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Mathematical Modelling And Position Control Of Brushless Dc (Bldc) Motor

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ABSTRACT

The aim of the paper is to design a simulation model of Permanent Magnet Brushless DC (PMBLDC) motor and to control its position. In the developed model, the characteristics of the speed, torque, back EMF, voltages as well as currents are effectively monitored and analysed. The PID controller is used to control the position of a Permanent magnet brushless DC motor by changing the current flow to control the average voltage and thereby the average current. Most useful application is in controlling of CNC machine.

KEY WORDS:-*Brushless dc (BLDC) motor, position control, mathematical modelling, PID controller*

INTRODUCTION

The economic constraints and new standards legislated by governments place increasingly higher requirements on electrical systems. New generations of equipment must have higher performance parameters such as better efficiency and reduced electromagnetic interference .System flexibility must be high to facilitate market modifications and to reduce development time. All these improvements must be achieved while, at the same time, decreasing system cost. Brushless motor technology makes it possible to achieve these specifications. Such motors combine high reliability with high efficiency, and for a lower cost in comparison with brush motors. The Brushless DC Motor (BLDC) motor is conventionally defined as a permanent magnet synchronous motor with a trapezoidal back Electro Motive Force (EMF) waveform shape.

A system based on the Direct Current (DC) motor provides a good, simple and efficient solution to satisfy the requirements of a variable speed drive. Although DC motors possess good control characteristics and ruggedness, their performance and applications in wider areas is inhibited due to sparking and commutation problems. Induction motor do not possess the above mentioned problems, they have their own limitations such as low Power factor and non-linear speed torque characteristics. With the advancement of technology and development of modern control techniques, the Permanent Magnet Brushless DC (PMBLDC) motor is able to overcome the limitations mentioned above and satisfy the requirements of a variable speed drive.

Electric motors influence almost every aspect of modern living. Refrigerators, vacuum cleaners, air conditioners, fans, computer hard drives, automatic car windows, and multitudes of other appliances and devices use electric motors to convert electrical energy into useful mechanical energy. In addition to running the common place appliances that we use every day, electric motors are also responsible for a very large portion of industrial processes.

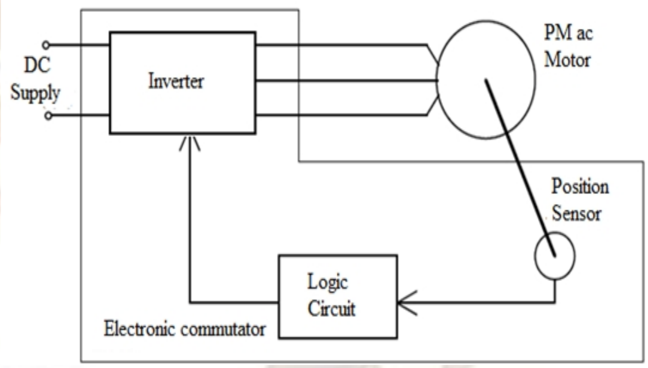

FIGURE (1): PERMANENT MAGNET BLDC MOTOR

POSITIONING APPLICATIONS

Most of the industrial and automation types of application come under this category. The applications in this category have some kind of power transmission, which could be mechanical gears or timer belts, or a simple belt driven system. In these applications,

The dynamic response of speed and torque are important. Also, these applications may have frequent reversal of rotation direction. The load on the motor may vary during all of these phases, causing the controller to be complex.

These systems mostly operate in closed loop. There could be three control loops functioning simultaneously: Torque Control Loop, Speed Control Loop and Position Control Loop. Optical encoder or synchronous resolvers are used for measuring the actual speed of the motor. In some cases, the same sensors are used to get relative position information. Otherwise, separate position sensors may be used to get absolute positions.

Computer Numeric Controlled (CNC) machines are a good example of this. Process controls, machinery controls and conveyer controls have plenty of applications in this category.

MATHEMATICAL MODELLING

Brushless DC Motors are permanent magnet motors where the function of commutatorand brushes were implemented by solid state switches. BLDC motors come in single-phase, 2-phase and 3-phase configurations. Corresponding to its type, the stator has the same number of windings. Out of these, 3-phase motors are the most popular and widely used. Because of the special structure of the motor, it produces a trapezoidal back electromotive force (EMF) and motor current generate a pulsating torque.

