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An Algorithm For Interval Continuous –Time MIMO Systems Reduction Using Least Squares Method

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Abstract

A new algorithm for the reduction of Large Scale Linear MIMO (Multi Input Multi Output) Interval systems is proposed in this paper. The proposed method combines the Least squares methods shifting about a point 'a' together with the Moment matching technique. The denominator of the reduced interval model is found by Least squares methods shifting about a point 'a' while the numerator of the reduced interval model is obtained by Moment matching Technique. The reduced order interval MIMO models retain the steady-state value and stability of the original interval MIMO system. The algorithm is illustrated by a numerical example.

Keywords: Least Squares Mean, Large scale Interval system, order reduction, MIMO Systems

1. Introduction

Shoji et al [1] proposed a procedure of using least squares moment matching technique for the order reduction of fixed parameters systems. This method was refined by Lucas and Beat and was extended to include the use of Markov parameters [2]. If the system transfer function contains a pole of magnitude less than one, then numerical problems can arise owing to a rapid increase in the magnitude of successive time moments .This gives an ill conditioned set of linear equations to solve for the reduced denominator .To overcome this problem, it is sometimes possible to use a linear shift s→ (s+a) such that the pole of smallest magnitude has the modulus of approximately one, this tends to reduce the sensitivity of the method. However, the focus of the work so far appears to concentrate mainly on the basic idea of extending this technique for order reduction of High Order Interval MIMO systems.

2. The proposed Reduction Procedure:

Let the transfer function of MIMO Interval system with 'q' outputs and 'u' inputs given as

$$G(s) = \frac{1}{D(s)}\begin{bmatrix} x_{11}(s) & x_{12}(s) & \dots\dots & x_{1u}(s) \\ x_{21}(s) & x_{22}(s) & \dots\dots & x_{2u}(s) \\ \dots. & \dots. & \dots. & \dots. \\ \dots & \dots. & \dots. & \dots. \\ x_{q1}(s) & x_{q2}(s) & \dots\dots & x_{qu}(s) \end{bmatrix}$$

Where the denominator D(s) is in the format of $D(s)=[b_0^-, b_0^+]+[b_1^-, b_1^+]s+\cdots+[b_n^-, b_n^+]s^n$

The Transfer function of each output can be defined as

$$G_1(s) = \frac{x_{11}(s)+x_{12}(s)+\cdots+x_{1u}(s)}{D(s)} \dots \quad 1^{st} \text{ Output}$$

$$G_2(s) = \frac{x_{21}(s)+x_{22}(s)+\cdots+x_{2u}(s)}{D(s)} \dots \quad 2^{nd} \text{ Output}$$

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$$G_q(s) = \frac{x_{q1}(s)+x_{q2}(s)+\cdots+x_{qu}(s)}{D(s)} \quad \dots \quad q^{th} \text{ Output}$$

The sum of numerators of the entire 'm' individual outputs are done by using the rules of Interval Arithmetic defined as follows

i) Addition: [a,b]+[c,d]=[a+c , b+d]
ii) Subtraction: [a,b]-[c,d]=[a-d , b-c]

The general transfer function of each output is defined as

$$G_q(s) = \frac{[a_0^-, a_0^+]+[a_1^-, a_1^+]s+\cdots+[a_{n-1}^-, a_{n-1}^+]s^{n-1}}{[b_0^-, b_0^+]+[b_1^-, b_1^+]s+\cdots+[b_n^-, b_n^+]s^n} \quad \dots(1)$$

Where $[a_i^-, a_i^+]$ for i = 0 to n-1 and $[b_i^-, b_i^+]$ for i = 0 to n are the interval parameters. Now this n^{th} order q^{th} output Interval original system is transferred to four fixed n^{th} order transfer functions using Kharitonov's theorem. Thus the four n^{th} order system transfer functions are defined each as

$$G_P(s) = \frac{A_{P0}+A_{P1}s+A_{P2}s^2+\cdots+A_{Pn-1}s^{n-1}}{B_{P0}+B_{P1}s+B_{P2}s^2+\cdots+B_{pn}s^n} \quad \dots(2)$$

