A Criss-Cross Metamaterial Based Electrically Small Antenna

[1]Kirti Inamdar, [2]Dr. Y. P. Kosta, [3]Dr. S. Patnaik
[1]Research Scholar, ECED, SVNIT, Surat
[2]Director, MEF, Rajkot
[3]Professor & Head, St. Xavier's Institute of Engg., Mumbai

ABSTRACT

Metamaterials (MM) have been able to make their position strong in the world of electromagnetic in the past years. Researchers have come up with several novel shapes which behave as metamaterials. The characteristic parameters of permittivity ε and the permeability μ were extracted by doing several experimentations and found them to be negative. This paper presents such a new shape namely Criss-Cross whose negative behaviour has been discussed. The mathematical modelling for finding the transmission and reflection coefficient of the wave in such medium has also been derived. Further, it has been used to reduce the size of a rectangular patch antenna.

Keywords: Metamaterial, negative index, permeability, permittivity, Jerusalem Cross.

I. Introduction

The material properties of permittivity (ε), permeability (μ) and chirality are all macroscopic concepts emanating from the microscopic electrical response of the matter. When a matter is electrically excited, the response of it gets polarized. This polarization is the addition of all types of polarizations such as electric, magnetic, atomic, ionic, directional and interfacial polarizations. All these add together to give the macroscopic material properties. MM are analyzed by considering the polarizability of the artificial (introduced/embedded) molecules in isolation. So it is required to apply electromagnetic mixing theory rules that will help to develop and estimate the bulk properties of such artificially created materials. One such theoretical development has been done by A. Shivola [1] and has been followed by the author for the formulation of MM theory. In this paper, a general formulation with complete solution for electromagnetic waves interacting with a stratified medium is being covered and then has been specialized to negative isotropic medium. The reflection and transmission of TE and TM waves by a negative isotropic medium has also been derived.

II. NEGATIVE ISOTROPIC MEDIA (NIM)

1. Backward Waves in NIM Media

The MM is always realized in array form. The NIM is defined as one in which all the MM inclusions are of same shape and placed equidistant from each other. For such a medium, the Maxwell's equations are of the same form as given below [2].

$$D = \epsilon E \qquad (2.1.1)$$

$$B = \mu H \qquad (2.1.2)$$

where D, E, B and H are the known standard vectors.

In NIM, the plane EM wave will be of the form

$$\begin{bmatrix} E\ (r,t) \\ H\ (r,t) \end{bmatrix} = \begin{bmatrix} \vec{E} \\ \vec{H} \end{bmatrix} \cos(\vec{k}.\vec{r}.\omega t) \qquad (2.1.3)$$

where k is the wave number for the wave in region r varying with time t.

So the Maxwell's equation become

$$\vec{k} X \vec{E} = \omega\mu\vec{H} \qquad (2.1.4)$$

$$\vec{k} X \vec{H} = -\,\omega\varepsilon\vec{E} \qquad (2.1.5)$$

$$\vec{k}.\vec{E} = 0 \qquad (2.1.6)$$

$$\vec{k}.\vec{H} = 0 \qquad (2.1.7)$$

The power density is given by

$$S = \vec{E}\ X\ \vec{H} = \frac{1}{\omega^2\mu\varepsilon}(kXE)\ X\ (kXH)$$

$$= \frac{-1}{\omega\mu}(kXE)\ X\ E = \frac{k}{\omega\mu}|E|^2$$

$$= \frac{-1}{\omega\varepsilon}\ H\ X\ (kXH) = \frac{k}{\omega\varepsilon}|H|^2 \qquad (2.1.8)$$

In case of NIMs, since both ε and μ are negative, the Poynting's vector is in the opposite direction of $\vec{k}$. Therefore the group and phase velocities with the three vectors $\vec{k}, \vec{E}$ *and* $\vec{H}$ form a Left Handed medium and the power propagates in a direction opposite to $\vec{k}$, thus the plane wave in NIM is a backward wave.