Three phase BLDC motor equations:-

$$Va=iaRa+La\frac{dia}{dt}+Mab\frac{dib}{dt}+Mac\frac{dic}{dt}+ea$$

$$Vb=ibRb+Lb\frac{dib}{dt}+Mba\frac{dia}{dt}+Mbc\frac{dic}{dt}+eb$$

$$Vc=icRc+Lc\frac{dic}{dt}+Mcb\frac{dib}{dt}+Mca\frac{dia}{dt}+ ec$$

R: Stator resistance per phase, assumed to be equal for all phases
L: Stator inductance per phase, assumed to be equal for all phases.
M: Mutual inductance between the phases.
ia,ib,ic: Stator current/phase.
Va,Vb,Vc: are the respective phase voltage of the winding
The stator self-inductances are independent of the rotor position, hence:
La=Lb=Lc=L
And the mutual inductances will have the form:
Mab=Mac=Mbc=Mba=Mca=Mcb=M
Assuming three phase balanced system, all the phase resistances are equal:
Ra=Rb=Rc=R
Rearranging the above equations

$$Va=iaR+L\frac{dia}{dt}+M\frac{dib}{dt}+M\frac{dic}{dt}+ea$$

$$Vb=ibR+L\frac{dib}{dt}+M\frac{dia}{dt}+M\frac{dic}{dt}+eb$$

$$Vc=icR+L\frac{dic}{dt}+M\frac{dib}{dt}+M\frac{dia}{dt}+ ec$$

Neglecting mutual inductance

$$Va=iaR+L\frac{dia}{dt}+ea$$

$$Vb=ibR+L\frac{dib}{dt}+eb$$

$$Vc=icR+L\frac{dic}{dt}+ec$$

TRAPEZOIDAL BACK EMF

When a BLDC motor rotates, each winding generates a voltage known as back Electromotive Force or back EMF, which opposes the main voltage supplied to the windings according to Lenz's Law. The polarity of this back EMF is in opposite direction of the energized voltage. Back EMF depends mainly on three factors:

- Angular velocity of the rotor
- Magnetic field generated by rotor magnets
- The number of turns in the stator windings

Once the motor is designed, the rotor magnetic field and the number of turns in the stator windings remain constant. The only factor that governs back EMF is the angular velocity or speed of the rotor and as the speed increases, back EMF also increases. The potential difference across a winding can be calculated by subtracting the back EMF value from the supply voltage. The motors are designed with a back EMF constant in such a way that when the motor is running at the rated speed, the potential difference between the back EMF and the supply voltage will be sufficient for the motor to draw the rated current and deliver the rated torque. If the motor is driven beyond the rated speed, back EMF may increase substantially, thus decreasing the potential difference across the winding, reducing the current drawn which results in a drooping torque curve.

In general, Permanent Magnet Alternating current (PMAC) motors are categorized into two types. The first type of motor is referred to as PM synchronous motor (PMSM). These produce sinusoidal back EMF and should be supplied with sinusoidal current / voltage. The second type of PMAC has trapezoidal back EMF and is referred to as the Brushless DC (BLDC) motor. The BLDC motor requires that quasi-rectangular shaped currents are to be fed to the machine.

When a brushless dc motor rotates, each winding generates a voltage known as electromotive force or back EMF, which opposes the main voltage supplied to the windings. The polarity of the back EMF is opposite to the energized voltage. The stator has three phase windings, and each winding is displaced by 120 degree. The windings are distributed so as to produce trapezoidal back EMF. The principle of the PMBLDC motor is to energize the phase pairs that produce constant torque. The three phase currents are controlled to take a quasi-square waveform in order to synchronize with the trapezoidal back EMF to produce the constant torque. The back EMF is a function of rotor position (θ) and hasthe amplitude E= Ke* ω (Ke is the back EMF constant).

The instantaneous back EMF in BLDC is written as:

$E_a= f_a(\theta)*K_{a*}\omega$
$E_b = f_b(\theta)*K_{b*}\omega$

$E_c = f_c(\theta) * K_c * \omega$

Where, "ω" is the rotor mechanical speed and "θ" is the rotor electrical position.

Themodelling of the back EMF is performed under the assumption that all three phaseshave identical back EMF waveforms. Based on the rotor position, the numerical expression of the back EMF can be obtained Therefore, with the speed command and rotor position, the symmetric three-phase back EMF waveforms can be generated at every operating speed.

The respective back EMF in the windings is represented by the equations:

$$ea = \begin{cases} \left(\frac{6E}{\pi}\right)\theta & \left(0 < \theta < \frac{\pi}{6}\right) \\ E & \left(\frac{\pi}{6} < \theta < \frac{5\pi}{6}\right) \\ -\left(\frac{6E}{\pi}\right)\theta + 6E & \left(\frac{5\pi}{6} < \theta < \frac{7\pi}{6}\right) \\ -E & \left(\frac{7\pi}{6} < \theta < \frac{11\pi}{6}\right) \\ \left(\frac{6E}{\pi}\right)\theta - 12E & \left(\frac{11\pi}{6} < \theta < 2\pi\right) \end{cases}$$

$$eb = \begin{cases} -E & \left(0 < \theta < \frac{\pi}{2}\right) \\ \left(\frac{6E}{\pi}\right)\theta - 4E & \left(\frac{\pi}{2} < \theta < \frac{5\pi}{6}\right) \\ E & \left(\frac{5\pi}{6} < \theta < \frac{9\pi}{6}\right) \\ -\left(\frac{6E}{\pi}\right)\theta + 10E & \left(\frac{9\pi}{6} < \theta < \frac{11\pi}{6}\right) \\ -E & \left(\frac{11\pi}{6} < \theta < 2\pi\right) \end{cases}$$

