Effects Of Radiation On Mhd Mixed Convection Flow Of A Micropolar Fluid Over A Heated Stretching Surface With Heat Generation/Absorption

P. Bala Anki Reddy*and D. Vijaya Sekhar**
*Department of Mathematics, Fluid dynamics division, VIT University, Vellore-632014, Tamil Nadu.
**Department of Mathematics, NBKRIST, Vidyanagar, Nellore, A.P., India.

Abstract

A theoretical analysis is performed to study the flow and heat transfer characteristics of magnetohydrodynamic mixed convection flow of a micropolar fluid past a stretching surface with radiation and heat generation/absorption. The transformed equations solved numerically using the shooting method. The effects of the velocity, the angular velocity, and the temperature for various values of different parameters are illustrated graphically. Also, the effects of various parameters on the local skin-friction coefficient and the local Nusselt number are given in tabular form and discussed. The results show that the mixed convection parameter has the effect of enhancing both the velocity and the local Nusselt number and suppressing both the local skin-friction coefficient and the temperature. It is found that local skin-friction coefficient increases while the local Nusselt number decreases as the magnetic parameter increases. The results show also that increasing the heat generation parameter leads to a rise in both the velocity and the temperature and a fall in the local skin-friction coefficient and the local Nusselt number. Furthermore, it is shown that the local skin-friction coefficient and the local Nusselt number decrease when the slip parameter increases.

Keywords: *Radiation, stretching surface, micropolar fluid, heat transfer, heat source.*

1.Introduction

The fluid dynamics due to a stretching surface and by thermal buoyancy is of considerable interest in several applications such as aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film in condensation processes, paper production, glass blowing, metal spinning and drawing plastic films. Free and mixed convections of a micropolar fluid over a moving surface have been studied by many authors [1–9] under different situations. Desseaux and Kelson [10] studied the flow of a micropolar fluid bounded by a linearly stretching sheet while Bhargava et. al. [11] studied the same flow of a micropolar flow over a non-linear stretching sheet. The theory of micropolar fluid was first introduced and formulated by Eringen [12]. Later
Eringen [13] generalized the theory to incorporate thermal effects in the so-called thermo micropolar fluid. Crane [14] first obtained an elegant analytical solution to the boundary layer equations for the problem of steady two dimensional flow due to a stretching surface in a quiescent incompressible fluid. Gupta and Gupta [15] extended the problem posed by Crane [14] to a permeable sheet and obtained closed-form solution, while Grubka and Bobba [16] studied the thermal field and presented the solutions in terms of Kummer's functions. The 3-dimensional case has been considered by Wang [17]. Chen [18] studied the case when buoyancy force is taken into consideration, and Magyari and Keller [19] considered exponentially stretching surface. The heat transfer over a stretching surface with variable surface heat flux has been considered by Char and Chen [20], Lin and Cheng [21], Elbashbeshy [22] and very recently by Ishak et al. [23], Mostaffa Mahmoud and Shimaa Waheed [24]. All of the above mentioned studies dealt with stretching sheet where the flows were assumed to be steady.

The aim of this work is to investigate the effect of radiation on the flow and heat transfer of a micropolar fluid over a vertical stretching surface in the presence of heat generation (absorption) and magnetic field, where numerical solutions are obtained using shooting method.

2. Formulation of the Problem

Consider a steady, two-dimensional hydromagnetic laminar convective flow of an incompressible, viscous, micropolar fluid with a heat generation (absorption) on a stretching vertical surface with a velocity $v_w(x)$. The flow is assumed to be in the x-direction, which is taken along the vertical surface in upward direction and y-axis normal to it. A uniform magnetic field of strength B_0 is imposed along y-axis. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field is assumed to be negligible. The gravitational acceleration g acts in the downward direction.

The temperature of the micropolar fluid far away from the plate is $T\infty$, whereas the surface temperature of the plate is maintained at Tw, where $T_w(x)=T_\infty + ax$, $a > 0$ is constant, and $Tw > T\infty$. The temperature difference between the body surface and the surrounding micropolar fluid generates a buoyancy force, which results in an upward convective flow. Under usual boundary layer and Boussinesq approximations, the flow and heat transfer in the presence of heat generation (absorption) are governed by the following equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(\nu+\frac{k}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\frac{k}{\rho}\frac{\partial N}{\partial y}+g\beta\left(T-T_\infty\right)-\frac{\sigma B_0^2}{\rho}u \qquad (2)$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=\frac{\gamma_0}{j}\frac{\partial^2 N}{\partial y^2}-\frac{k}{\rho j}\left(2N+\frac{\partial u}{\partial y}\right) \qquad (3)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{Q_0}{\rho c_p}\left(T-T_\infty\right) \qquad (4)$$