2. Reflection and Transmission of EM wave in a Negative stratified media

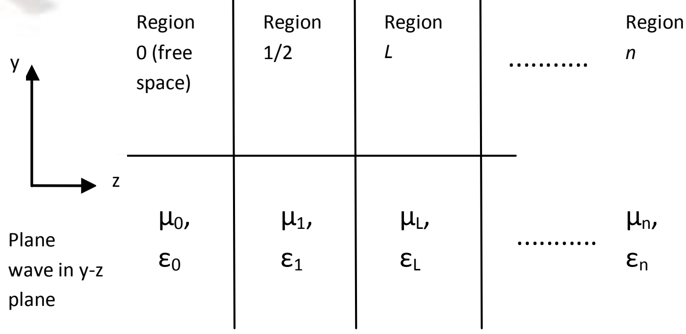


Fig. 2.1. Propagation of a plane wave through stratified media

Consider a plane wave incident on a isotropic stratified media with boundaries at z = d_1, d_2,….d_t as shown in Fig. 2.1. At region-0, the plane of incidence is parallel to y-z plane and the medium comprises of varying μ and ε. The Maxwell's equation for any region L can be separated into TE and TM components governed by E_{Lx} and H_{Lx} [3]. However, only TM components are presented here. They are given as

$$E_{Ly} = -\frac{1}{i\omega\varepsilon_L}\frac{\partial}{\partial z}H_{Lx} \quad (2.2.1)$$

$$E_{Lz} = \frac{1}{i\omega\varepsilon_L}\frac{\partial}{\partial y}H_{Ly} \quad (2.2.2)$$

$$\left(\frac{\partial}{\partial y^2}+\frac{\partial}{\partial t^2}+\omega^2\mu_L\varepsilon_L\right)H_{Lx} = 0 \quad (2.2.3)$$

The total field strength in the L region is

$$H_{Lx} = (E_L^+e^{ik_{Lz}z}+E_L^-e^{-ik_{Lz}z})e^{ik_yy} \quad (2.2.4)$$

$$E_{Ly} = \frac{k_y}{\omega\varepsilon_L}(E_L^+e^{ik_{Lz}z}-E_L^-e^{-ik_{Lz}z})e^{ik_yy} \quad (2.2.5)$$

$$E_{Lz} = \frac{k_y}{\omega\varepsilon_L}(E_L^+e^{ik_{Lz}z}+E_L^-e^{-ik_{Lz}z})e^{ik_yy} \quad (2.2.6)$$

The E_L^+ represents all the wave components with a propagating velocity component in $+\hat{z}$ direction and E_L^- that in $-\hat{z}$ direction. In the region where *L = 0,*

$$E_0^+ = E_0 \; and \; E_0^- = RE_0 \quad (2.2.7)$$

In the region say *t = L+1,*

$$E_t^+ = TE_0 \; and \; E_t^- = 0 \quad (2.2.8)$$

This is because region t is a semi-infinite region and has no wave propagating in $+\hat{z}$. T denotes the transmitted wave amplitude and R denotes the reflected wave amplitude. Applying and matching boundary conditions at the boundary, we get

$$\rho_{L(L+1)}=\frac{\varepsilon_Lk_{(L+1)z}}{\varepsilon_{L+1}k_{Lz}} \quad (2.2.9)$$

(2.2.9) is the reflection coefficient of the TM wave in the region *L* with effect of region *L+1*. Also,

$$E_L^+e^{ik_{Lz}d_{L+1}} = \frac{1}{2}(1+\rho_{L(L+1)})\{E_{l+1}^+e^{ik_{(L+1)z}d_{L+1}} + RL(L+1)EL+1-e-ikL+1ZdL+1 \quad (2.2.10)$$

$$E_L^-e^{-ik_{Lz}d_{L+1}} = \frac{1}{2}(1+\rho LL+1RLL+1El+1+eikL+1ZdL+1 \; EL+1-e-ikL+1ZdL+1 \quad (2.2.11)$$