$$ec = \begin{cases} E & \left(0 < \theta < \frac{\pi}{6}\right) \\ -\left(\frac{6E}{\pi}\right)\theta + 2E & \left(\frac{\pi}{6} < \theta < \frac{\pi}{2}\right) \\ -E & \left(\frac{\pi}{2} < \theta < \frac{7\pi}{6}\right) \\ \left(\frac{6E}{\pi}\right)\theta - 8E & \left(\frac{7\pi}{6} < \theta < \frac{9\pi}{6}\right) \\ E & \left(\frac{9\pi}{6} < \theta < 2\pi\right) \end{cases}$$

By putting E=1 in the above back EMF equations a back EMF function is obtained. The back EMF function is a function of the rotor position which is represented as $f_a(\theta)$, $f_b(\theta)$& $f_c(\theta)$ with limit values between -1 & 1 is defined as:

$$fa(\theta) = \begin{cases} \left(\frac{6}{\pi}\right)\theta & \left(0 < \theta < \frac{\pi}{6}\right) \\ 1 & \left(\frac{\pi}{6} < \theta < \frac{5\pi}{6}\right) \\ -\left(\frac{6}{\pi}\right)\theta + 6 & \left(\frac{5\pi}{6} < \theta < \frac{7\pi}{6}\right) \\ -1 & \left(\frac{7\pi}{6} < \theta < \frac{11\pi}{6}\right) \\ \left(\frac{6}{\pi}\right)\theta - 12 & \left(\frac{11\pi}{6} < \theta < 2\pi\right) \end{cases}$$

$$fb(\theta) = \begin{cases} -1 & \left(0 < \theta < \frac{\pi}{2}\right) \\ \left(\frac{6}{\pi}\right)\theta - 4 & \left(\frac{\pi}{2} < \theta < \frac{5\pi}{6}\right) \\ 1 & \left(\frac{5\pi}{6} < \theta < \frac{9\pi}{6}\right) \\ -\left(\frac{6}{\pi}\right)\theta + 10 & \left(\frac{9\pi}{6} < \theta < \frac{11\pi}{6}\right) \\ -1 & \left(\frac{11\pi}{6} < \theta < 2\pi\right) \end{cases}$$

$$fc(\theta) = \begin{cases} 1 & \left(0 < \theta < \frac{\pi}{6}\right) \\ -\left(\frac{6}{\pi}\right)\theta + 2 & \left(\frac{\pi}{6} < \theta < \frac{\pi}{2}\right) \\ -1 & \left(\frac{\pi}{2} < \theta < \frac{7\pi}{6}\right) \\ \left(\frac{6}{\pi}\right)\theta - 8 & \left(\frac{7\pi}{6} < \theta < \frac{9\pi}{6}\right) \\ 1 & \left(\frac{9\pi}{6} < \theta < 2\pi\right) \end{cases}$$

The induced EMFs do not have sharp corners, but rounded edges.

The quasi-square trapezoidal back EMF waveform and the phase current of the PMBLDC motor with respect to the rotor position is shown in the figure The graph is presented for one complete cycle rotation of 360 degrees

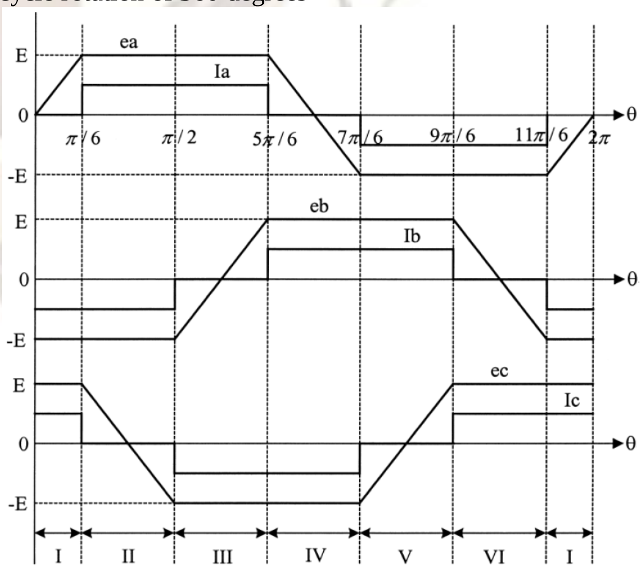

FIGURE (2): BACK EMF AND PHASE CURRENTS WAVEFORMS OF BLDC

TORQUE GENERATION

The Torque is the product of the theoretical motor constant 'K_t'the supplied current' I'. In a single pole system, usable torque is only produced for 1/3 of the rotation. To produce useful torque throughout the rotation of the stator, additional coils, or "phases" are added to the fixed stator. The developed torque by each phase is the product of the motor constant 'k_t'and the current 'I'.