Subject to the boundary conditions:

$$u=u_w(x)=cx,\ v=0, N=-m_0\frac{\partial u}{\partial y},\ T=T_w(x) \quad at\ y=0$$

$$u\to 0,\ N\to 0,\ T\to T_\infty \quad as\ \ y\to\infty \qquad (5)$$

where u and v are the velocity components in the x and y directions respectively, T is the fluid temperature, N is the component of the microrotation vector normal to the x-y plane, $\rho$is the density, j is the microinertia density, μ is the dynamic viscosity, k is the gyro-viscosity (or vortex viscosity), β is the thermal expansion coefficient, σ is the electrical conductivity, c_p is the specific heat at constant pressure, κ is the thermal conductivity, c is a positive constant of proportionality, x measures the distance from the leading edge along the surface of the plate, and γ_0 is the spin-gradient viscosity. Thermal radiation is simulated using the Rosseland diffusion approximation and in accordance with this, the radiative heat flux q_r is given by:

$$q_r=-\frac{4\sigma^*}{3k_1^*}\frac{\partial T^4}{\partial y}$$

Where σ^* is the Stefan–Boltzman constant and k_1^* is the Rosseland mean absorption coefficient.

We follow the recent work of the authors [25, 26] by assuming that γ_0 is given by

$$\gamma_0=\left(\mu+\frac{k}{2}\right)j=\mu\left(1+\frac{K}{2}\right)j \qquad (6)$$

This equation gives a relation between the coefficient of viscosity and micro inertia, where $K = k/\mu$ (> 0) is the material parameter, $j = \nu/c$, $\sqrt{j}$ is the reference length, and m_0 ($0 \le m_0 \le 1$) is the boundary parameter. When the boundary parameter $m_0 = 0$, we obtain $N = 0$ which is the no-spin condition, that is, the microelements in a concentrated particle flow close to the wall are not able to rotate (as stipulated by Jena and Mathur (27)). The case $m_0 = 1/2$ represents the weak concentration of microelements. The case corresponding to $m_0 = 1$ is used for the modelling of turbulent boundary layer flow (see Peddieson andMcnitt (28)).

We introduce the following dimensionless variables:

$$\eta=\left(\frac{c}{\nu}\right)^{1/2}y, N=cx\left(\frac{c}{\nu}\right)^{1/2}g, u=cxf', v=-(c\nu)^{1/2}f, \theta=\frac{T-T_\infty}{T_w-T_\infty}, M=\frac{\sigma B_0^2}{c\rho},$$

$$\lambda=\frac{g\beta a}{c^2},\ R=\frac{3k_1^*k_1}{16\sigma^*T_\infty^3}, \Pr=\frac{\mu c_p}{k}, \gamma=\frac{Q_0}{\rho c c_p}, K=\frac{k}{\rho\nu}, R=\frac{3k_1^*k}{16\sigma^*T_\infty^3} \qquad (7)$$

Through (7), the continuity (1) is automatically satisfied and (2) – (4) will give then

$$(1+K)f'''+ff''-f'^2+Kg'-Mf'+\lambda\theta=0 \qquad (8)$$

$$\left(1+\frac{K}{2}\right)g''+fg'-f'g-K\left(2g+f''\right)=0 \qquad (9)$$

$$\frac{1}{\Pr}\left(1+\frac{1}{R}\right)\theta''+f\theta'-f'\theta+\gamma\theta=0 \qquad (10)$$

The corresponding boundary conditions are

$$f'=1, f=0,\ g=-m_0f'',\ \theta=1 \quad \text{at} \quad \eta=0$$
$$f'=0,\ g\to 0, \theta\to 0 \quad \text{as} \quad \eta\to\infty \qquad (11)$$

where the primes denote the differentiation with respect to η, M is the magnetic parameter, λ is the buoyancy parameter, γ is the heat generation (> 0) or absorption (<0) and R is the radiation parameter.

The physical quantities of interest are the local skin-friction coefficient Cf_x and the local Nusselt number Nu_x, which are respectively proportional to $-f''(0), -\theta'(0)$ are worked out and their numerical values presented in a tabular form.

3. Solution of the problem

The governing boundary layer equations (8) - (10) subject to boundary conditions (11) are solved numerically by using shooting method. First of all higher order non-linear differential equations (8) - (10) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique. From the process of numerical computation, the skin-friction coefficient, the Nusselt number and Sherwood number which are respectively proportional to $-f''(0)$ and $-\theta'(0)$ are also sorted out and their numerical values are presented in a tabular form.

