Effect Of Magnetic Field On Steady Blood Flow Through An Inclined Circular Tube

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ABSTRACT

The present paper is devoted to study the flow of an incompressible viscous, electrically conducting fluid (blood) in a rigid inclined circular tube in the presence of transverse magnetic field. The blood is considered to be Newtonian fluid. The analytical expression for velocity profile and flow rate are obtained. The velocity profile and flow rate for various values of Hartmann number and inclination angle have been shown graphically. The bluntness appears in velocity profile and this bluntness decrease by increasing magnetic field.

Keywords : Incompressible fluid, transverse magnetic field, Hartmann number.

I. INTRODUCTION

The study of the rheological properties of blood can allow a better understanding of blood circulation. This depends on numerous factors such as the driving force of the heart, the shape, as well as mechanical and physiological behaviour of the vascular walls. The experimental and theoretical studies of blood flow phenomena are very useful for the diagnosis of a number of cardiovascular diseases and development of pathological patterns in human physiology and for other practical applications.

The study of magneto hydrodynamic flow problems through a tube has found an applications in many fields like MHD power generation, blood flow measurements, etc. The application of MHD principles in medicine and biology is of interest in the literature of bio-mathematics (Vardanyan [1], Sud et. al. [2], [3]). Blood can be regarded as a magnetic fluid appeared in which red blood cells, magnetic in nature due to iron oxide in it. The movement of blood in an externally applied magnetic field is governed by magneto hydrodynamic principles. By Lenz's law, the Lorentz's force will oppose the motion of conducting fluid.The MHD principles may be used to deaccelerate the flow of blood in a human arterial system and therefore it is useful in the treatment of certain cardiovascular disorders (Korchevskii and Marochunik [4]) and in the diseases with accelerated blood circulation like haemorrhages and hypertension etc.

Magnetic field interactions with blood flow have been studied by many workers. The idea of electromagnetic fields in medical research was firstly given by Kolin [5] and later Korchevskii and Marochunik [4] discussed the possibility of regulating the movement of blood in human system by applying magnetic field. Vardanyan [6] studied the effect of magnetic field on blood flow theoretically and obtained steady solutions for velocity profile and flow rate by neglecting the induced fields.

A large number of theoretical and experimental attempts have been made in the literature to explain the blood flow behavior when it flows through the vessels of circulatory system of living beings. Many investigators have used single-phase homogeneous Newtonian viscous fluid (Womersley [7], McDonald [8], Whitmore [9], Copley and Stainsby [10], Attinger [11], Fung [12], Lew and Fung [13].)

Various mathematical models have been investigated by several workers to examine the behaviour of blood flow under the influence of magnetic fields (Chaturani and Bhartiya [14],Tiwari [15], Abdel Wahab M. and Salem S.I. [16], Sanyal et al [17], Verma, S.R. ([18],[19],[20]), Verma and Srivastava [21].) Chaturani and Bhartiya [14] studied two layered magneto hydrodynanic flow through parallel plates with applications to blood flow and noticed that magnetic field help in reducing the cell injury and the dialysis time. Sanyal et. al. [17] investigated the characteristics of blood flow in a rigid inclined circular tube with periodic body acceleration under the influence of a uniform magnetic field and point out that flow velocity deviates with various parameters and this deviation of flow velocity can be regulated by a proper use of magnetic field. Chaturani and Upadhya [22] studied the gravity flow of fluid with couple stress along an inclined plane with application to blood flow. Many researchers (Rathod and Thippeswamy [23], Leonid et. al.[24], Chen and Tzuoo [25], Astarita et. al. [26], Verma, S.R. [27]) investigated the nature of blood flow in a inclined vessel or surface.

In the present paper, it is proposed to develop a mathematical model to study the characteristics of steady flow of blood through an inclined circular tube in the presence of transverse magnetic field. In the analysis we consider blood as a Newtonian fluid. The analytical expression of blood flow velocity and flow rate are obtained. The effect of magnetic field, inclination angle on blood flow, flow rate of blood has been discussed.