The sum of the torques is
$T_a + T_b + T_c$

Assumption made is all the phases are perfect symmetry

$Kt(motor)=K_{t(a)}=K_{t(b)}=K_{t(c)}$

$i_{motor} = i_a = i_b = i_c$

At any given angle θ, the applied torque as measured on the rotor shaft is

$T_{motor} = 2^* K_{t(motor)} * i_{motor}$

The key to effective torque and speed control of a BLDC motor is based on relatively simple torque and back EMF equations, which are similar to those of the DC motor. The generated electromagnetic torque is given by

$T_e = [e_a i_a + e_b i_b + e_c i_c] / \omega$ (in N.m)

The electromagnetic torque is also related with motor constant and the product of the current with the electrical rotor position which is given as

$T_e = K_t\{ f_a(\theta) i_a + f_b(\theta) i_b + f_c(\theta) i_c\}$

The equation of motion for simple system is,

$J(d\omega/dt) + B\omega = T_e - T_l$

Where,

T_l is the load torque, J is motor inertia, B is damping constant.

The relation between angular velocity and angular position (electrical) is given by

$d\theta/dt = (P/2) * \omega$

Where, P is numbers of Poles,

The Simulink diagram based on the mathematical equations as described above is designed in MATLABSIMULINK as shown in the figure The mat lab function block in the figure is described the back EMF function. The equations of back EMF functionis to be fed into "S-Function Block" in Mat lab Simulink which passes the program written in M-file to the Mat lab workspace.

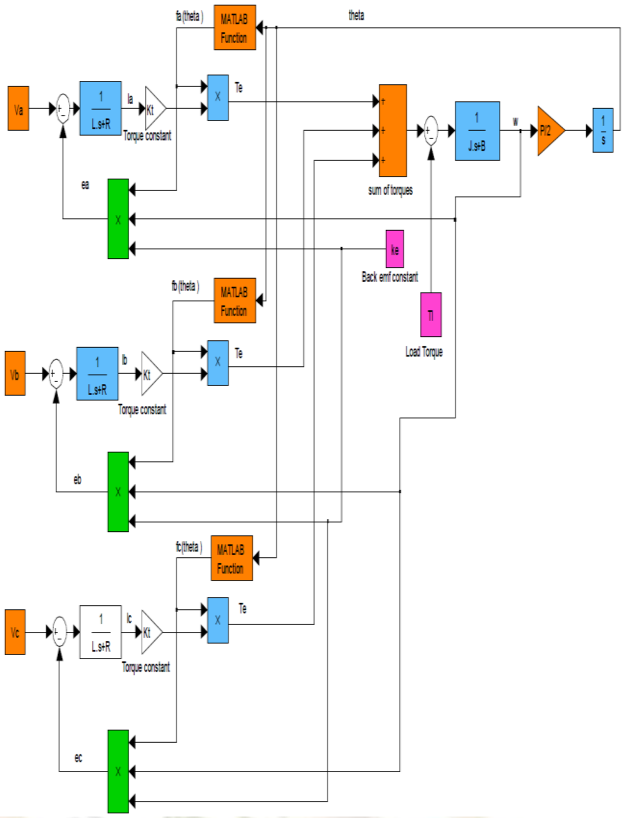

FIGURE (3): MATHEMATICAL MODEL REPRESENTATION OF BLDC MOTOR

POSITION CONTROL

In most of the industrial processes like electrical, mechanical, construction, petroleum industry, iron & steel industry, power sectors, development sites, paper industry, beverages industry the need for higher productivity is placing new demands on mechanisms connected with electrical motors. They lead to different problems in work operation due to fast dynamics and instability. That is why control is needed by the system to achieve stability and to work at desired set targets. The position control of electrical motors is most important due to various non-linear effects like load and disturbance that affects the motor to deviate from its normal operation. The position control of the motor is to be widely implemented in machine automation.

The position of the motor is the rotation of the motor shaft or the degree of the rotation which is to be controlled by giving the feedback to the controller which rectifies the controlled output to achieve the desired position. The application includes robots (each joint in a robot requires a position servo), computer numeric control (CNC) machines, and laser printers. The common characteristics of all such systems is that the variable to be controlled (usually position or velocity) is fed back to modify the command signal. The BLDC motor employs a dc power supply switched to the stator phase windings of the motor by power devices, the switching sequence being determined from rotor position. The phase current of BLDC motor, in typically rectangular shape, is synchronized with the back EMF to produce constant torque at a constant speed. The mechanical commutator of the brush dc motor is replaced by electronic switches, which supply current to the motor windings as a function of the rotor position.

To control the position of motor shaft, the simplest strategy is to use a proportional controller with gain K. Figure shows the position control of PMBLDC motor in which the motor output angular velocity is integrated to obtain the actual position of the motor. The output is feedback to the input and the error signal which is the differencebetween set point and actual motor position acts as the command signal for the PID controller.