4. Results and Discussion

To study the behavior of the velocity, the angular velocity, and the temperature profiles, curves are drawn in Figures 1–18. The effect of various parameters, namely, the magnetic parameter M, the material parameter K, the radiation parameter R, the buoyancy parameter λ, the heat generation (absorption) parameter γ, and the Prandtl number Pr have been studied over these profiles.

Figures 1–3 illustrate the variation of the velocity f', the angular velocity g, and the temperature θ profiles with the magnetic parameter M. Figure 1 depict the variation of f' with M. It is observed that f' decreases with the increase in M along the surface. This indicates that the fluid velocity is reduced by increasing the magnetic field and confines the fact that application of a magnetic field to an electrically conducting fluid produces a drag like force which causes reduction in the fluid velocity. The profile of the angular velocity g with the variation of M is shown in Figure 2. It is clear from this figure that g increases with an increase in M near the surface and the reverse is true away from the surface. Figure 3 shows the resulting temperature profile θ for various values of M. It is noted that an increase of M leads to an increase of θ.

Figure 4 illustrates the effects of the material parameter K on f'. It can be seen from this figure that the velocity decreases as the material parameter K rises near the surface and the opposite is true away from it. Also, it is noticed that the material parameter has no effect on the boundary layer thickness. The effect of K on g is shown in Figure 5. It is observed that initially g decreases by increasing K near the surface and the reverse is true away from the surface. Figure 6 demonstrates the variation of θ with K. From this figure it is clear that θ decreases with an increase in K.

It was observed from Figure 7 that the velocity increases for large values of λ while the boundary layer thickness is the same for all values of λ. Figure 8 depicts the effects of λ on g. The angular velocity g is a decreasing function of λ near the surface and the reverse is true at larger distance from the surface. Figure 9 shows the variations of λ on θ. It is found that θ decreases with an increase in λ.

Figure 10 shows the effect of the heat generation parameter ($\gamma > 0$) or the heat absorption parameter ($\gamma < 0$) on f'. It is observed that f' increases as the heat generation parameter ($\gamma > 0$) increases, but the effect of the absolute value of heat absorption parameter ($\gamma < 0$) is the opposite. The effect of the heat generation parameter ($\gamma > 0$) or the heat absorption parameter ($\gamma < 0$) on g within the boundary layer region is observed in Figure 11. It is apparent from this figure that g increases as the heat generation parameter ($\gamma > 0$) decreases, while g increases as the absolute value of heat absorption parameter ($\gamma < 0$) increases near the surface and the reverse is true away from the surface. Figure 12 displays the effect of the heat generation parameter ($\gamma > 0$) or the heat absorption parameter ($\gamma < 0$) on θ. It is shown that as the heat generation parameter ($\gamma > 0$) increases, the thermal boundary layer thickness increases. For the case of the absolute value of the heat absorption parameter ($\gamma < 0$), one sees that the thermal boundary layer thickness decreases as γ increases.

The effect of the Prandtl number Pr on the velocity, the angular velocity, and the temperature profiles is illustrated in Figures 13, 14, and 15. From these figures, it can be seen that f' decrease with increasing Pr, while g increases as the Prandtl number Pr increases near the surface and the reverse is true away from the surface. The temperature θ of the fluid decreases with an increase of the Prandtl number Pr as shown in Figure 15. This is in agreement with the fact that the thermal boundary layer thickness decreases with increasing Pr. Figures 16, 17, and 18 depict the effect of the radiation parameter on f', g, and θ, respectively. It is seen that f' and θ decrease as α increases, while g increases as α increases.

The local skin-friction coefficient in terms of $-f''(0)$ and the local Nusselt number in terms of $-\theta'(0)$ for various values of M, λ, K, γ, R and Pr are tabulated in Table 1. It is obvious from this table that local skin-friction coefficient and the local Nusselt number increases with the increase of the magnetic parameter M. It is noticed that as K or λ increases, the local skin-friction coefficient as well as the local Nusselt number decreases. It is found that the local Nusselt number decreases with the increase of the heat generation parameter ($\gamma > 0$), while it increased with the increase of the absolute value of the heat absorption parameter ($\gamma < 0$). It is found that both the local skin friction coefficient and the local Nusselt number increase with increasing Pr or R

