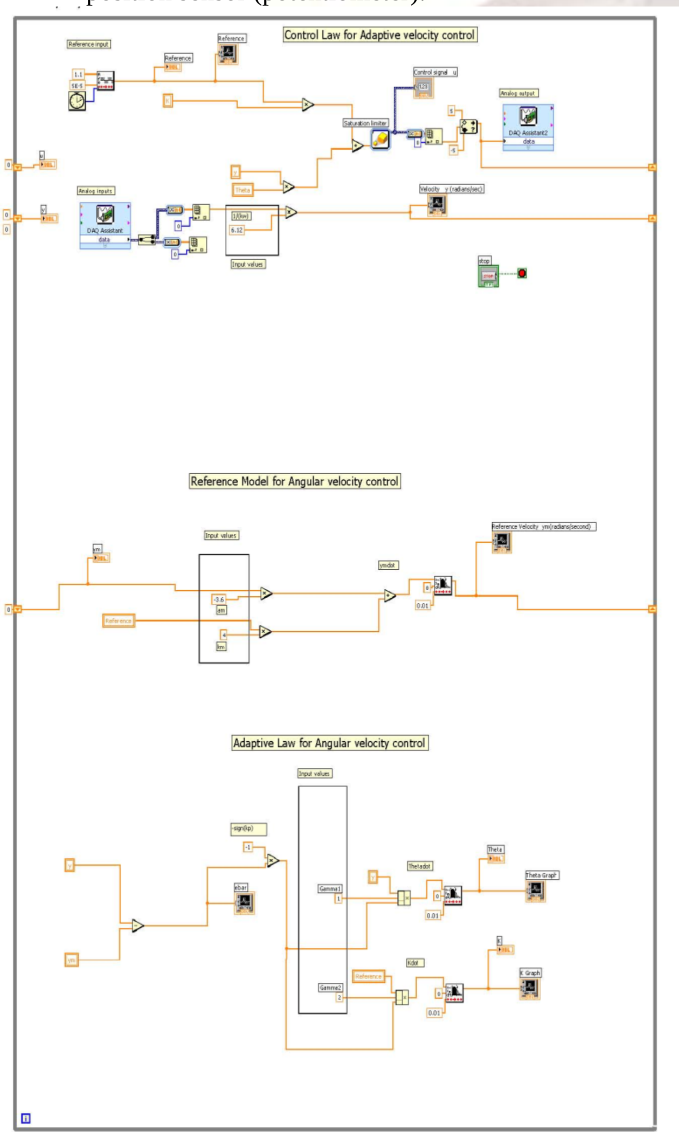

Figure 24: The block diagram view of the Adaptive control for angular velocity

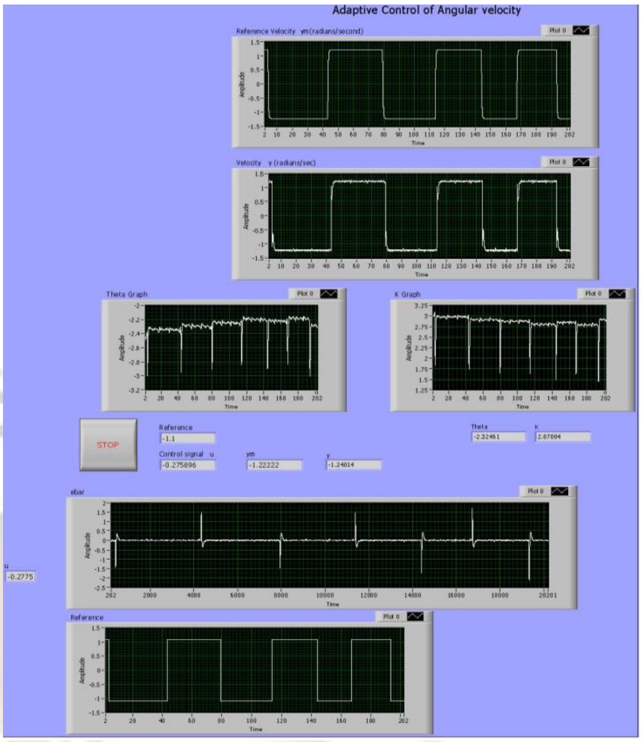

Figure 25: Front panel view of velocity control for $\gamma_1 = 3$ and $\gamma_2 = 4$

