

Shaft Design under Fatigue Loading By Using Modified Goodman Method

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ABSTRACT

In this paper, shaft employed in an Inertia dynamometer rotated at 1000rpm is studied. Considering the system, forces, torque acting on a shaft is used to calculate the stresses induced. Stress analysis also carried out by using FEA and the results are compared with the calculated values. Shaft is having varying cross sections due to this stress concentration is occurred at the stepped, keyways, shoulders, sharp corners etc. caused fatigue failure of shaft. So, calculated stress concentration factor from which fatigue stress concentration factor is calculated. Endurance limit using Modified Goodman Method, fatigue factor of safety and theoretical number cycles sustained by the shaft before failure is estimated and compared results with FEA.

Keywords – Dynamometer, Factor of Safety, Fatigue, FEA Goodman, Stress Concentration.

I. Introduction

A shaft is a rotating member, usually of circular cross-section for transmitting power. It is supported by bearings and supports two flywheels. It is subjected to torsion, and bending in combination. Generally shafts are not of uniform diameter but are stepped, keyways, sharp corners etc. The stress on the shaft at a particular point varies with rotation of shaft there by introducing fatigue. Even a perfect component when repeatedly subjected to loads of sufficient magnitude, will eventually propagate a fatigue crack in some highly stressed region, normally at the surface, until final fracture occurs. Extensive work has been carried out by failure analysis research community investigating the nature of fatigue failures using analytical, FEA and experimental methods [1]. According to Osgood all machine and structural designs are problems in fatigue [2]. Failure of an elevator shaft due torsion-bending fatigue was given in [3]. The failure of a shaft due improper fastening of support was explained in [4]. Accurate stress concentration factors for shoulder fillet in round and flat bars for different loading conditions are given in [5]. Failure analysis of a locomotive turbocharger main-shaft and rear axle of an automobile was discussed in [6,7]. Celealettin Karaagac and M.Evren Toygar considered an agitator shaft with a circumferential groove for

which the fatigue life has been estimated [8]. Michele Zappalorto, Filippo Berto and Paolo Lazzarin predicted the notch stress concentration factors of round bars under torsion [9].

II. Theory

Inertia dynamometer is operated to generate necessary inertia/torque is used to check performance characteristics of engine. Here, shaft is simply supported in bearing at two ends with subjected load of two flywheels fixed & removable and self weight of shaft as shown in Fig.1 and the specifications in Table1.

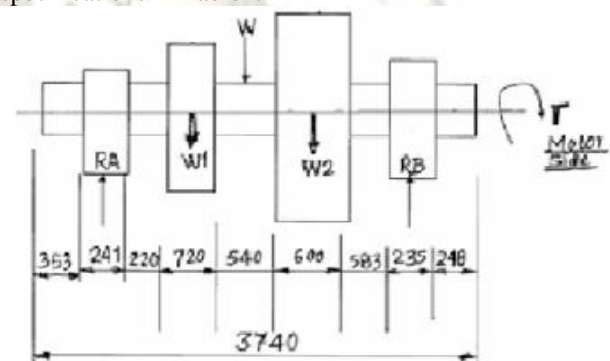


Figure.1 Free body diagram of inertia dynamometer

Table.1 Specifications of Inertia Dynamometer

R_A	Reaction at Bearing on other side point A. (kg)	5013
R_B	Reaction at Bearing on the Motor Side point B.(kg)	6427
W	Weight of Shaft (kg)	3100
W_1	Weight of Fixed Flywheel(kg)	1340
W_2	Weight of Removable Flywheel(kg)	7000
T_{max}	Maximum Torque on Shaft (kg-	1800
T_{min}	Minimum Torque on Shaft (kg-	449
N	Rotational Speed(rpm)	1000
d	Minimum shaft diameter (mm)	330