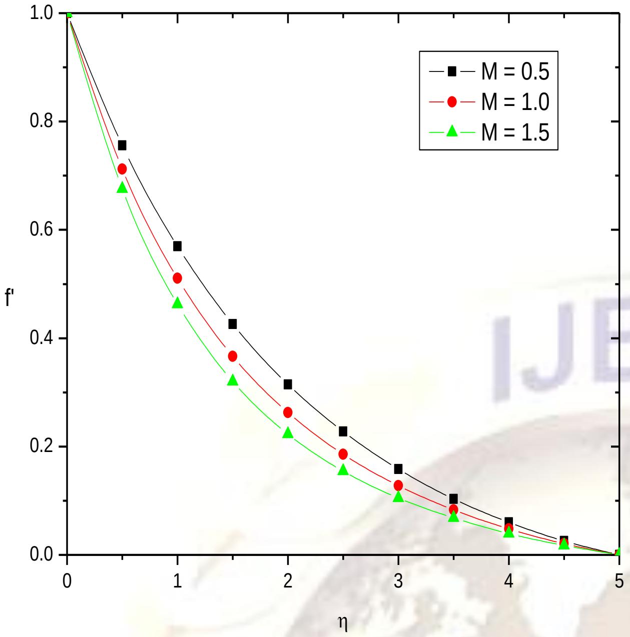

Fig. 1. Velocity profiles for various values of M.

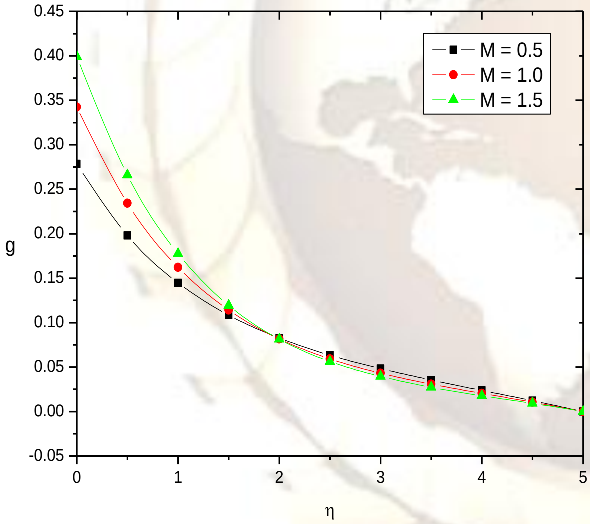

Fig. 2. Angular Velocity profiles for various values of M.

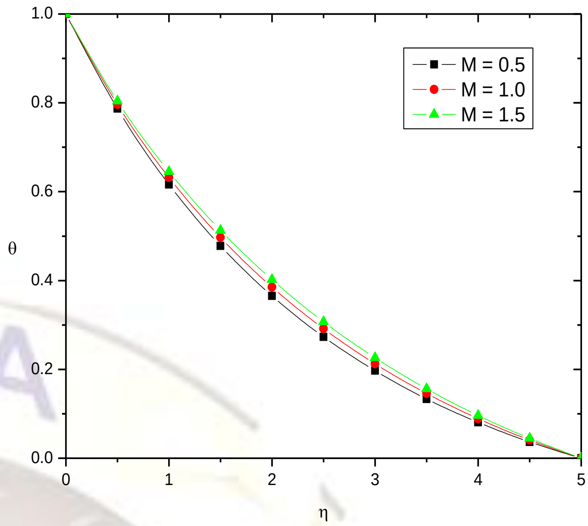

Fig. 3. Temperature profiles for various values of M.

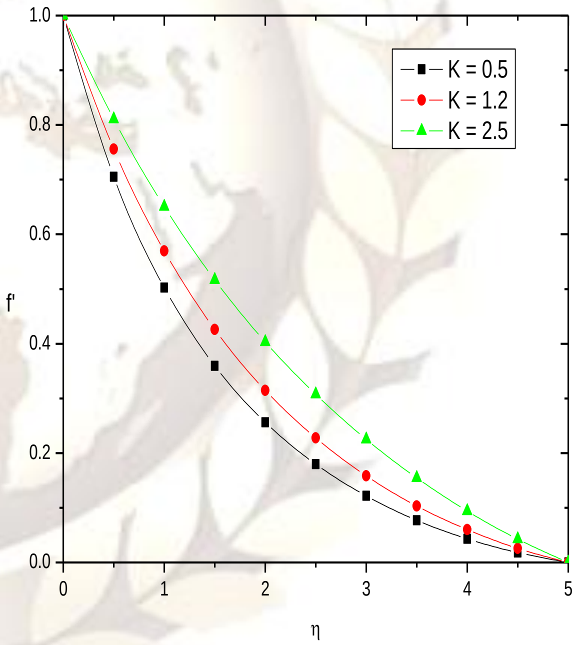

Fig. 4. Velocity profiles for various values of K.