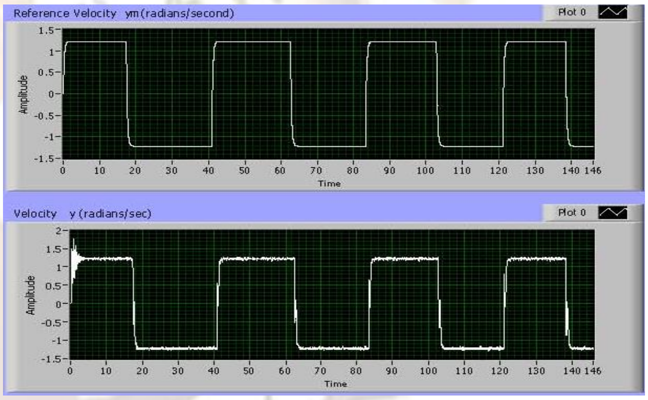

Figure 26: Front panel view of velocity control for $\gamma_1 = 7$ and $\gamma_2 = 10$

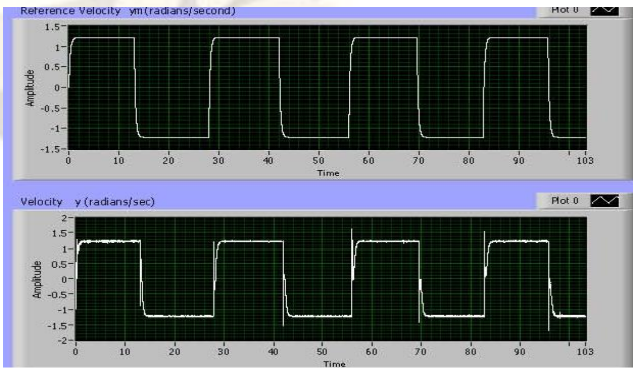

Figure 27: Front panel view of velocity control for $\gamma_1 = 20$ and $\gamma_2 = 30$

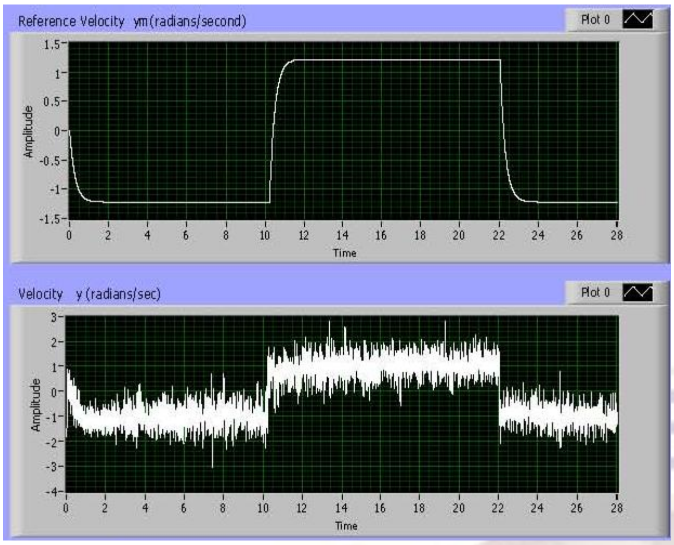

Figure 28: Front panel view of velocity control for γ_1 =100 and γ_2 =50

Case (b): Here we will use unit step signal as the reference signal and see the effect. Here the adaptive law gains γ_1 and γ_2 will also be changed as follows:

$$\begin{bmatrix}\gamma_1\\ \gamma_2\end{bmatrix}=\begin{bmatrix}1\\ 2\end{bmatrix},\ \begin{bmatrix}\gamma_1\\ \gamma_2\end{bmatrix}=\begin{bmatrix}10\\ 20\end{bmatrix},\ \begin{bmatrix}\gamma_1\\ \gamma_2\end{bmatrix}=\begin{bmatrix}100\\ 200\end{bmatrix}$$