Where p=1, 2, 3, 4 and n = order of the original system. Replace the $G_p(s)$ by $G_p(s+a)$ where the value of 'a' obtained by the harmonic mean of the real parts of the roots of $G_p(s)$ defined as:

$$\frac{1}{a} = \sum_{i=1}^{n}\left[\frac{1}{|P_i|}\right]\Big/ n \quad \dots(3)$$

Where P_i are the poles of $G_p(s)$

Step1:

Expand $G_p(s+a)$ about s=0, to obtain the time moments (C_i) given by

$$G_p(s+a) = \sum_{i=0}^{\infty} c_i s^i \quad \dots(4)$$

Similarly, if $G_p(s + a)$ is expanded about s= ∞ , then the Markov parameters m_j are obtained by:

$$G_p(s+a) = \sum_{j=1}^{\infty} m_j s^{-j} \qquad \dots (5)$$

Let the corresponding r^{th} order reduced model is synthesized as $R_p(s+a) = \frac{N_p(s+a)}{D_p(s+a)}$

$$R_p(s+a) = \frac{d_{p0}+d_{p1}s+d_{p2}s^2+\cdots+d_{pr-1}s^{r-1}}{e_{p0}+e_{p1}s+e_{p2}s^2+\cdots+e_{pr}s^r} \quad ..(6)$$

To Obtain Reduced Order models (Retaining only Time moments):

Step 2: Equating the equation (4) and (6) to retain the time moments of the original system which generates the following set of equations:

$$\begin{bmatrix} c_r & c_{r-1} & \cdots & c_1 \\ c_{r+1} & c_r & \cdots & c_2 \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & & \cdots \\ c_{2r-1} & c_{2r-2} & \cdots & c_r \end{bmatrix} * \begin{bmatrix} e_{p0} \\ e_{p1} \\ . \\ . \\ e_{pr-1} \end{bmatrix} = \begin{bmatrix} -c_0 \\ -c_1 \\ . \\ . \\ -c_{r-1} \end{bmatrix} \quad \dots (7)$$

or ,H e = c, in matrix vector form where the coefficients of 'e_{pr}' , the parameters of the reduced denominator obtained by least squares sense using the generalized inverse method

$e = (H^T H)^{-1} H^T c$ …(8)

If the coefficients e_i given by the equation (8) do not constitute a stable denominator, another row is to be added to the existing equation set so that the model assumes a matching of the next time moment of the original system and so until a stable denominator obtained.

$[c_{2r}\ c_{2r-1} \dots c_{r+1}]$ and $[-c_r]$ …(9)

Once the coefficients $e_{p0}, e_{p1}, e_{p2}, \dots, e_{pr}$ of vector 'e' obtained from equation (8) then $D_p(s+a)$ is obtained as

$$D_P(s+a) = e_{p0} + e_{p1}s + e_{p2}s^2 + \cdots + e_{pr}s^r$$

Step 3: Apply inverse shift of s→(s-a) and finally the r^{th} order reduced denominator obtained as:

$$D_p(s) = E_{p0} + E_{p1}s + E_{p2}s^2 + \cdots + E_{pr}s^r$$

Thus the Reduced Model

$$R_p(s) = \frac{N_p(s)}{D_p(s)} = \frac{D_{p0} + D_{p1}s + D_{p2}s^2 + \cdots + D_{pr-1}s^{r-1}}{E_{p0} + E_{p1}s + E_{p2}s^2 + \cdots + E_{pr}s^r}$$

…(10)

Step4: Calculate the reduced numerator $N_p(s)$ as before by matching proper number of time moments of $G_P(s)$ to that of reduced model $R_p(s)$.

