Attempts to Construct a Time Operator

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ABSTRACT

Time enters the Schrodinger equation as an external parameter, and not a dynamical variable. It is not a standard quantum mechanical observable.This survey explores various attempts made in order to treat time as a dynamical variable(observable) and hence measure it.

I. Introduction

The role of time is a source of confusion and controversy in quantum mechanics [1]. In the Schrodinger equation time represents a classical external parameter, not a dynamical variable. The time measured in experiments, however, does not correspond to an external parameter, it is actually an intrinsic property of the system under consideration, which represents the duration of a physical process; the life time of unstable particles is a well-known example. Quantum mechanics was initially formulated as a theory of quantum micro-systems interacting with classical macro systems [2]. Quantum mechanics allows the calculation of dynamical variables ofsystems at specified instants in time using the Schrodinger equation [3]. The theory also deals with probability distributions of measurable quantities at definite instants in time [4]. The time of an event does not correspond to a standard observable in quantum mechanics [5].

Asking the question of when a given situation occurs, time is no longer an external parameter. Time, in such a situation, becomes dynamical. However, such a time observable does not have the properties of a "standard" quantum mechanical observable. This research is dedicated to exploring various attempts made in order to treat time as a dynamical variable (observable). All attempts use essentially one of two approaches, namely those of direct and indirect measurement of time. Direct approaches use theoretical toy model experiments while indirect approaches are of mathematical nature.

To determine the time of arrival or the tunneling time, the measurement of the required quantities must always be done, directly or indirectly. The notion of measurement emerges from interpretations of quantum mechanics, however the time problem arises in all of them. The interaction of a quantum Microsystems with a classical macro system is described in terms of quantum measurements [2]. As time is treated as an external parameter in standard quantum theory, quantum observation theory talks about observations made at given instants in time [3]. The system in standard quantum theory interacts with a measuring device through the timedependent interaction Hamiltonian. Quantum mechanics is actually designed to answer the question "where is a particle at time t?" In standard quantum mechanics, The probability corresponds to a measurement result of a particle being at a given location at one specific time. The above mentioned micro-system is taken to be in a superposition of states of its variables. Suppose the macro-system interacts with one of the micro-system's variables, then the macro-system only sees one of the many possible values of the variable [2]. The interaction itself projects the state of the micro-system into a state with the given value. In terms of wave-functions, the interaction (act of measurement) causes the wave-function of the Microsystems (a superposition of states) to "collapse" into one state with a specific value (eigenvalue). Dirac mentioned that the superposition is one of two most important concepts in quantum mechanics; the other one is Schrodinger equation [6].

Even though several alternative interpretations have been devised (Bohm, many-worlds, etc.), they all have one problem in common: how can the exact time at which a measurement occurs be determined? Rovelli [2] illustrates how the problem of time arises in each interpretation. If a system is viewed as having a wave-function which collapses during a measurement, is the collapse immediate? If a system is viewed in terms of values of its dynamical variables which become definite when observed, how to determine exactly when this occurs in an experiment? If a system's wave-function is taken a branch, when does this occur? If a wave-function does not branch and the observer selects one of its components and sticks with the choice, when does the selection occur? If there exist probabilities for sequences of events to happen, when does such an event occur? The above questions indicate the universality and challenging concept of time. We review some attempts to set up the time operators. Last but not least the, problem of time in quantum gravity is outlined, where time, if it is a fundamental variable, must also be a dynamical variable. Quantum gravity has the interesting feature that the philosophical question of what time actually is raised. If one would know what time is fundamentally, then perhaps the problems encounterin determining time in quantum mechanics could be solved, as one would then know what one is actually looking for. In what follow, we only highlight the subjects, and to understand more each part needs to be explored in details. Another approach to the time problem is the

decoherent histories approach to quantum mechanics [7,8, 9, 10]. This formalism makes use of the fact that what one considers to be a closed quantum, is never completely closed, as there always
the physical quantity corresponding to the time operator.
of canonically conjugate variables, the second variable experiences a shift in value. This shift cannot be calculated exactly without interfering with the measurement of the first variable