Here it can be seen as like before, when γ_1 and γ_2 are being increased, the convergence becomes faster and reaches the saturation limit for larger values. It can be clearly seen the convergence timing by observing the step response rather than observing the response for square wave reference signal. The corresponding results are shown in figure 29, 30 and 31 respectively for changing values of γ_1 and γ_2.

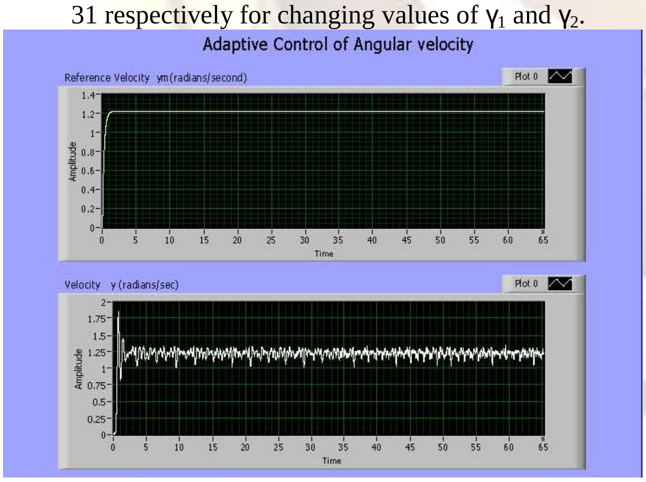

Figure 29: Front panel view of velocity control for γ_1 =1 and γ_2 =2

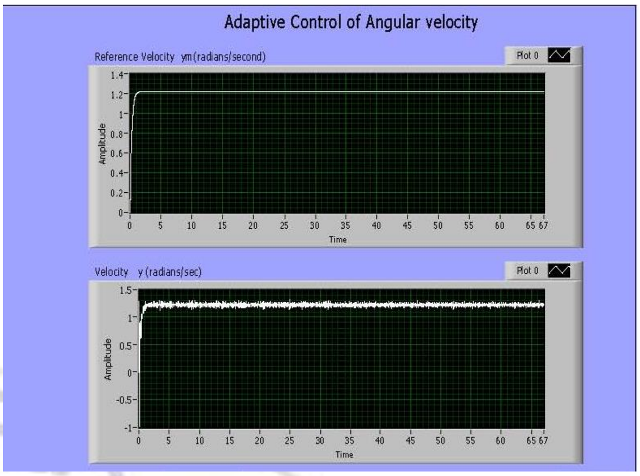

Figure 30: Front panel view of velocity control for γ_1 =10 and γ_2 =20

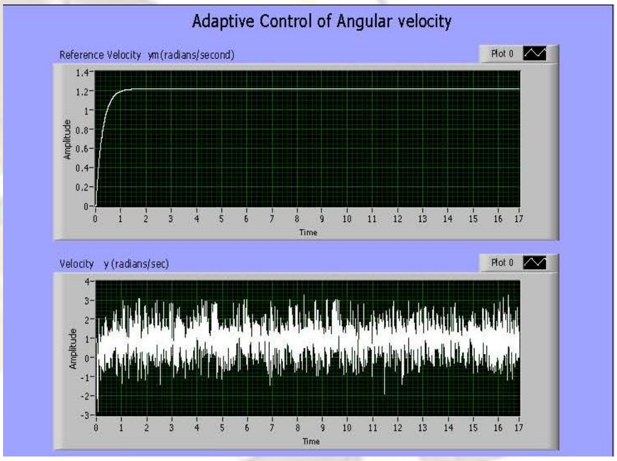

Figure 31: Front panel view of velocity control for γ_1 =100 and γ_2 =20

4.3 Comparing Experimental results with Simulation results

For comparing the accuracy of the real time implementation of the adaptive controller, it can be compared with the corresponding simulation results. During the time of calibration we measured the time constant τ and static state gain K of the first order plant for output velocity. The Simulink model for this adaptive control system is shown in figure 29. The same reference model as case (a) is being used for this simulation and γ_1 =3 and γ_2=4 are chosen as adaptive gains. The corresponding simulation result for position output is shown in figure 30. By comparing the simulation results with the experimental results, it can be concluded that real-time adaptive control system works quite fine. Although simulation result seems showing better performance, but there may