Step5: From all the reduced denominator polynomials of 'q' outputs a common reduced interval denominator is obtained. While reduced interval numerator is obtained individually for each output.

Step6: The Gain of the reduced interval model of each output is adjusted with its Original Interval system to match steady state by considering s=0

To Obtain Biased Reduced order Models:

The above step can also be generalized by including Markov parameters in the least square fitting process, as follows:

$$\left.\begin{aligned} d_{p0} &= e_{p0}c_0 \\ d_{p1} &= e_{p1}c_0 + e_{p0}c_1 \\ d_{p2} &= e_{p2}c_0 + e_{p1}c_1 + e_{p0}c_2 \\ &\ \ . \quad . \quad . \quad . \quad . \\ &\ \ . \quad . \quad . \quad . \quad . \\ &\ \ . \quad . \quad . \quad . \quad . \\ d_{pr-1} &= e_{pr-1}c_0 + \cdots + e_{p0}c_{r-1} \\ 0 &= e_{pr-1}c_1 + \cdots + e_{p0}c_r \\ 0 &= e_{pr-1}c_2 + \cdots + e_{p0}c_{r+1} \\ &\ \ . \quad . \quad . \quad . \quad . \\ &\ \ . \quad . \quad . \quad . \quad . \\ 0 &= e_{pr-1}c_t + \cdots + e_{p0}c_{r+t-1} \end{aligned}\right\} \quad ..(11)$$

and

$$\left.\begin{aligned} d_{pr-1} &= m_1 \\ d_{pr-2} &= m_1 e_{pr-1} + m_2 \\ &\ \ . \quad . \quad . \quad . \\ &\ \ . \quad . \quad . \quad . \\ d_{pt} &= m_1 e_{pt+1} + m_2 e_{pt+2} + \cdots + m_{r-t} \end{aligned}\right\} \quad ..(12)$$

Where the c_i and m_j are the time moments and Markov parameters of the system respectively. Elimination of d_j(j= t, t+1, …,r-1) in equation (12) by substituting into (11) gives the reduced denominator coefficients as the solution of:

$$\begin{bmatrix} c_{r+t-1} & c_{r+t-2} & \cdots & \cdots & \cdots & \cdots & c_t \\ c_{r+t-2} & c_{r+t-3} & \cdots & \cdots & \cdots & c_t & c_{t-1} \\ . & . & \cdots & \cdots & \cdots & . & . \\ c_{r-1} & c_{r-2} & \cdots & \cdots & \cdots & c_1 & c_0 \\ c_{r-2} & c_{r-3} & \cdots & \cdots & \cdots & c_0 & -m_1 \\ c_{r-3} & c_{r-4} & \cdots & \cdots & c_0 & -m_1 & -m_2 \\ . & . & & & . & . & . \\ . & . & & & . & . & . \\ . & . & \cdots & \cdots & . & . & . \\ c_t & c_{t-1} & \cdots & c_0 & -m_1 & \cdots & -m_{r-t-1} \end{bmatrix} X \begin{bmatrix} e_{p0} \\ e_{p1} \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ e_{pr-1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ m_1 \\ m_2 \\ m_3 \\ . \\ . \\ m_{r-t} \end{bmatrix} \quad \dots..(13)$$

or H e = m in matrix vector form and e can be calculated $e = (H^T H)^{-1} H^T m$ …..(14) are the coefficients of the reduced model denominator. If this estimate still does not yield a stable reduced denominator then H and m in (14) are extended by another row, which corresponds to using the next

Markov parameter from the full system in Least Squares match.

Step7: Repeat step3, step4, step5 and step6 respectively to obtain biased reduced order models.