II. The Relation of the Uncertainty Principle to a Time Operator

Bohr also realized that the two uncertainty principles (9) and (7) can be interpreted in two different ways the first is as limitations on the accuracy of a measurement and the second is as statistical laws referring to a large sequence of The difficulty in giving meaning to the relation (7) is due to the quantity Δt. In the way it is interpreted above, the uncertainty relation (7) implies the existence of a self-adjointoperator; canonically conjugate to the Hamiltonian$\hat{H}$, which itself is self-adjoint. Ifthis time operator $\hat{T}$ exists, then the quantity Δt can be interpreted in the same way as Δx' or Δp_x ,and the uncertainty principle can be applied to the physical observableCorresponding to T. To obtain the uncertainty relation for energy and time, the commutator of the Hamiltonian and the time operator is assumed to be of the form:

$$[\hat{H},\hat{T}]=i\hbar \tag{11}$$

The form of (11) is such that $\hat{T}$ and $\hat{H}$ are canonically conjugate to each other. It also implies that both operators have a continuous spectrum. This in turn means that neither of the two can be a Hamiltonian, as such an operator is defined to have a semi- bounded spectrum. From this line of reasoning the supposed time operator $\hat{T}$ cannot exist. This problem is encountered when one uses (11) to derive the uncertainty relation (7) in the same way as (9) is derived from $[\hat{x},\hat{p}] = i\hbar$ [14].

III. Attempts to construct a Time Operator

Standard quantum theory, as proposed by Pauli [21], requires that measurable. Quantities (observables) are represented by self-adjoint operators, which act on the Hilbert space of physical states [4]. The probability distribution of the measurement outcomes of an observable are obtained as "an orthogonal spectral decomposition of the corresponding self-adjoint operator" [4]. The indirect measurement of time basically is the quest of finding a self-adjoint operator whose Eigen states are orthogonal. As the time operator is one of the canonically conjugate pair of time and energy, the time operator must be defined in such a way as to preserve the semi bounded spectrum of the Hamiltonian. Pauli pointed out [2 L] that the existence of a self-adjoint time operator is incompatible with the semi-bounded character of the Hamiltonian spectrum. By using a different argument based on the time-translation property of the arrival time concept, Allclock has found the same negative conclusion [22-24]. The negative conclusion can also be traced back to the semi-infinite nature of the Hamiltonian spectrum.
Kijowski [25] tried different approaches to find a time operator. He chose to (interpret the uncertainty relation (11) in a statistical way.
Grot, Rovelli and Tate [26] construct a time of arrival operator as the solution to the problem of calculating the probability for the TOA of a particle at a given point. They argue, using the principle of superposition, that a time operator T can be defined, whose probability density can be calculated from the spectral decomposition of the wave-function l. ψ (x) into Eigen states of $\hat{T}$ (in the usual way). They found an uncertainty relation which approaches (11) to arbitrary accuracy. Oppenheim, Reznik and Unruh [27] follow the method by Grot at al. They used coherent states to create a positive operator valued measure (POVM). The standard method to find an operator is by using the correspondence principle, which states that the corresponding classical equations are quantized using specific quantization rules. Taking the Hamiltonian of a classical system H(p,q) where p and q are canonical variables (H,T), where H is the Hamiltonian and T is its conjugate variable. These variable satisfy Hamilton's equation:

$$\frac{dT}{dt} = \{H,T\} = 1 \tag{12}$$

where T is the interval of time and the curly brackets denote a Poisson bracket. This relation can be translated to quantum mechanics through canonical quantization. This is a procedure where classical expressions remain valid in the quantum picture by effectively substituting Poisson brackets by commutatores:

$$\{H,T\} = \frac{1}{i\hbar}[\hat{H},\hat{T}] \tag{13}$$

In the Heisenberg picture Hand T are hence interpreted as self-adjoint operators. Further it seems natural to require that the time operator satisfies an eigenvalue equation in the usual way:

$$\hat{T}(t)|t_A\rangle = t_A|t_A\rangle \tag{14}$$

In all of physic, except in General Relativity, Physical systems are supposed to be situated in a three-dimensional Euclidean space. The points of this space will be given by Cartesian coordinated **r** = (x,y,z). Together with the time parameter t they from the coordinates of a continuous space - time background. The (3 + 1) dimensional space-time must be distinguished from the 2N- dimensional phase space of the space of the system, and space-time coordinates (r,t) must be distinguished from the dynamical variables ($q_k P_k$) characterizing material systems in space-time.