2.1 Selection and Use of Failure Theory

Here select the Distortion energy theory for fatigue failure analysis to find maximum stress values because there is combined loading of bending and torsion. Distortion energy theory is used when the factor of safety is to be held in close limits and the

cause of failure of the component is being investigated. This theory predicts failure most accurately for ductile material. But design calculations involved in this theory are slightly complicated as compared with other theories of failure. The below equation (1-2) is for design of shaft for fluctuating load (Case-II reversed bending with reversed torsion) calculated von-Mises stress.

$$\sigma_{eq} = \sqrt{(\sigma_{bmax})^2 + 3(\tau_{max})^2} \quad (1)$$

From, Design of safety,

$$\sigma_{eq} \leq \frac{S_{yt}}{f_s} \quad (2)$$

Where,

σ_{eq} = Equivalent stress/von Mises stress(MPa)

σ_{bmax} = Maximum bending stresses(MPa)

τ_{max} = Mximum shear stresses(MPa)

S_{yt} = Yield strength (MPa)

f_s = factor of Safety.

2.2 Modified Goodman Method

The modifying endurance limit of the shaft is found using Modified Goodman equation by taking mean stress (σ_m) in to account. Modified Goodman curve is a plot between the mean stress along X-axis and amplitude stress along Y-axis as shown in fig.2 and The equation (3) is represented as below

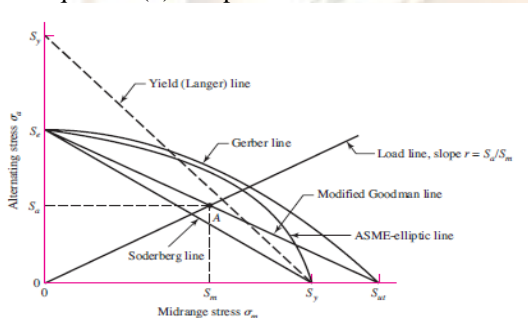


Figure.2 Fatigue diagram showing various criteria of failure.

By Modified Goodman Equation,

$$\frac{S_a}{S_e} + \frac{S_m}{S_{yt}} = 1 \quad (3)$$

Factor of safety is calculated for High cycle Fatigue applications from above eqn (3).

We know that;

$$\begin{aligned} &\text{fatigue failure safety is,} \\ &\text{factor of safety} < 1 \text{ Design is fail} \\ &\text{factor of safety} > 1 \text{ Design is safe.} \end{aligned} \quad (4)$$

where,

S_e = Modified /Corrected Endurance limit (MPa)

S_{yt} = Yield tensile strength (MPa)

2.3 Fluctuating Stresses-Design for Finite and Infinite Life

There are two types of problems in fatigue design- (i) components subjected to completely reversed stresses, and (ii) components subjected to

fluctuating stresses. the mean stress is zero in case of completely reversed stresses. But, in case of fluctuating stresses, there is always a mean stress and the stresses can be purely tensile, purely compressive or mixed depending upon the magnitude of the mean stress. Such a problems are solved by Modified Goodman diagram, which will be discussed in section 2.2.The design problems for fluctuating stresses are further divided into two groups-(i) design for infinite life, and (ii) design for finite life.

2.3.1 Design for Infinite Life

When the component is to be designed for infinite life, the endurance limit becomes the criteria of failure. The amplitude stress induced in such components should be lower than the endurance limit in order to withstand the infinite number of cycles. Such a components are designed with the help of following equation (5):

$$\sigma_a = \frac{S_e}{(f_s)} \quad (5)$$

The S-N curve as shown in fig.3.The curve is valid for steels

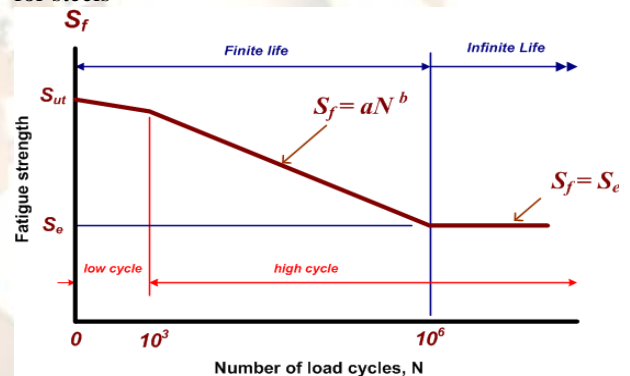


Figure.3 S-N curve for steel.