3. Illustrative Example

$$G(s) = \frac{1}{D(s)}\begin{bmatrix} x_{11}(s) & x_{12}(s) \\ x_{21}(s) & x_{22}(s) \end{bmatrix}$$

$$D(s) = [13, 15]\, s^2 + [44, 48]s + [30,31]$$

$$x_{11}(s) = [16,18]s + [14,15]$$
$$x_{21}(s) = [7,8]s + [11,12]$$
$$x_{22}(s) = [12,15]s + [14,15]$$
$$x_{12}(s) = [9,12]s + [20,21]$$

$$G_1(s) = \frac{x_{11}(s) + x_{12}(s)}{D(s)}$$

$$G_2(s) = \frac{x_{21}(s) + x_{22}(s)}{D(s)}$$

The transfer function of the Original system (First Output) is

$$G_1(s) = \frac{[25,30]s + [34,36]}{[13,15]s^2 + [44,48]s + [30,31]}$$

The corresponding Kharitonov's transfer functions are

$$C_1(s) = \frac{25s + 34}{15s^2 + 44s + 30}; HM = 1.363608$$
$$C_2(s) = \frac{30s + 34}{15s^2 + 48s + 30}; HM = 1.250030$$
$$C_3(s) = \frac{25s + 36}{13s^2 + 44s + 31}; HM = 1.409088$$
$$C_4(s) = \frac{30s + 36}{13s^2 + 48s + 31}; HM = 1.291678$$

Applying the proposed reduction method, the reduced First Order models retaining the 4 Time Moments of the original system are:

$R_1(s) = \frac{1.65585}{s+1.461051}$ I.S.E=0.000738

$R_2(s) = \frac{1.942597}{s+1.714056}$ I.S.E=0.005894

$R_3(s) = \frac{1.884469}{s+1.622737}$ I.S.E=0.004358

$R_4(s) = \frac{2.194297}{s+1.889534}$ I.S.E=0.012399

The transfer function of the Original system (Second Output) is

$$G_2(s) = \frac{[19,23]s + [25,27]}{[13,15]s^2 + [44,48]s + [30,31]}$$

The corresponding Kharitonov's transfer functions are

$$C_1(s) = \frac{19s + 25}{15s^2 + 44s + 30}; HM = 1.363608$$
$$C_2(s) = \frac{23s + 25}{15s^2 + 48s + 30}; HM = 1.250030$$
$$C_3(s) = \frac{19s + 27}{15s^2 + 44s + 31}; HM = 1.409088$$
$$C_4(s) = \frac{23s + 27}{13s^2 + 48s + 31}; HM = 1.291678$$

Applying the proposed reduction method, the reduced First Order models retaining the 4 Time Moments of the original system are:

$R_1(s) = \frac{1.258960}{s+1.510752}$ I.S.E=0.000366

$R_2(s) = \frac{1.493948}{s+1.792738}$ I.S.E=0.002671

$R_3(s) = \frac{1.432597}{s+1.644833}$ I.S.E=0.002417

$R_4(s) = \frac{1.685639}{s+1.935363}$ I.S.E=0.006571

From all the 8 reduced first order models, the lowest and highest values are picked to obtain a common denominator for both the outputs. Similarly, for the case of numerator the lowest and highest coefficients are picked separately for 1^{st} output and 2^{nd} output from the four reduced models each. Adjusting the gains of original and reduced models, substituting s=0, the reduced order models using the proposed method are obtained as

$$R_1(s) = \frac{[1.65585\,, \quad 2.194297]}{[1\ 1]s + [1.461051, \quad 1.935363]}$$

$$R_2(s) = \frac{[1.2717493908, \quad 1.6856387]}{[1\ 1]s + [1.461051\,, \quad 1.935363]}$$

Thus the overall Reduced Interval MIMO model obtained as

$$R(s) = \frac{1}{D_r(s)}\begin{bmatrix} [1.65585, \quad 2.194297] \\ [1.2717493908, \quad 1.6856387] \end{bmatrix}$$

Where $D_r(s) = [1\ 1]s + [1.461051, \quad 1.935363]$

Comparison with Stable Pade Approximation for linear MIMO Structured Uncertain systems" of Ismail et.al [6]:

The reduced Interval MIMO model obtained by O.Ismail [6].

$$R_o(s) = \frac{1}{[44\ 48]s + [30\ 31]}\begin{bmatrix} [34\ 36] \\ [25\ 27] \end{bmatrix}$$

The step responses of the original system and that of reduced models obtained by the proposed Least squares method and that of Ismail et.al. are compared in Fig. 1-4 for Ist output and IInd output respectively.