A point particle is a material system having a mass, a velocity, an acceleration, while r is the coordinate of a fixed point of empty space. It is assumed that three- Dimensional space is isotropic (rotation symmetric) and homogeneous (translation symmetric) and that there is translation symmetry in time. In special relativity the space-time symmetry is enlarged by Lorentz transformations which mix **x** and t, transforming them as the components of a four-vector.

The generators of translation in spaceand time are the total momentum P and the total energy H, respectively. The generator of space rotations is the total angular momentum **J**.

It is worth noting that the universal time coordinate t should not be mixed with dynamical position variables. The important question to ask is: Do physical systems exist that have a dynamical variable which resembles the time coordinate t in the same -way as the position variable q of a point particle resembles the space coordinate **x** ? The answer is yes! Such systems are clocks. A clock stands, ideally, in the same simple relation to the universal time coordinate t as a point particle stands to the universal space coordinate **x**. We may generally define an ideal clock as a physical system which has a dynamical variable which behaves under time translations in the same way as the time coordinate t. Such a variable, which we shall call a "clockvariable" or, more generally, a "time-variable", may be a pointer position or an angle or even a momentum. Just as a position-variable indicates the position of a system in space, a clock-variable indicates the 'position' of a system in time t. In quantum mechanics the situation is essentially not different. The theory supposes a fixed, unquantized space-time background, the points of which are given by c-number coordinates **x**, t. The space time symmetry transformations are expressed in terms of these coordinates.

Dynamical variables of physical systems, on the other hand, are quantized: they are replaced by self-adjoint operators on Hilbert space. All formulas of the preceding section remain valid if the poisson-brakets are replaced by commutators according to $\{,\} \to (i\hbar)^{-1}[,]$.

So, the idea, that t can be seen as the canonical variable conjugate to the Hamiltonian, leads one to expect t to obey the canonical commutation relation $[t,H]= i\hbar$. But iftis the universal time operator it should have continuous eigenvalues running from $-\infty$ to $+\infty$ and, from this, the same would follow for the eigenvalues of any H. But we know that discrete eigenvalues of H may occur. From this Pauli concluded [21]: ... that the introduction of an operator t is basically forbidden and the time must necessary be considered as an ordinary number ("c-number") ... "Thus, the 'unsolvable' problem of time in quantum mechanics has arisen. Note that it is crucial for this argument that t is supposed to be a universal operator, valid for all systems: according to Pauli the introduction of such an operator is basically forbidden because some systems have discrete energy eigenvalues. From our previous discussion it should be clear that the universal time coordinate t is the partner of the space coordinates x. Neither the space coordinates nor is the time coordinate quantized in standard quantum mechanics. So, the above problem simply doesn't exist! If one is to look for a time operator in quantum mechanics one should not try to quantize the universal time coordinate but consider time-like (in the literal sense) dynamical variable of specific physical system, i.e. clocks. Since a clockvariable is an ordinary dynamical variable quantization should not, in principle, be especially problematic. One must, however, be prepared to encounter the well-known quantum effects mentioned above: a dynamical system may have a continuous timevariable, or a discrete one or no time-variable at all. Recently, some efforts have. been performed to overcome Pauli's argument [28]. The proposed time operator is canonically conjugate to $i\hbar\partial$ rather than to H, therefore Pauli's therefore Pauli's theorem no longer applies. It is argued that "the reasons for choosing time as a parameter lie not so much in ontology as in methodology and epistemology. The time operator idea needs to be more explored in an accurate way.

VI.Conclusion and further comments

This survey explores various ways of defining time in standard quantum mechanics and some different ways of measuring it. The approaches of measuring time yield a whole spectrum of results along with a range with a range of difficulties encountered. All methods yield results which have a strict limit on their accuracy and. Generality. This reflects the quantum nature of the problem.