2.4 Material Properties

The CAD model of the shaft with its components is shown in fig.4.The material of the shaft under consideration is Fe410W-A (EN10025)-IS2062.Mechanical properties are shown in Table.2. The Material S-N curve shown in the fig.5. From material curve it is seen that at certain stress range the curve becomes straight, this means that material is having infinite life below that stress. Also, validate design stresses to compare alternating / mean stress value $[\log_{10}(\sigma_a)] = 7.94 Pa$ of material through graph with induced stresses of inertia dynamometer. The stresses should be within limit than stress value of material by this it is concluding in further section whether induced / working stresses are safe or not.

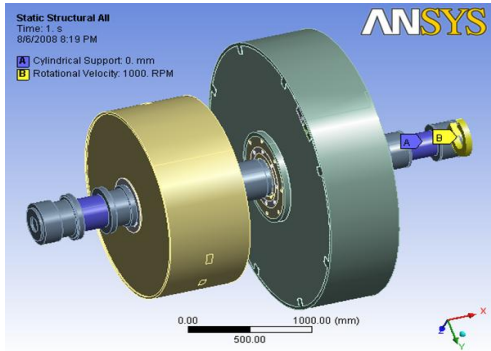


Figure.4 CAD model of inertia dynamometer

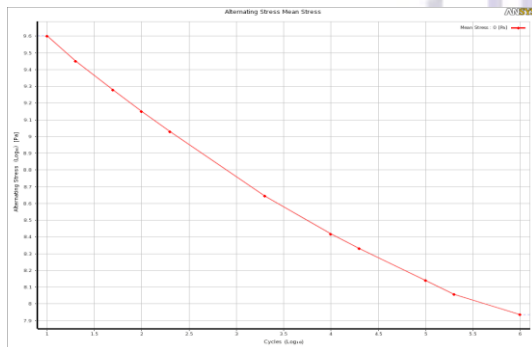


Figure.5 Material Curve for EN10025
Table.2 Material Properties of Shaft

Physical Properties	Values
Ultimate Strength (σ_{ut})	410 Mpa (N/mm^2)
Yield Strength (S_{Yt})	230 Mpa (N/mm^2)
Young's Modulus (E)	$2.1 \times 10^5 N/mm^2$
Poisson's Ratio (ν)	0.3
Density (ρ)	$7850 kg/m^3$

III. 3. Fatigue Analysis by Analytical Approach

In this section, calculated the von-Mises (equivalent) stress (σ_{eq}), factor of safety (f_s), Number of cycles(N) resp.

Initially, calculated Maximum and minimum bending moments at different points,

Maximum Bending Moment at D = $57.84 \times 10^6 N\text{-mm}$

Minimum Bending Moment at E = $35.11 \times 10^6 N\text{-mm}$.

$$(M_b)_{Max} = 57.84 \times 10^6 N\text{-mm.}$$

$$(M_b)_{Min} = 35.11 \times 10^6 N\text{-mm.}$$

Then,

I.To find Mean & Amplitude Stresses.

$$(M_b)_m = \frac{1}{2}((M_b)_{max} + (M_b)_{min})$$

(6)

$$(M_b)_m = 46.475 \times 10^6 N\text{-mm.}$$

$$(M_b)_a = \frac{1}{2}((M_b)_{max} - (M_b)_{min})$$

(7)

$$(M_b)_a = 22.73 \times 10^6 N\text{-mm.}$$

$$(M_t)_{max} = 1800 \text{ kg-m} = 18 \times 10^6 \text{ N-mm (given)}$$

$$(M_t)_{min} = 449 \text{ kg-m} = 4.49 \times 10^6 \text{ N-mm (given)}$$

$$\sigma_{xm} = \frac{32(Mb) m}{\pi d^3}$$

(8)