II. MATHEMATICAL FORMULATION AND ANALYSIS

Let us consider a one-dimensional laminar steady blood flow through a uniform straight and inclined rigid circular tube in the presence of transverse magnetic field. The equations governing the flow in cylindrical polar coordinates as -

$$\frac{\partial u}{\partial z} = 0 \tag{1}$$

$$\frac{\partial p}{\partial r} = 0 \tag{2}$$

$$-\frac{\partial p}{\partial z} + \mu \nabla^2 u + \rho\, g\, Sin\theta - \sigma\, B_0^2 u = 0 \tag{3}$$

$$Where \quad \nabla^2 = \frac{1}{r}\left(\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right)$$

Here u is the axial velocity, ρ is the density of blood, μ is the coefficient of viscosity of blood, g is the acceleration due to gravity, θ is inclination angle, $\frac{\partial p}{\partial z}$ is pressure gradient, Bo is the transverse component of magnetic field and σ is the electrical conductivity of the medium.

The boundary conditions are

$$u = 0 \quad \text{at} \quad r = R_0 \tag{4}$$

$$\frac{\partial u}{\partial r} = 0 \quad \text{at} \quad r = 0 \tag{5}$$

where Ro is the radius of tube.

Let us introduce the following transformation

$$y = \frac{r}{R_0} \tag{6}$$

Then the governing equation (3) becomes

$$\frac{\partial^2 u}{\partial y^2} + \frac{1}{y}\frac{\partial u}{\partial y} - M^2 u = \frac{R_0^2}{\mu}\left(\frac{\partial p}{\partial z} - \rho\, g\, Sin\theta\right) \tag{7}$$

Where $M = B_0\, R_0 \sqrt{\frac{\sigma}{\mu}}$ is the Hartmann number.

The corresponding boundary conditions (4) and (5) takes the form

$$u = 0 \quad \text{at} \quad y = 1 \tag{8}$$

$$\frac{\partial u}{\partial y} = 0 \quad \text{at } y = 0 \tag{9}$$

The expression for velocity u obtained as the solution of equation (7) under the boundary conditions (8) and (9) is given as

$$u = \frac{R_0^2}{\lambda^2 \mu}\left(\frac{\partial p}{\partial z} - \rho\, g\, Sin\theta\right)\left[1 - \frac{J_0(\lambda\, y)}{J_0(\lambda)}\right] \tag{10}$$

where $\lambda^2 = -M^2$ and Jo is the Bessel's function of zero order.

Volume flow rate Q is given by

$$Q = \int_0^{R_0} 2\,\pi\, u\, r\, dr \tag{11}$$

Using equation (10) in (11), the expression for flow rate is obtained as

$$Q = \frac{2\pi R_0^4}{\lambda^2 \mu}\left(\frac{\partial p}{\partial z} - \rho\, g\, Sin\theta\right)\left[\frac{1}{2} - \frac{1}{\lambda}\frac{J_1(\lambda)}{J_0(\lambda)}\right] \tag{12}$$

Where J_1 is the Bessel's function of first order.

III. RESULTS AND DISCUSSION

The theoretical results such as velocity profile, flow rate for the effect of magnetic field on steady laminar flow of blood through a rigid inclined circular tube are obtained in this analysis. Numerical solutions of velocity profile and flow rate are shown through tables and graphs for different value of y, Hartmann number M and inclination angle θ.

Velocity profile and flow rate equations (10) and (12) can be written as after neglecting higher order terms of small quantities as

$$u = \frac{R_0^2}{\mu}\left(\rho\, g\, Sin\theta - \frac{\partial p}{\partial z}\right)\left[\left(\frac{1}{4} - \frac{3}{64}M^2 - \frac{1}{2304}M^4\right) - \left(\frac{1}{4} - \frac{1}{16}M^2 + \frac{3}{256}M^4\right)y^2 - \left(\frac{1}{64}M^2 - \frac{1}{256}M^4\right)y^4 - \left(\frac{1}{2304}M^4\right)y^6\right] \tag{13}$$

$$Q = \frac{\pi R_0^4}{\mu}\left(\rho\, g\, Sin\theta - \frac{\partial p}{\partial z}\right)\left[\frac{1}{8} - \frac{1}{48}M^2 - \frac{1}{4608}M^4\right] \tag{14}$$