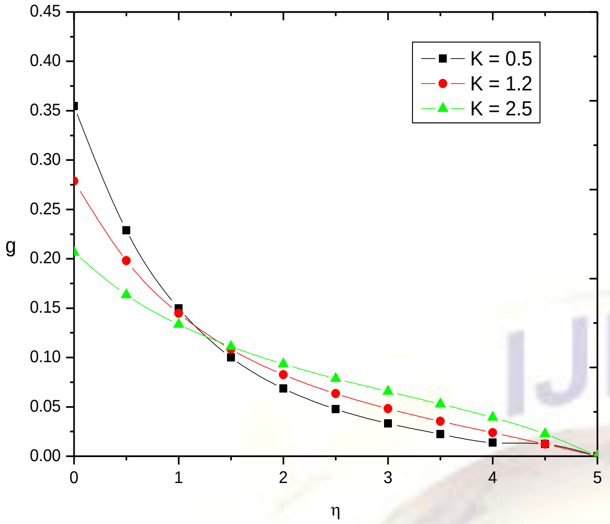

Fig. 5. Angular velocity profiles for various values of K.

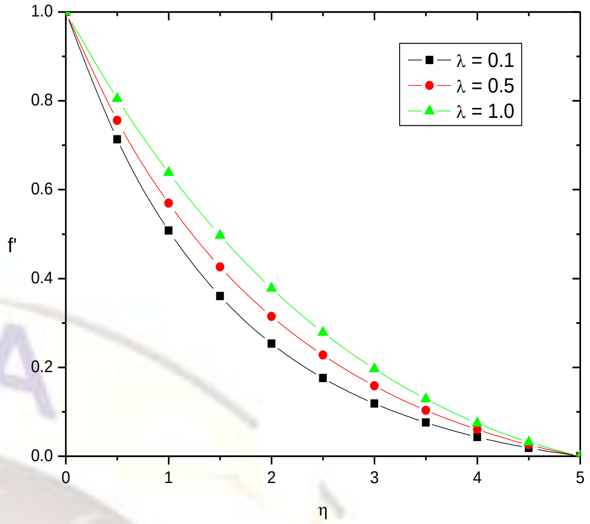

Fig. 7. Velocity profiles for various values of λ.

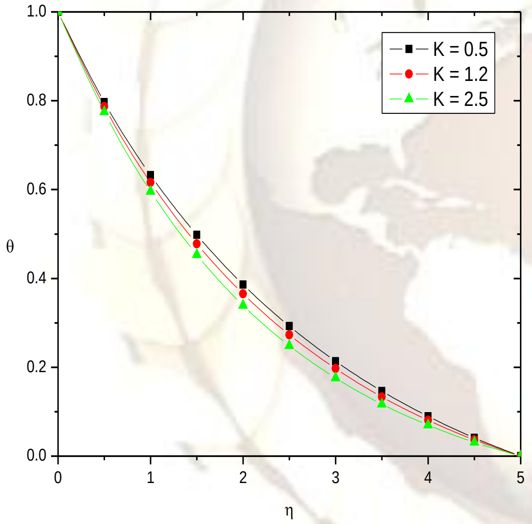

Fig. 6. Temperature profiles for various values of K.

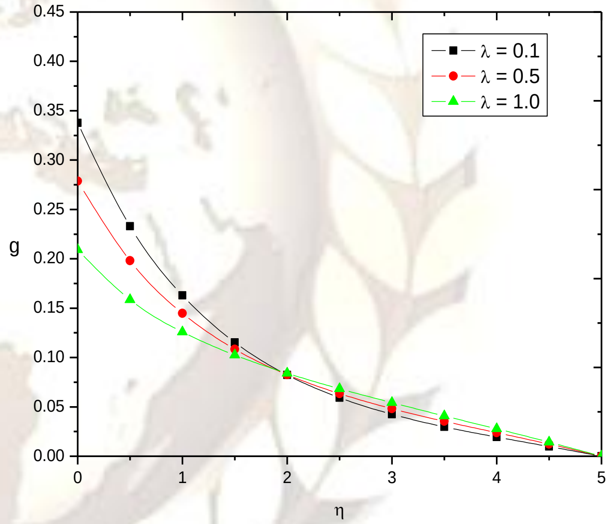

Fig. 8. Angular velocity profiles for various values of λ.