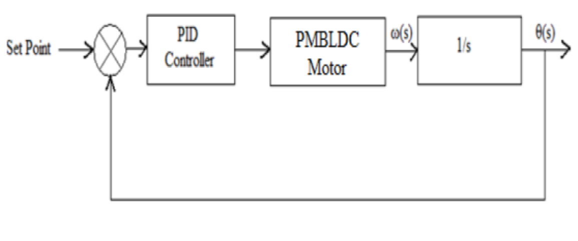

FIGURE (4): POSITION CONTROL OF PMBLDC MOTOR

LOOP TUNING

Tuning a control loop is the adjustment of its control parameters (gain/proportional band,integral gain/reset, derivative gain/rate) to the optimum values for the desired control response. Stability (bounded oscillation) is a basic requirement, but beyond that, different systems have different behaviour, Differentapplications have differentrequirements, and requirements may conflict with one another. Some processes have a degree of non-linearity and so parameters that work well at full-load conditions don't work when the process is starting up from no-load; this can be corrected by gain scheduling (using different parameters in different operating regions). PID controllers often provide acceptable control using default tunings, but performance can generally be improved by careful tuning, and performance may be unacceptable with poor tuning. PID tuning is a difficult problem, even though there are only three parameters and in principle is simple to describe, because it must satisfy complex criteria within the limitations ofPID control. There are accordingly various methods for loop tuning, and more sophisticated techniques are the subject of patents; this section describes some traditional manual methods for loop tuning. If the PID controllerparameters (the gains of the proportional, integral and derivative terms) are chosen incorrectly, the controlled process input can be unstable, i.e. its output diverges, with or without oscillation, and is limited only by saturation or mechanical breakage. Instability is caused by excess gain, particularly in the presence of significant lag. Generally, stability of response is required and the process must not oscillate for any combination of process conditions and set points, though sometimes marginal stability (bounded oscillation) is acceptable ordesired.

The optimum behaviour on a process change or set point change varies depending on the application. Two basic requirements are regulation (disturbance rejection - staying at a given set point) and command tracking (implementing set point changes) - these refer to how well the controlled variable tracks the desired value. Specific criteria for command tracking include rise time and settling time. Some processes must not allow an overshoot of the process variable beyond the set point if, for example, this would be unsafe. Other processes must minimize the energy expended in reaching a new set point. There are several methods for tuning a PID loop. The most effective methods generally involve the development of some form of process model, then choosing P, I, and D based on the dynamic model parameters. Manual tuning methods can be relatively inefficient, particularly if the loops have response times on the order of minutes or longer. The choice of method will depend largely on whether or not the loop can be taken "offline" for tuning, and the response time of the system. If the system can be taken offline, the best tuning method often involves subjecting the system to a step change in input, measuring the output as a function of time, and using this response to determine the control parameters.

MANUAL TUNING

If the system must remain online, one tuning method is to first set K_i and K_d values to zero. Increase the K_p until the output of the loop oscillates, then the K_p should be set to approximately half of that value for a "quarter amplitude decay" type response. Then increase K_i until any offset is correct in sufficient time for the process. However, too much Ki will cause instability. Finally, increase Kd, if required, until the loop isacceptably quick to reach its reference after a load disturbance. However, too much K_d will cause excessive response and overshoot. A fast PID loop tuning usually overshoots slightly to reach the set point more quickly; however, some systems cannot accept overshoot, in which case an over-damped closed-loop system is required, which will require a K_p setting significantly less than half that of the K_p setting causing oscillation.

SIMULATION: - MATLAB Simulink model of Permanent Magnet Brushless DC Motor and its position control using PID controller.

REFERENCE CURRENTS

Rotor position (Degrees)	Reference currents		
	Ia	Ib	Ic
0-30	0	-I	I
30-90	I	-I	0
90-150	I	0	-I
150-210	0	I	-I
210-270	-I	I	0
270-330	-I	0	I
330-360	0	-I	I