For the purpose of computation of numerical results, we have made use of the following values :

Ro = 100 x 10^{-4} cm, g = 980 cm/sec^2,

ρ = 1.06 gm/cm^3,

$$\frac{\partial p}{\partial z} = -67.5\times 10^3 \ dyne/cm^3$$

The variation of velocity profile with y for different value of M is shown in Fig.1. From the figure it is clear that the velocity for $\theta = 30^0$ maximum velocity occurs for M = 0.0 and then gradually decreases as Hartmann number M increases. It is also clear that the bluntness appears in velocity profile and this bluntness decreases by increasing magnetic field.

Fig.2 shows that variation of velocity profile with M for different value of y and $\theta = 30^0$, the velocity profile decreases as M increases and figure shows that velocity is maximum at the axis and decreases as y increases. The bluntness is same as Fig.1.

For fixed y = 0.6, the variation of velocity with M and θ are given through Table-1. From table it is clear that velocity increases as inclination angle θ increases but very slowly for fixed M. Velocity profile decreases as M increases for any fixed θ.

Table-2 shows the variation of velocity with θ and y for fixed Hartmann number M = 1.0. From numerical values, we can see that velocity increases as θ increases by decreases as y increases and back flow exist near the wall. There is no backflow when M = 0.

In the absence of Magnetic field M = 0.0, the variation of velocity with θ and y is computed and given through Table-3. From that we see that velocity decreases with y and become zero at the wall and increases as inclination angle θ increases. There in no back flow at the wall in this case.

The variation of flow rate with inclination angle θ and Hartmann number M is given through Table-4. Flow rate increases with θ for any fixed M but decreases with increasing M. Therefore by using an external magnetic field, a remarkable thing is that we can regulate the blood flow and treatment some of diseases such as bleeding and clotting.

From above discussion it may be noticed that the transverse magnetic field effects largely on the axial flow velocity of blood and by taking appropriate value of M, we may regulate the axial flow velocity. By applying proper magnetic field attached with those instruments we can regulate the blood flow and enhance patient's activities. Movement of hemoglobin in blood vessels is accelerated while calcium and cholesterol deposits in blood are decreased. Even the other unwanted materials adhered to the inner side of blood vessels, which provoke high blood pressure, are decreased and made to vanish. The blood is cleansed and circulation is increased. The activity of the heart increases and pains disappear.