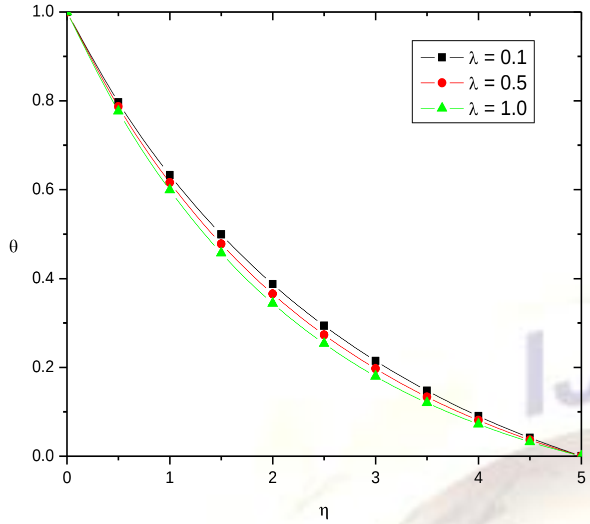

Fig. 9. Temperature profiles for various values of λ
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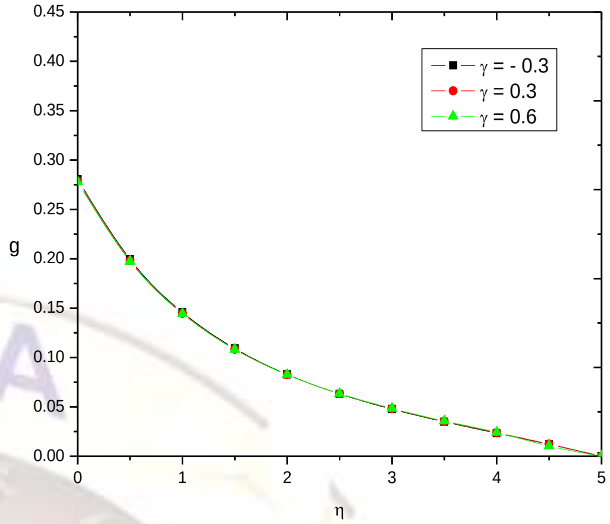

Fig. 11. Angular velocity profiles for various values of γ.

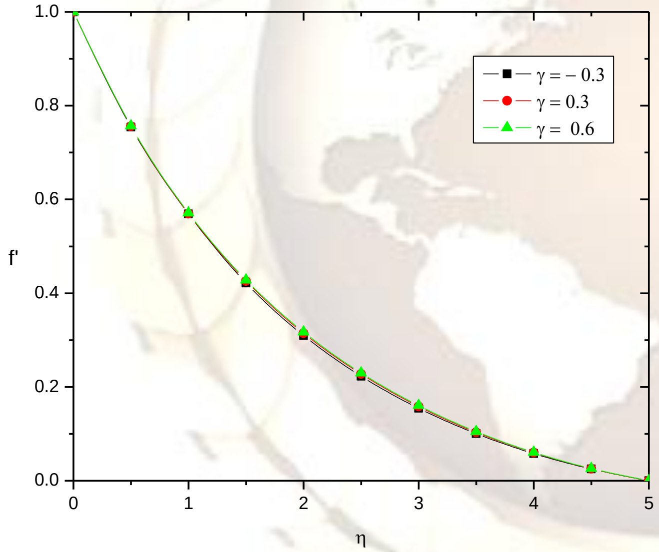

Fig. 10. Velocity profiles for various values of γ.

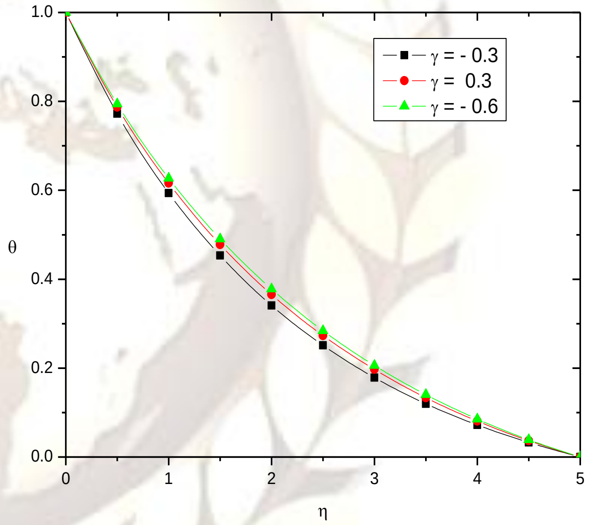

Fig. 12. Temperature profiles for various values of γ.