FIGURE (5): TABLE SHOWING REFFERENCE CURRENTS

SIMULINK MODEL OF PMBLDC MOTOR

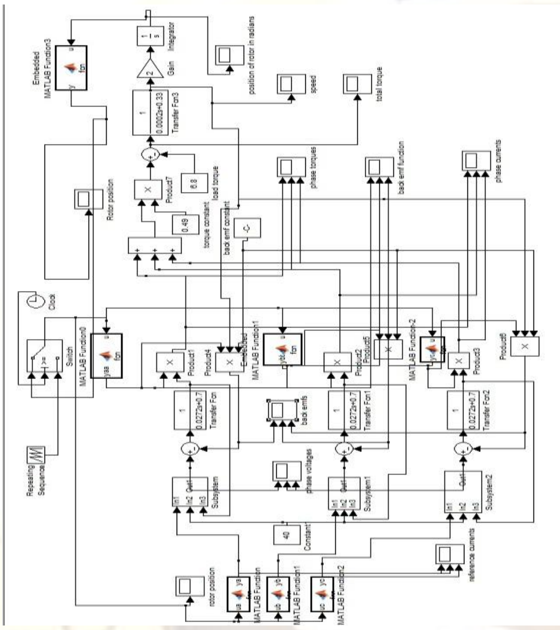

FIGURE (6): SIMULINK MODEL OF MATHAMATICAL MODEL OF PMBLDC

MATHEMATICAL MODELLING RESULTS

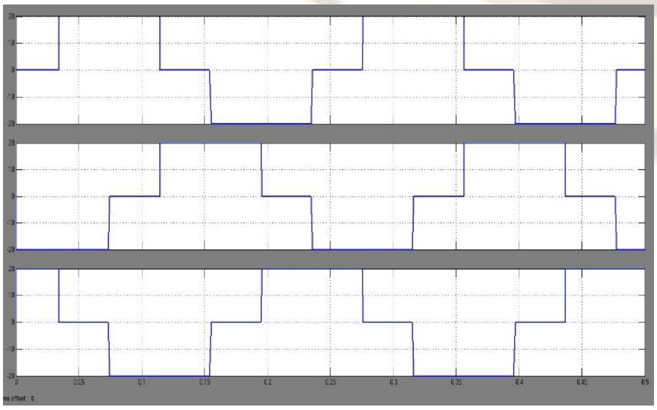

FIGURE (7): REFERENCE CURRENTS (X-AXIS: TIME, Y-AXIS: CURRENT)

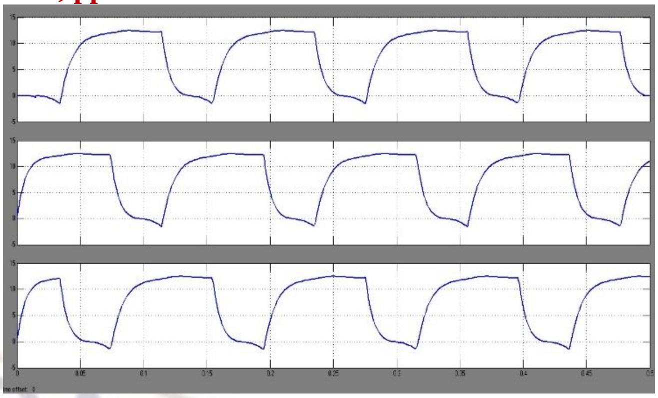

FIGURE (8): PHASE CURRENTS (X-AXIS: TIME, Y-AXIS: CURRENT)

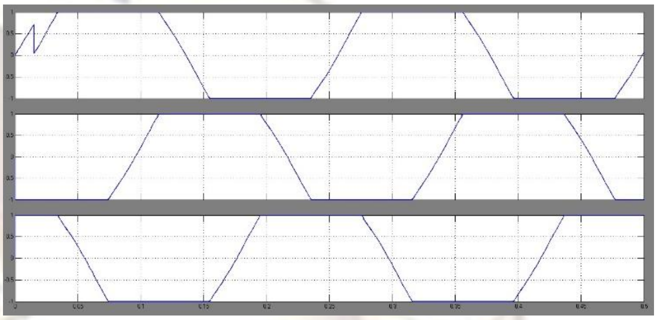

FIGURE (9): BACK-EMF FUNCTION (X-AXIS: TIME, Y-AXIS: BACK-EMF)

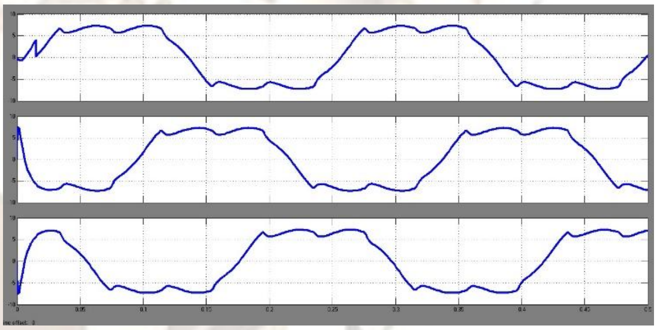

FIGURE (10): PHASE BACK-EMF'S (X-AXIS: TIME, Y-AXIS: BACK-EMF)

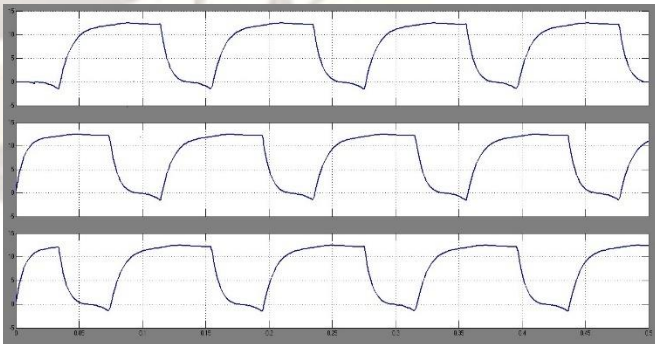

FIGURE (11): PHASE TORQUES (X-AXIS: TIME, Y-AXIS: TORQUE)