be lot of issues, like external noise, sensor noise etc. which are excluded in the simulation design. Besides, the linear approximate model of the DC motor has been used for the simulation which may be the approximation of the real plant model. So the simulation results quite differ from the real-time adaptive control system.

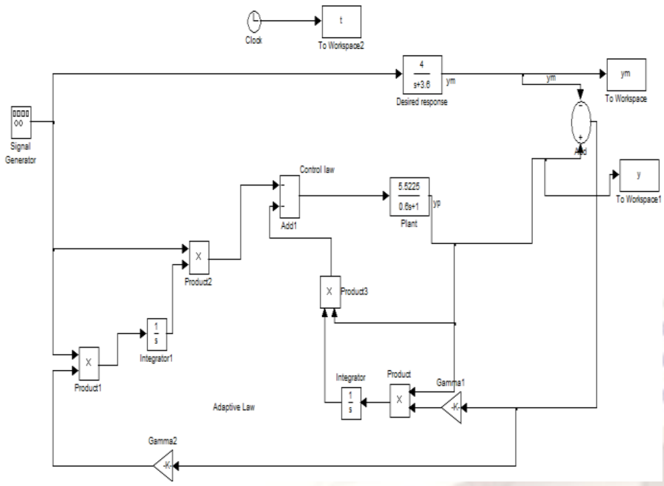

Figure 29: Simulink design for adaptive control system for angular displacement of DC motor

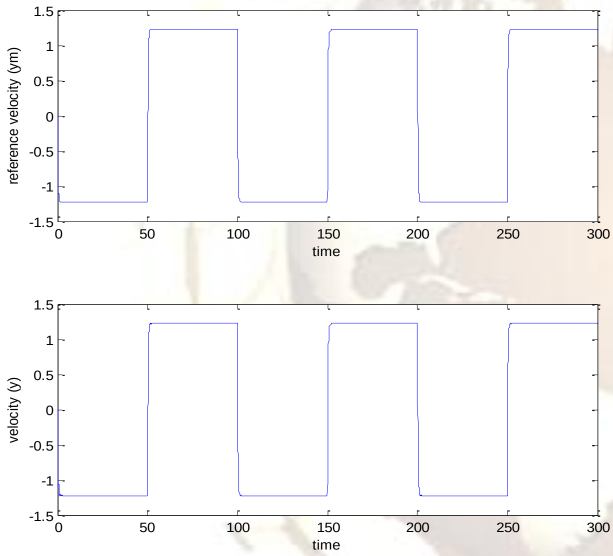

Figure 30: Output response for angular position control

V. CONCLUSION

In this paper, our target was to analyse the methods and results of adaptive control system implemented on D.C. motor setup. There some steps to design the system. First before designing the control system, we have to be very well known about the plant model working principle and about the sensors. Then calibration of sensors is the most important initial step of system design. After that a computer aided design has been done for implementing the control system. The effects of changing different parameters have been observed. For angular position control of D.C. motor the plant model is second order system and angular velocity control of D.C. motor the plant model is a first order system and corresponding adaptive systems have been designed. Finally as we have obtained the time constant and static gain of the plant, so the results extracted from the hardware setup have been compared with the simulation results by building Simulink model in matlab.

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