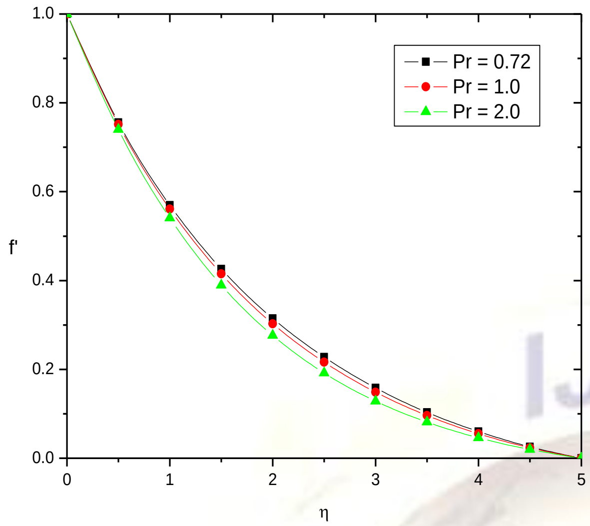

Fig. 13. Velocity profiles for various values of Pr.

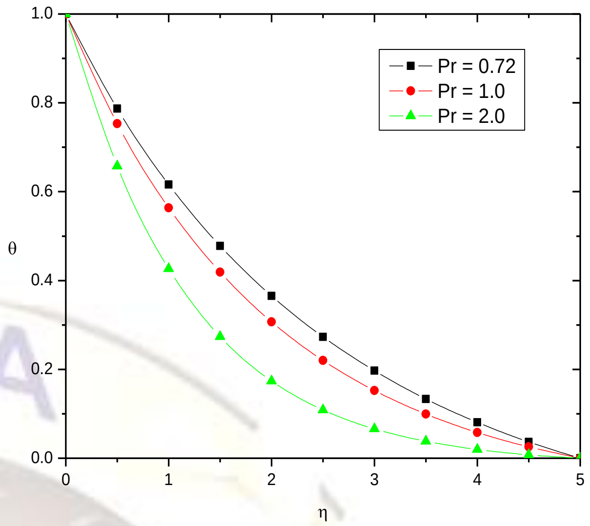

Fig. 15. Temperature profiles for various values of Pr.

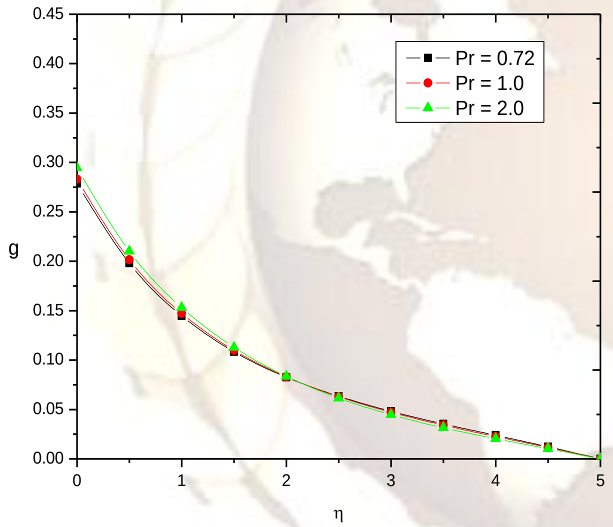

Fig. 14. Angular velocity profiles for various values of Pr.

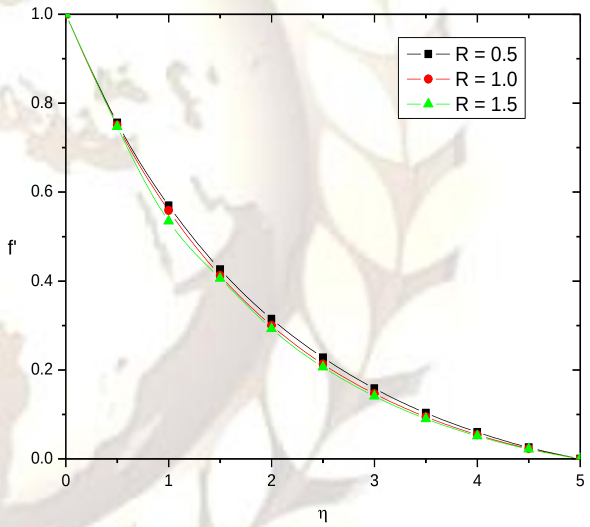

Fig. 16. Velocity profiles for various values of R.