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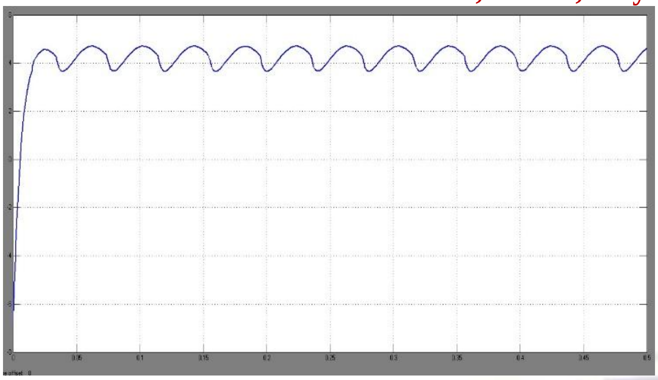

FIGURE (12): TOTAL TORQUE (X-AXIS: TIME, Y-AXIS: TORQUE)

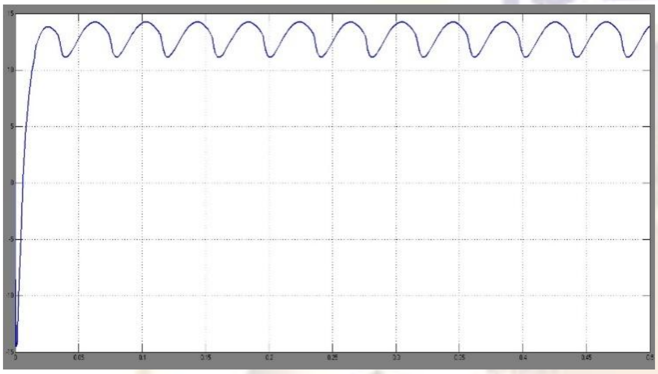

FIGURE (13): SPEED (X-AXIS: TIME, Y-AXIS: SPEED)

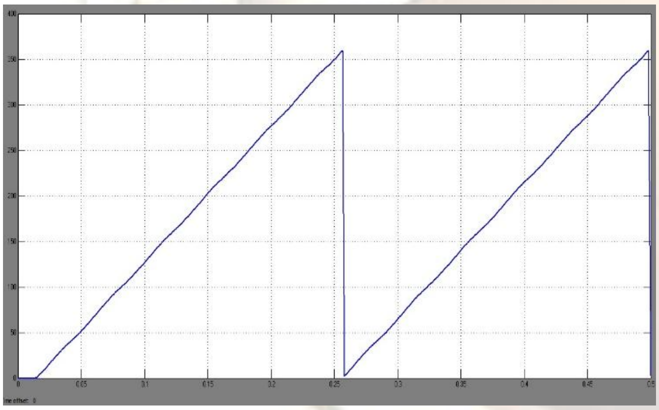

FIGURE (14): ROTOR POSITION (X-AXIS: TIME, Y-AXIS: DEGREES)

SIMULINK MODEL OF POSITION CONTROL OF PMBLDC MOTOR

Figure (15) below shows the position control of PMBLDC motor with PID controller which is manually tuned to obtain the desired rotor position. The PID values used is to average the current which is fed to the inverter. The value at which the position is obtained at K_p=0.4, K_i=0.05 and K_d=0.01. Subsequently graphical rotor position with time is shown below for the various angles.