The main difficulty in defining a quantum time operator lies in non-existence, in general, of a self-adjoint operator conjugate to the Hamiltonian, a problem which can be traced back to the semi-bounded nature of the energy spectrum. In turn, the lack of a self-adjoint time operator implies the lack of a properly and unambiguously defined probability distribution of arrival time. There are two possibilities to overcome the problem. If one decides that any proper time operator must be strictly conjugate to the Hamiltonian, then one has to perform the search for a self-adjoint operator. If, on the contrary, one imposes self-adjoint property as a desirable requirement for any observable, then one necessarily has to give up the requirement that such an operator be conjugate to the Hamiltonian. The two main equations of motion, the Schrodinger and wheeler- Dewitt equation reflect two different presupposed natures of time: in the schrodinger equation, time corresponds to an external parameter and in the wheeler-Dewitt equation thereis no time. This research explores the concept of trying to tum a time parameter into an observable, a dynamical variable. Why was time in quantum mechanics represented by a parameter in the first place? A possible answer is that it is due to our perception. It is meaningful, for us, to talk about events happening at a certain time. This lets us put events into a chronological order in our minds. We do not think about an event happening to us. Another question is, why does one want time to be an observable? One major reason is our notion of change: we seem to perceive that time changes. Another motivation for the study of time in quantum mechanics is the problem of time in quantum gravity. Quantum cosmology represents an analogy to closed quantum systems, as both cosmology and closed quantum systems are describing the same type of situation, the difference being the size scale. . Saunders states: "quantum cosmology is the most clear-cut and important failing of the Copenhagen interpretation" [31]. Perhaps the lack of understanding of time in quantum gravity is due to a fundamental reason, based on the two quantum gravity components: quantum mechanics and general relativity. The problem does not lie in general relativity however, so it must be rooted in the formulation of quantum mechanics.

Quantum theory of measurement is based on measurements occurring at given instants of time. A measurement corresponds to a classical event. Dirac said "the aim of quantum mechanics was to account for the observables: behavior in the simplest possible ways" [6]. Kant [32] held Newtonian absolute space and space- time for an "idea of reason". Saunders states" In particular, we need a global time coordinate' which enters in to the fundamental equations; it is no good if this involves ad hoc or ill-defined approximations, available at only certain length scales or cosmological epochs" [33]. His idea of a universal definition of time sounds very appealing. Does this universal concept of time require the reformulation of quantum mechanics? Tunneling time might .also be a candidate to shed some light on to the mystery of time. Quantum mechanical tunneling is "one of the most mysterious phenomena of quantum mechapics" and at the same time it is one of the basic and important processes in Nature, party responsible for our existence [34]. The question of the duration of a tunneling process is an open problem. Experiments to record the tunneling time were motivated by the many different theories trying to describe this phenomenon. Questions arise such as, "is tunneling instantaneous?","is it subluminal or superluminal (faster than the speed of light)?"Chiao published a paper with experimental evidence that tunneling is superluminal [34]. If this is true, what implications does superluminal tunneling have on our understanding of the nature of time? What does it mean to say that something happens faster than instantaneously?
Does time undergo a change in nature when it "enters" a classical forbidden region? If so, what is it and what does it change to?
In quantum gravity, the evolution of the gravitational field does not correspond to evolution in physical time. The internal time on a manifold i~ not ':l:~ absolute quantity. Barbour [35] claims that an instant in time corresponds to a configuration and Deutsch, in his interpretation of quantum mechanics, claims that a change in time corresponds to a change in his interpretation of quantum mechanics, claims that a change in time corresponds to a change in the number of Deutsch worlds [37]. Is it possible that the notion of absolute time be a hint towards timelessness? If time does not exist then the various different formulations of the nature of time only appear through our perception and we cannot follow these back to a universal truth. Perhaps there does exist a universal concept of time, which is far too abstract to grasp. Whatever time may be, the time discussed in this overview raise various questions, which perhaps are trying to point us into .a certain direction. Trying to answer questions about the concepts of the time of arrival, the time-energy uncertainty relation, tunneling time and time in quantum gravity show us that perhaps nothing is more important than to first of all understand the basic building block-time-without which no structure can be perfectly built. The problem of time still stands to be resolved, the quest for this research still continues.

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