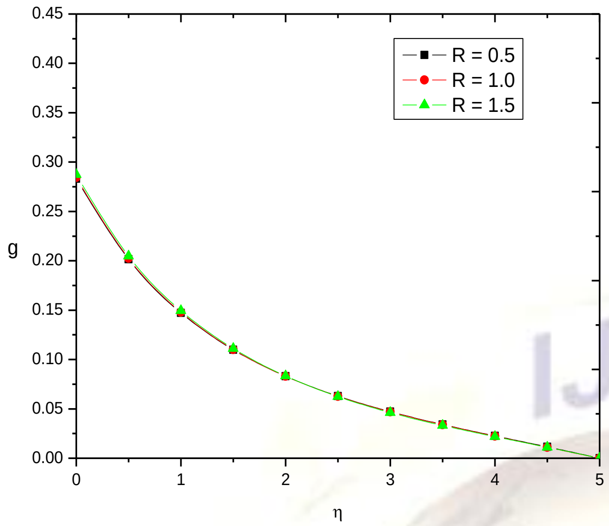

Fig. 17. Angular velocity profiles for various values of R.

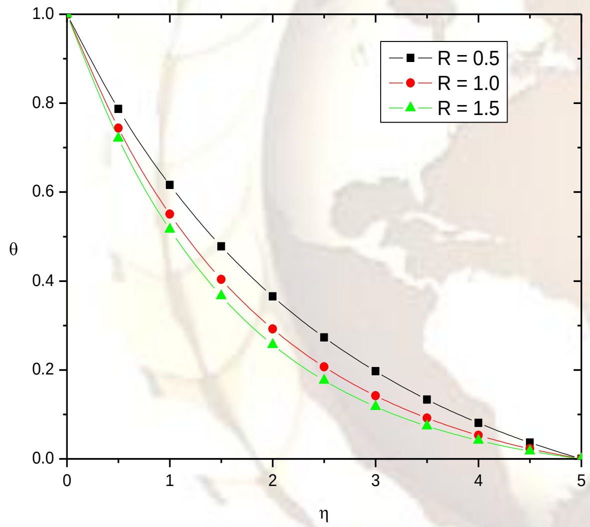

Fig. 18. Temperature profiles for various values of R.

Table 1.

M	λ	K	γ	R	Pr	$-f''(0)$	$-\theta'(0)$
0.5	0.5	1.2	0.3	0.5	0.72	0.748402	0.297623
1.0	0.5	1.2	0.3	0.5	0.72	0.90297	0.377953
0.5	1.0	1.2	0.3	0.5	0.72	0.556624	0.200548
0.5	0.5	2.5	0.3	0.5	0.72	0.648954	0.215111
0.5	0.5	1.2	-0.3	0.5	0.72	0.768284	0.304027
0.5	0.5	1.2	0.6	0.5	0.72	0.745397	0.296626
0.5	0.5	1.2	0.3	1.0	0.72	0.763454	0.302369
0.5	0.5	1.2	0.3	0.5	1.0	0.760363	0.301384

5. Conclusions

In the present work, the effects of heat generation (absorption) and a transverse magnetic field on the flow and heat transfer of a micropolar fluid over a vertical stretching surface with radiation have been studied. The governing fundamental equations are transformed to a system of nonlinear ordinary differential equations which are solved by using shooting technique. The velocity, the angular velocity, and the temperature fields as well as the local skin-friction coefficient and the local Nusselt number are presented for various values of the parameters governing the problem. From the results, we can observe that, the velocity decreases with increasing the magnetic parameter, and the absolute value of the heat absorption parameter, while it increases with increasing the buoyancy parameter, the heat generation parameter, and the Prandtl number. Also, it is found that near the surface the velocity decreases as the slip parameter and the material parameter increase, while the reverse happens as one moves away from the surface. The angular velocity decreases with increasing the material parameter, the slip parameter, the buoyancy parameter, and the heat generation parameter, while it increases with increasing the magnetic parameter, the absolute value of the heat absorption parameter, and the Prandtl number near the surface and the reverse is true away from the surface. In addition the temperature distribution increases with increasing the slip parameter, the heat generation parameter, and the magnetic parameter, but it decreases with increasing the Prandtl number, the buoyancy parameter, the material parameter, and the absolute value of the heat absorption parameter. Moreover, the local skin-friction coefficient increases with increasing the magnetic parameter and the absolute value of the heat absorption parameter, while the local skin-friction decreases with increasing the buoyancy parameter, the slip parameter, and the heat generation parameter. Finally, the local Nusselt number increases with increasing the buoyancy parameter, and the absolute value of the heat absorption parameter, and decreases with increasing the magnetic parameter, the slip parameter, and the heat generation parameter.

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