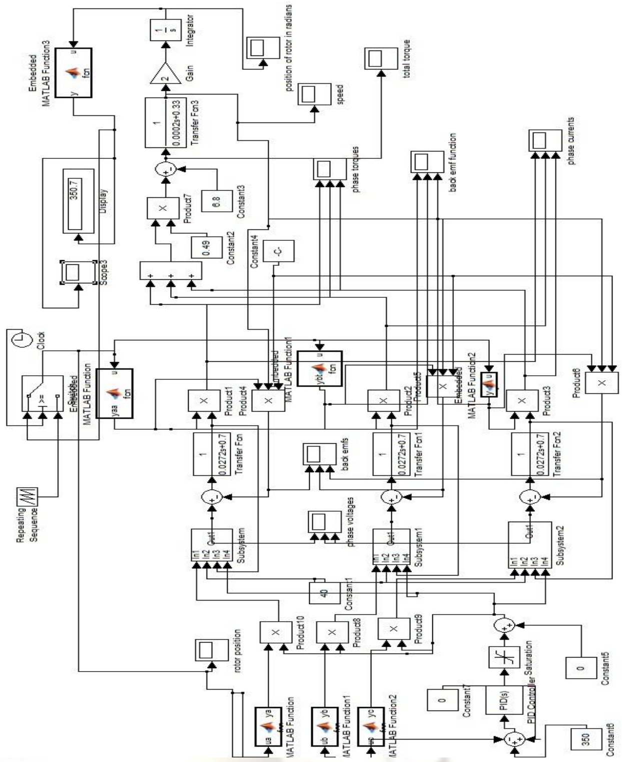

FIGURE (15): SIMULINK MODEL OF POSITION CONTROL OF PMBLDC

POSITION CONTROL AT DIFFERENT VALUES OF THETA

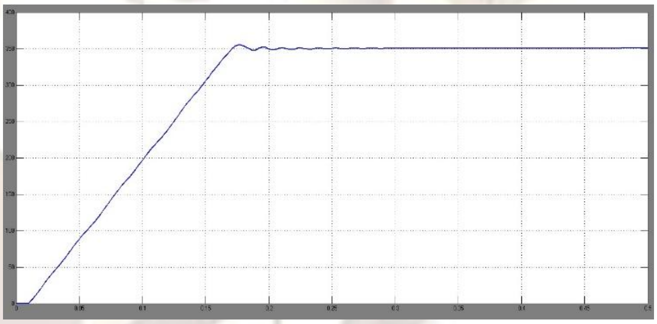

FIGURE (16): ROTOR POSITION FOR θ=350 (X-AXIS: TIME, Y-AXIS: DEGREES)

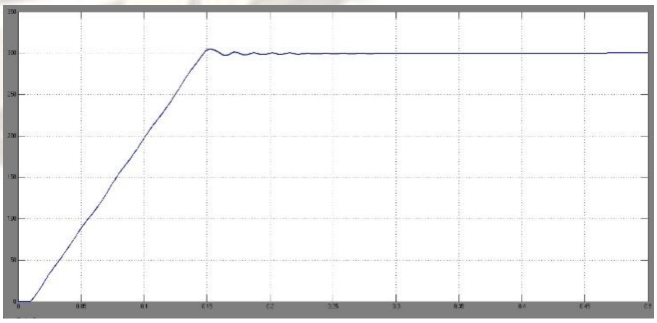

FIGURE (17): ROTOR POSITION FOR θ=300 (X-AXIS: TIME, Y-AXIS: DEGREES)

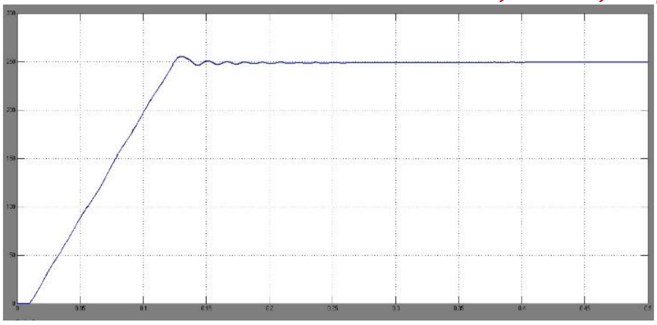

FIGURE (18): ROTOR POSITION FOR θ=250 (X-AXIS: TIME, Y-AXIS: DEGREES)

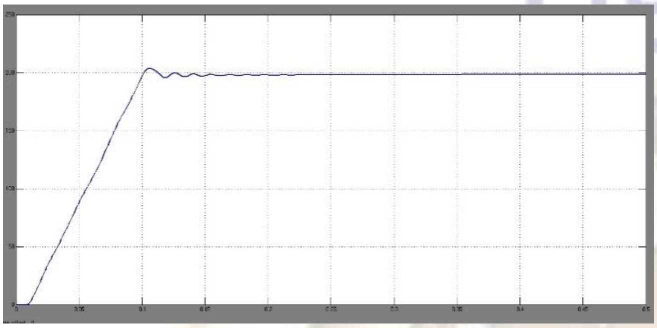

FIGURE (19): ROTOR POSITION FOR θ=200 (X-AXIS: TIME, Y-AXIS: DEGREES)

CONCLUSION

Electric machines are used to generate electrical power in power plants and provide mechanical work in industries. The DC machine is considered to be a basic electric machine. The Permanent Magnet BrushlessDC (PMBLDC) motors are one of the electrical drives that are rapidly gaining popularity, due to their high efficiency, good dynamic response and low maintenance. The brushless DC (BLDC) motors and drives have grown significantly in recent years in the appliance industry and the automotive industry. BLDC drives are very preferable for compact, low cost, low maintenance, and high reliability system.

In this paper, a mathematical model of brushless DC motor is developed. The mathematical model is presented in block diagram representation form. The simulation of the Permanent Magnet Brushless DC motor is done using the software package MATLAB/SIMULINK and its phase voltage, phase current, back emf and torque waveform are analysed. A PID controller has been employed for position control of PMBLDC motor.

FUTURE SCOPE

- Tuning of PID controller for position control using Artificial Intelligence techniques.
- Implementation of real time hardware system for PMBLDC motor position control.

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