where $R_{L(L+1)} = \frac{1-\rho_{L(L+1)}}{1+\rho_{L(L+1)}}$

(2.2.11) represents the reflection coefficient for the wave in the region *L* caused by the boundary separating regions L and L+1. Also,

$$\rho_{(L+1)L} = \frac{1}{\rho_{L(L+1)}} \quad (2.2.12)$$

Therefore,

$$R_{(L+1)L} = -R_{L(L+1)} \quad (2.2.13)$$

This implies that the reflection coefficient in the region *L+1*,which is $R_{(L+1)L}$ is caused by the boundary separating regions *L+1* and *L* is equal to the negative of $R_{L(L+1)}$.

If the ratio of (2.2.11) and (2.2.10) is taken and solved by using continued fraction approach, one can find the reflection coefficient of the wave in such negative index media given by

$$R = \frac{e^{izk_{0z}d_1}}{R_{01}} + \frac{\left[1-\left(1/R_{01}^2\right)e^{iz(k_1z+k_{0z})d_1}\right]}{\left(1/R_{01}\right)e^{izk_{1z}d_1}} + \frac{e^{izk_{0z}d_2}}{R_{12}}\frac{\left[1-\left(1/R_{12}^2\right)e^{iz(k_2z+k_{1z})d_2}\right]}{\left(1/R_{12}\right)e^{izk_{2z}d_2}} \dots\dots\dots\frac{e^{izk_{(n-1)z}d_n}}{R_{(n-1)n}} + \frac{\left[1-\left(1/R_{(n-1)n}^2\right)e^{iz(k_nz+k_{(n-1)z})d_n}\right]}{\left(1/R_{(n-1)1}\right)e^{izk_{nz}d_n}} + R_{nt}e^{izk_{nt}d_{(n+1)}} \quad (2.2.14)$$

Thus the general solution of the EM wave in MM is presented in this section. Next step is to find the material parameters i.e. ε and μ. Traditional effective material analysis is shown in [6] and [7] and is famously known as NRW (Nicolson-Ross and Weir) approach. The authors have used the same for the Criss-Cross structure.

III. CRISS-CROSS METAMATERIAL

The Criss-Cross shape is derived from the Jerusalem Cross. In this shape, a cross is placed on upper face of substrate and the same cross is twisted by 45° and placed on the back side of the substrate. This gives birth to a criss-cross shape. The dimensions of this unit cell are 6.35 x 6.35mm made up of copper with thickness of 0.017mm. The width of each strip is 1.016mm. The substrate used is Rogers RT/ Duroid 5880 with ε_r = 2.2 and a thickness of 1.016mm. Its S parameters were obtained in HFSS [8]. The parameters ε and μ were obtained by using equations in earlier section. As mentioned in earlier chapter, the coding of all equations was done in MATLAB. In first step, [4] and [5] were implemented. The results obtained were matching to a good extent with that published in paper. This helped to testify the correctness and accuracy of the code. Then in the same code, the S parameters of the criss-cross shape were imported and thus results were obtained. Fig. 3.1 is the Criss-Cross unit cell developed in HFSS with structural details as mentioned above. The results for Criss-

Cross unit cell are only presented in this paper. If one observe the ε and μ graph in Fig. 3.4 & Fig. 3.5 respectively, it is exhibiting negativity is the frequency band of 5 to 9GHz. The negative parameter bandwidth obtained is thus 61.53% which is quite high as compared to Square SRR structure (17-18%) [4] or other cross structures (31%) [5].