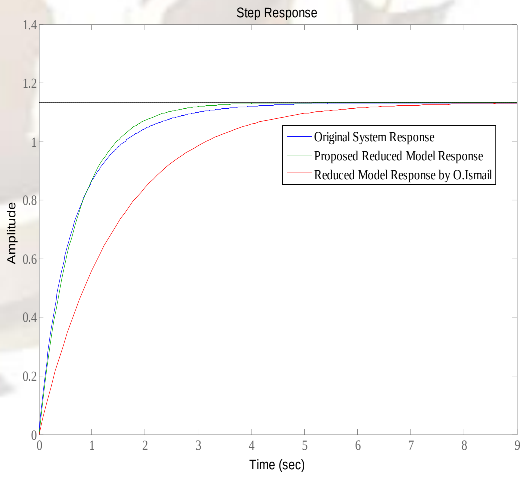

Fig.1 Lower Bound Response (Ist Output)

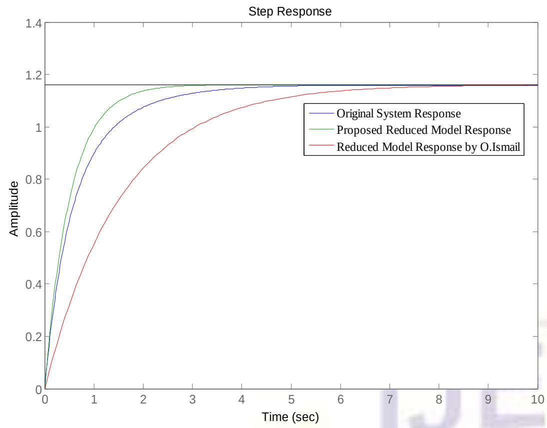

Fig.2 Upper Bound Response (Ist Output)

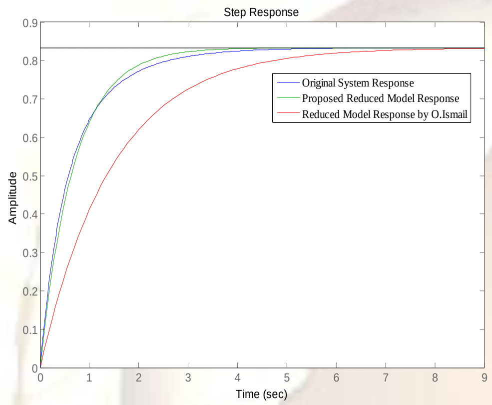

Fig.3 Lower Bound Response (IInd Output)

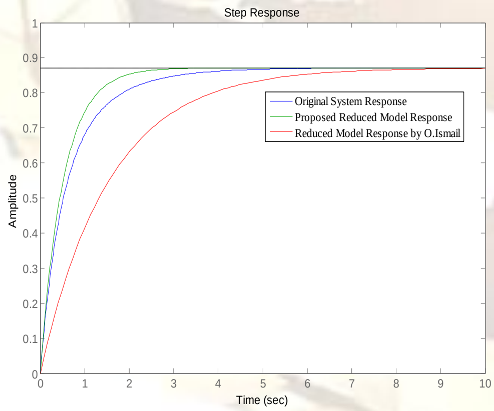

Fig.4 Upper Bound Response (IInd Output)

Conclusion: An Algorithm for the reduction of high order interval MIMO systems is proposed using least square and general least square moment matching at a shifting point ‘a’. This method leads to stable reduced models for linear continuous time interval MIMO systems.

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