$$\sigma_{xm} = 13.17 \text{ N/mm}^2$$

$$\sigma_{xa} = \frac{32(Mb) a}{\pi d^3}$$

(9)

$$\sigma_{xa} = 6.44 \text{ N/mm}^2$$

$$\tau_{xym} = \frac{16(Mt) m}{\pi d^3}$$

(10)

$$\tau_{xym} = 2.24 \text{ N/mm}^2$$

$$\tau_{xya} = \frac{16(Mt) a}{\pi d^3}$$

(11)

$$\tau_{xya} = 0.636 \text{ N/mm}^2$$

$$\sigma_m = \sqrt{(\sigma_{xym} * Kf)^2 + 3(\tau_{xym} * Kfs)^2} \quad (12)$$

$$\sigma_m = 38.26 \text{ N/mm}^2 = 38.26 * 10^6 \text{ Pa}$$

Also, calculate log stress values,

$$\log(\sigma_m) = \log(38.26 * 10^6)$$

$$\log(\sigma_m) = 7.5 \text{ Pa} \quad (13)$$

$$\sigma_a = \sqrt{(\sigma_{xa} * Kf)^2 + 3(\tau_{xa} * Kfs)^2} \quad (14)$$

$$\sigma_a = 18.47 \text{ N/mm}^2 = 18.47 * 10^6 \text{ Pa}$$

$$\log(\sigma_a) = \log(18.47 * 10^6)$$

$$\log(\sigma_a) = 7.2 \text{ Pa} \quad (15)$$

Since, $\sigma_m = \sigma_{eq} = 38.26 \text{ N/mm}^2$

So, we can say that,

The von-Mises/equivalent stresses are 38.26 N/mm^2

$$\tan \theta = \frac{\sigma_a}{\sigma_m} \text{ \& also, } \tan \theta = \frac{S_a}{S_m} \quad (16)$$

$$\tan \theta = 0.483$$

$$\theta = 25.76^\circ \cong 26^\circ$$

$$\therefore \tan \theta = 0.4877$$

II.To find Endurance Strength ' S_e ,

$$S_e = K_{load} \times K_{size} \times K_{surf} \times K_{temp} \times K_{reliability} \times K_d \times K_{stress\ concentration} \times S'_e \quad (17)$$

$$\text{But, } S'_e = 0.5 S_{ut} \quad (18)$$

$$\therefore S'_e = 0.5 (410) = 205 \text{ N/mm}^2$$

$$K_{load} = 1, K_{size} = 0.75, K_{temp} = 1 \text{ for } T \leq 450^\circ C$$

$$K_{surf} = A \cdot S_{ut}^b = 4.51(410)^{0.265} = 0.92, ,$$

$$K_{reliability} = 90\% = 0.897, K_d = \frac{1}{K_f} = \frac{1}{2.5} = 0.35.$$

$$S_e = 1 * 0.75 * 0.92 * 1 * 0.897 * 0.35 * 205$$

By Modified Goodman Equation,

$$S_e = 44.4082 \text{ N/mm}^2$$

$$\frac{S_a}{S_e} + \frac{S_m}{S_{yt}} = 1 \quad (19)$$

Put above values,

$$\frac{0.4877S_m}{44.4082} + \frac{S_m}{230} = 1$$

$$S_m = 65.23 \text{ N/mm}^2$$

$$S_a = 31.81 \text{ N/mm}^2$$

$$\therefore \text{FOS} = \frac{S_a}{\sigma_a} = 1.72$$

$$\therefore \text{fatigue factor of safety} = f_s = 1.72$$

Step III : To find Number of Cycles from S-N Construction

$$S_f = \overline{AO} = \frac{\overline{AD} \cdot \overline{OC}}{\overline{DC}}$$