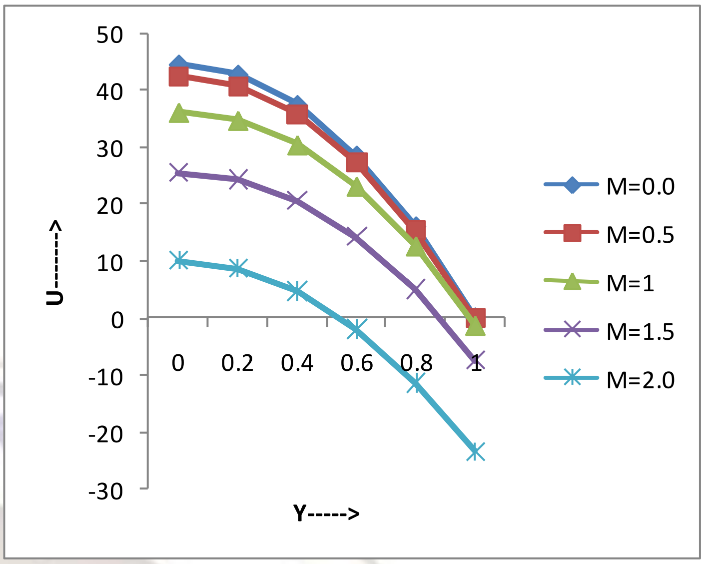

Figure 1: Variation of u with y for different M at θ=30º

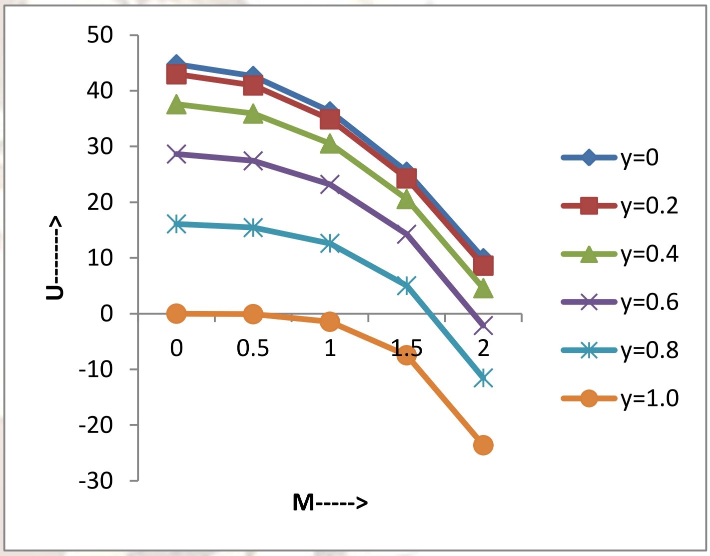

Figure 2: Variation of u with M for different y at θ=30º

Table: 1

Variation of u (cm/sec) with M and θ for y = 0.6

θ → M ↓	0^0	15^0	30^0	45^0	60^0
0.0	28.42	28.54	28.65	28.73	28.80
0.5	27.213	27.321	27.42	27.51	27.57
1.0	23.0207	23.1124	23.1979	23.2713	23.3276
1.5	14.1410	14.1974	14.2499	14.2949	14.3295
2.0	-2.1138	-2.1222	-2.1300	-2.1368	-2.1420

Table: 2
Variation of u (cm/sec) with θ and y for M = 1.0

θ→ y↓	0^0	15^0	30^0	45^0	60^0
0.0	36.023	36.167	36.300	36.415	36.503
0.2	34.6022	34.7400	34.8684	34.9787	35.0634
0.4	30.3036	30.4243	30.5367	30.6333	30.7074
0.6	23.0207	23.1124	23.1979	23.2713	23.3276
0.8	12.5228	12.5727	12.6192	12.6591	12.6898
1.0	-1.4459	-1.4516	-1.4570	-1.4616	-1.4652

Table: 3
Variation of u (cm/sec) with θ and y for M = 0.0

y→ θ↓	0.0	0.2	0.4	0.6	0.8	1.0
0^0	44.4073	42.6310	37.3021	28.4207	15.9866	0.0
15^0	44.5842	42.8008	37.4507	28.5339	16.0503	0.0
30^0	44.7490	42.9590	37.5892	28.6394	16.1096	0.0
45^0	44.8905	43.0949	37.7081	28.7299	16.1606	0.0
60^0	44.9912	43.1992	37.7993	28.7995	16.1997	0.0

Table: 4
Variation of Q with θ and M

θ→ M↓	0^0	15^0	30^0	45^0	60^0
0.0	0.00696	0.00699	0.00702	0.007044	0.007062
0.5	0.00667	0.00670	0.00673	0.00675	0.00677
1.0	0.00579	0.00582	0.00584	0.00586	0.00588
1.5	0.00429	0.00431	0.00433	0.00434	0.00435
2.0	0.00213	0.00214	0.002148	0.002154	0.00216

REFERENCES

[1] Vardanyan, V.A. (1973). Effect of magnetic field on blood flow. *Biophysics, 18*, p. 515.

[2] Sud, V.K., Suri, P.K. and Mishra, R.K. (1974). Effect of magnetic field on oscillating blood flow in arteries. *Study Biophysica*, *46*, p. 163.

[3] Sud,V.K., Suri, P.K. and Mishra, R.K. (1978). Laminar flow of blood in an elastic tube in the presence of magnetic field. *Studia Biophysica*, *69*, p. 175.