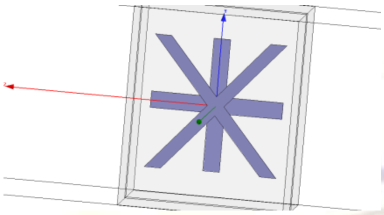
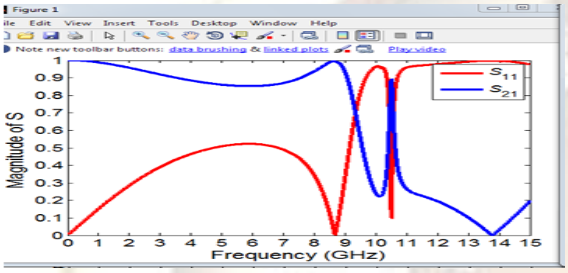


Fig. 3.1 A Criss-Cross shape metamaterial unit cell



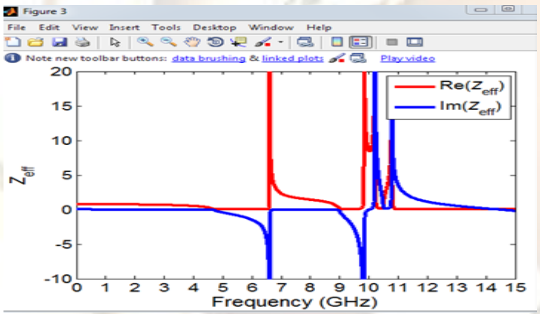
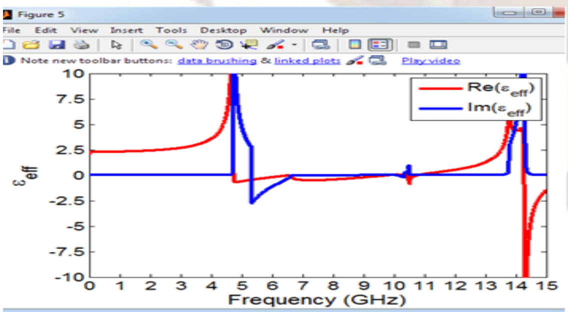


Fig. 3.2 S-parameter for Criss-Cross Unit cell in MATLAB Fig. 3.3. Impedance plot for Criss-Cross Unit cell in MATLAB



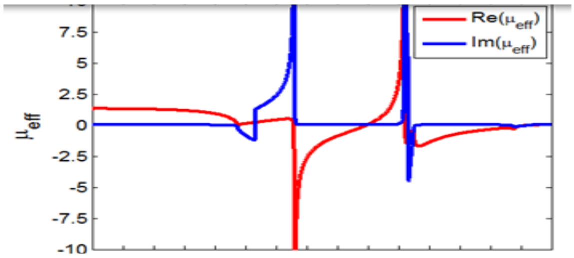
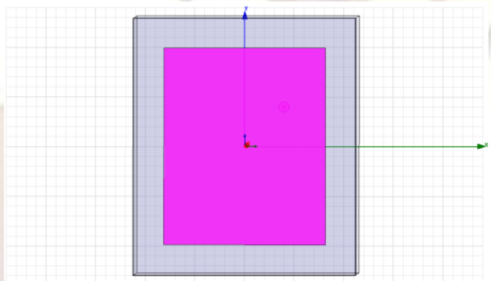


Fig. 3.4. Effective ε plot for Criss-Cross Unit cell in MATLAB Fig. 3.5. Effective μ plot for Criss-Cross Unit cell in MATLAB

IV. CRISS-CROSS BASED PATCH ANTENNA

After confirmation of negative band exhibition of this shape, a 3 X 3 array of the unit cell was made and was applied to the substrate of the rectangular patch antenna. The MM is exhibiting negativity is the frequency band of 5 to 9 GHz. As per the thumb rule, the unit cell size less than quarter wavelength behaves like MM therefore, at higher frequency, its behaviour as MM is not effective. Therefore, band 5 to 7 GHz is considered to be the effective negative medium band and due to this the patch was designed for f_0 = 6 GHz. The length (l) of the patch is 16.3 mm and its width (w) is 19.8 mm. The substrate dimensions are l = 22.4 mm and w = 25.9 mm with a thickness of 1.016 mm with ε_r = 2.2. Fig. 4.1 shows the conventional patch antenna and Fig. 4.2 its tri-band response.