$$S_f = \frac{\sigma_a * S_{ut}}{S_{ut} - \sigma_m}$$

$$= 20.371 \text{ MPa} \quad (20)$$

$$0.9 S_{ut} = 0.9(410) = 369 \text{ Mpa}$$

$$\log_{10}(0.9 S_{ut}) = 2.5670$$

$$\log_{10}(S_e) = 1.6474$$

$$\log_{10}(S_f) = 1.3090$$

$$\log_{10}(N) = \frac{(6-3)(2.5670-1.3090)}{(2.5670-1.6474)}$$

$$\log_{10}(N) = 4.10395 + 3 = 7.10395$$

$$N = 12.70 \times 10^6 \text{ cycles}$$

Since, Infinite number of cycles more than 1×10^6 cycles.

IV. Fatigue Analysis by FEA Approach

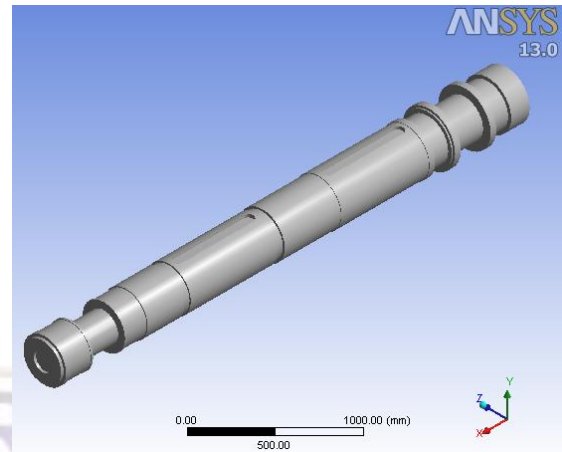


Figure.5 CAD geometry of shaft

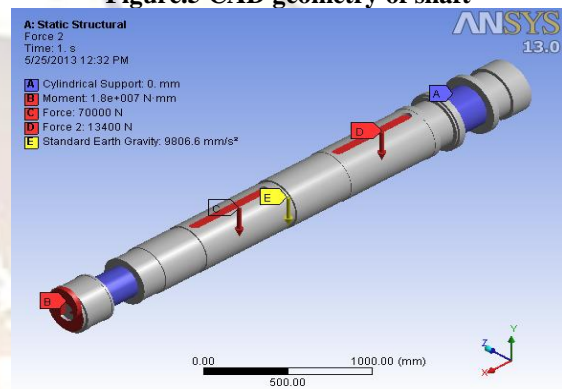


Figure 6.Loading condition of shaft

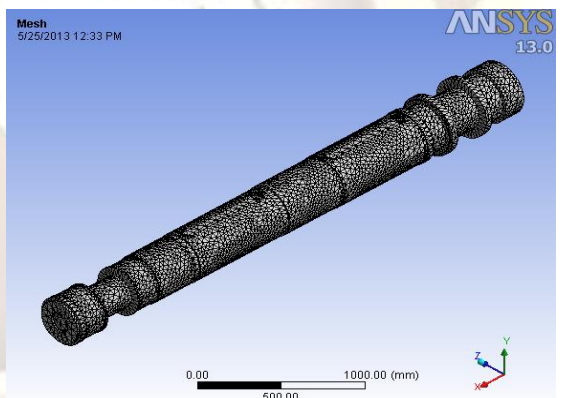


Figure 7.Meshing of shaft
 (Element- tetrahedral10 node, No. of node- 64938, No. of elements- 38057)