[4] Korchevskii, E.M. and Marochunik, L.S. (1965). Magneto hydrodynamic version of movement of blood. *Biophysics, 10*, P. 411.

[5] Kolin, A. (1936). Electromagnetic flow meter : principle of method and its applications to blood flow measurement. *Proc. Soc. Exp. Biol. Med.*, *35*, P. 53.

[6] Vardanyan, V.A. (1973). Effect of magnetic field on blood flow. *Biophysics, 18*, p. 515.

[7] Womersley, J.R. (1954). Method for the calculation of velocity, rate of flow and viscous drag in arteries when the pressure gradient is known, *J, Physiol. 127*, p. 38.

[8] McDonald, D.A. (1960). Blood flow in arteries. *Arnold, London*.

[9] Whitmore, R.L. (1963). Hemorhelology and hemodynamics. *Birorheology 1*, p. 201.

[10] Copley, A.L. and Stainsby, G. (1960). Flow properties of blood. *Pergamon press, Oxford.*

[11] Attinger, E.O. (1964). Pulsatile blood flow. *McGraw - Hill, New York.*

[12] Fung, Y.C. (1964). Biomechanics, its scope, history and some problems of continuum mechanics in physiology. *Applied Mechanics Review 21*, p. 1.

[13] Lew, H.S. and Fung, Y.C. (1970). Entry flow into blood vessels at arbitrary Reynolds number. *Journal of Biomechanics 3*, P. 23.

[14] Chaturani, P. and Bhartiya Saxena, S. (2001). Two-layered magneto hydrodynamic flow through parallel plates with applications. *Indian J. pure appl. Math. 32(1)*, p. 55.

[15] Tiwari Vandana (1998). Effect of magnetic field on a simple blood flow in a arterial region. *J.M.A.C.T. 31*, p. 183.

[16] Abdel Wahab M. and Salem S.I. (2012). Magneto hydrodynamic blood flow in a narrow tube. *World Research Journal of Biomaterials 1, No.-1*, p. 1.

[17] Sanyal, D.C., Das, K. and Debnath, S. (2007). Effect of magnetic field on pulsatile blood flow through an inclined circular tube with periodic body acceleration. *Journal of Physical Sciences*, *11*, p. 43.

[18] Verma, S.R. (2003). Effect of magnetic field on Couette flow of Newtonian fluid with application to blood flow. *J. PAS, 9*, p. 185.

[19] Verma, S.R. (2005). Theoretical study of

MHD flow of fluid through parallel plates with application to blood flow. *Proc. XV Ann. Conf. PAS.*, p. 72.

[20] Verma, S.R. (2011). Two-layered two-phase magneto hydrodynamic fluid model of blood flow through narrow circular vessels. *J. Tensor society 5*, p. 27.

[21] Verma, S.R. and Srivastava Anuj (2012). Effect of magnetic field on unsteady blood flow through a narrow tube. *International J.S.F.M. 3*, No.1, p. 85.

[22] Chaturani, P. and Upadhya, V.S. (1977). Gravity flow of fluid with couple stress along an inclined plane with application to blood flow. *Biorhedogy, 14,* p. 237.

[23] Rathod, V.P. and Thippeswamy, G. (1999). Gravity flow of pulsatile blood through closed rectangular inclined channel with micro-organisms. *The Mathematical Education, 33, No.-1,* p. 40.

[24] Leonid, B., Laure, P., Dias, F. and Bridges, T.J. (1999). Linear pulse structure and signaling in a film flow on an inclined plane. *J. Fluid Mech., 396,* p. 37.

[25] Chen, T.S. and Tzuoo, K.L. (1982). Vortex instability of free convection flow over horizontal and inclined surface. *J. Heat Transfer, 104,* p. 637.

[26] Astarita, G., Mariucxi, G. and Polumbo, G. (1964). Non- Newtonian gravity flow on inclined surface. *Ind. Engg. Chem. Fundam., 3,* p. 333.

[27] Verma, S.R.(2006).Theoretical analysis of steady blood flow along an inclined plane influenced by the gravity force. *Jnanabh. 36*, p. 183.