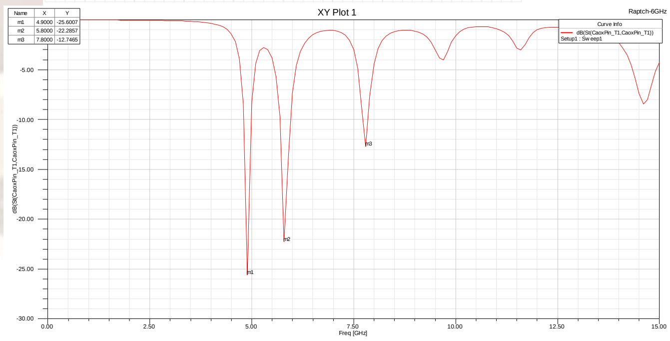
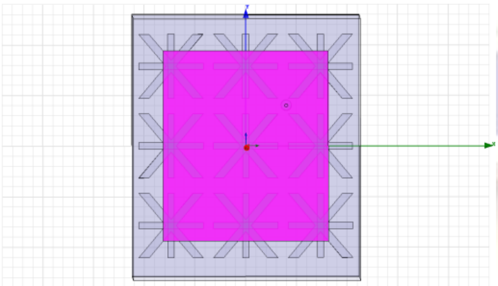


Fig. 4.1. A 6 GHz conventional patch antenna
Fig. 4.2. S-parameter for the conventional patch antenna.

At f_1 = 4.9 GHz, S_{11} = -25.6dB, At f_2 = 5.8 GHz, S_{11} = -22.28dB

and at f_3 = 7.8GHz, S_{11} = -12.74dB

Once MM are included in the substrate, the resonating frequency in each band shifted towards the left of the frequency axis as compared to the original one. In order to get the original frequency back, the physical parameters of the design like l, w, feed position and distance between unit cells was varied and optimised values were achieved. The S parameter response for these optimised values is shown in Fig. 4.4.



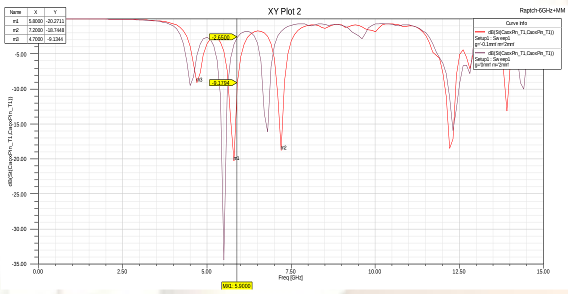


Fig. 4.3. A 6 GHz Criss-Cross MM based patch antenna Fig. 4.4 S-parameter for Criss-Cross MM based patch antenna.

At f_1 = 4.7 GHz, S_{11} = -9.13dB, At f_2 = 5.8 GHz, S_{11} = -20.27dB

and at f_3 = 7.2GHz, S_{11} = -18.74dB.

The length and width of the MM based patch antenna is reduced to l = 14.3 mm and w = 18.8 mm. This reduction in the area of the patch is due to introduction of MM.

V. CONCLUSION

The original dimensions of the patch were 16.3 X 19.8 mm whereas one with MM inclusions made in the substrate has dimensions of 14.3 X 18.8 mm. Therefore, the % reduction in area is 16.7%. The other performance parameters (like gain, BW, VSWR) of a conventional patch are though not improved after the introduction of MM but they are not deteriorated too. Thus we can say that the simple patch can resonate with reduced dimensions without disturbing the other performance parameters if it has MM inclusions in its substrate. Bandwidth of antenna can be improved by making some slots in the patch, stacking etc. Mathematical modelling of the Criss-Cross MM has been presented in this paper and thus its negative permittivity and permeability are retrieved. It can be said that this new shape can make its place in the DNG family and may open many more perspectives for research.

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