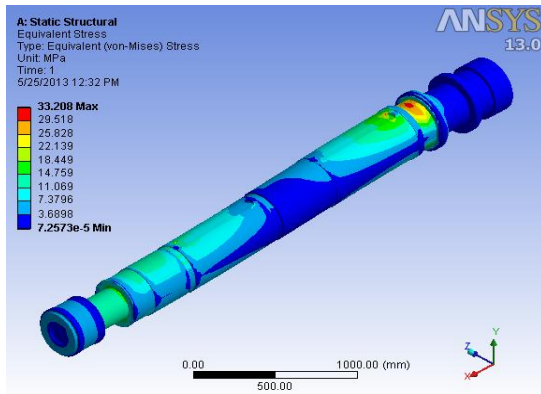


Figure 8. Equivalent / von-mises stresses of shaft

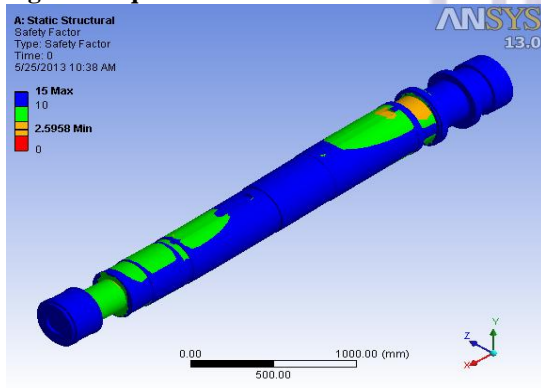


Figure 9. Fatigue factor of safety

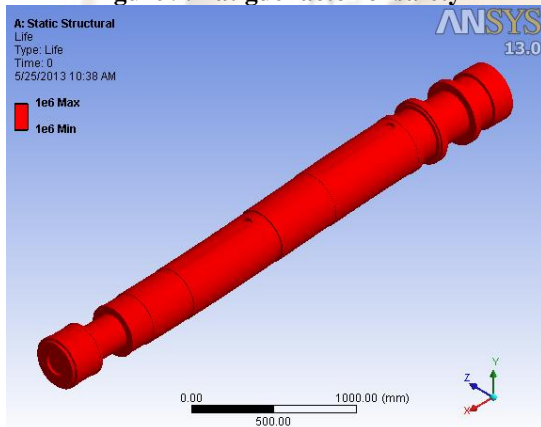


Figure 10. Number of fatigue life cycles

V. Results and Discussion

Table.3 Comparison of Analytical and FEA

Analytical Approach	FEA Approach
$\sigma_{eq} = 38.26 \text{ N/mm}^2$	$\sigma_{eq} = 33.208 \text{ N/mm}^2$
$f_s = 1.72$	$f_s = 2.59$
Infinite Life	Infinite Life

From above Table.3 it is clear that, the fatigue analysis of Inertia Dynamometer shaft gives close results by Analytical and FEA approach. From optimization point of view the FEA analysis is very useful tool. Also, to change the material having low

value of σ_{yt} then to reduced the value of fatigue factor of safety up to 1.5 for saving cost of material. Software used for fatigue analysis CATIA V5, ANSYS13 which gives moderate results.

VI. Conclusion

The fatigue life prediction is performed based on finite element analysis and analytical method. Using the constant amplitude loading, the fatigue life of the dynamometer shaft has been predicted. This study will help to understand more the behavior of the dynamometer shaft and give information for the manufacturer to improve the fatigue life of the dynamometer shaft using FEA tools. It can help to reduce cost, critical speed and times in research and development of new product. From eqⁿ (13) & (15), logarithmic alternating / mean stress values 7.2 Pa & 7.5 Pa which are less than material alternating stress value of 7.94 Pa from fig.(5). So, conclude that, the working alternating / mean stress within limit value and increased fatigue strength for infinite life below material endurance strength limit.

Also, it is clear from above results, Von-Mises stress value by analytical approach $\sigma_{eq} = 38.26 \text{ N/mm}^2$ which are nearly same by using FEA approach having difference of 10% in both results which is acceptable range.

The fatigue factor of safety calculated from eqⁿ(19) is 1.72 & by FEA approach 2.59 again these values are approximately same. The number of life cycles are calculated by using modified Goodman method from S-N construction are $12.70 * 10^6$ cycles and by using FEA it will show safely run at $1 * 10^6$ cycles. So, it will show infinite life cycle checked at High cycle fatigue.

VII. Acknowledgement

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