A.Jayalakshmi, C.Janaki / International Journal of Engineering Research and Applications (IJERA) ISSN: 2248-9622 www.ijera.com
Vol. 3, Issue 2, March -April 2013, pp.1286-1290

On wgrα-Closed and wgrα-Open Maps In Topological Spaces

1. A.Jayalakshmi & 2.C.Janaki

1. Department of Mathematics, Sree Narayana Guru College, Coimbatore, TN,India.
2 .Department of Mathematics, L.R.G.Govt.Arts.College for Women, Tirupur, TN,India.

Abstract

The aim of this paper is to introduce wgrα-closures and obtain a characterization of wgrα-continuous functions. Also we introduce and discuss the properties of wgrα-closed and wgrα-open maps.

1. Introduction

Generalized closed sets in a topological space were introduced by N.Levine[10] in order to extend many of the properties of closed sets. Regular open and regular α-open sets have been introduced and investigated by Stone[16] ,A.Vadivel and K.Vairamanickam[17] respectively. Balachandran, Sundram and Maki[4], Bhattacharyya and Lahiri[5], Arockiarani[2],Dunham[7], Gnanambal[8], Malghan [11], Palaniappan and Rao[13], Park J.K and Park J.H[14], Arya and Gupta[3], Devi[6] ,C.Janaki[9], A.Vadivel and K.Vairamanickam[17] have worked on generalized closed sets and studied some of their properties. Balachandran et al [4] introduced the concept of generalized continuous maps and generalized irresolute maps of a topological space. In this paper, we study the properties of wgrα-continuous functions and introduce the concept of a new class of maps called wgrα-closed and wgrα-open maps.

2. Preliminaries

In this section we recall some definitions which are used in this paper.

Definition: 2.1
A subset A of a topological space (X,τ) is called α-closed [14] if $A\subset int(cl(int(A))$.

Definition: 2.2
A subset A of a topological space (X,τ) is called gα-closed [6] if $\alpha cl(A)\subset U$,when ever $A\subset U$ and U is α-open in X.

Definition: 2.3
A subset A of a topological space (X,τ) is called swg-closed [12] if $cl(int(A))\subset U$, whenever $A\subset U$ and U is semi-open in X.

Definition: 2.3
A subset A of a topological space (X,τ) is called rwg-closed [12] if $cl(int(A))\subset U$, whenever $A\subset U$ and U is regular-open in X.

Definition: 2.4
A subset A of a topological space (X,τ) is called rgα-closed [17] if $\alpha cl(A)\subset U$, whenever $A\subset U$ and U is regular α-open in X.

Definition: 2.5
A subset A of a topological space (X,τ) is called ω-closed [1,15] if $cl(A)\subset U$, whenever $A\subset U$ and U is semi-open in X.

3. wgrα-continuous functions

Theorem: 3.1
If A and B are subsets of a space X. Then (i)wgrα-cl(X)=X and wgrα-cl(φ)=φ.(ii)$A\subset$ wgrα-cl(A).
(iii) If B is any wgrα-closed set containing A, then wgrα-cl(A)$\subset$B.
(iv) If $A\subset B$ then wgrα-cl(A)$\subset$wgrα-cl(B).
(v) wgrα-cl(A)=wgrα-cl(wgrα-cl(A)).

Proof
Straight forward.

Theorem: 3.2
If A and B are subsets of a space X, then wgrα-cl(A) $\cup$ wgrα-cl(B)$\subset$wgrα-cl $(A\cup B)$.

Theorem: 3.3
If A and B are subsets of a space X, then wgrα-cl(A $\cap$ B)$\subset$wgrα-cl(A)$\cap$ wgrα-cl(B).

Theorem: 3.4
Let A be subset of (X,τ) and $x\in X$. Then $x\in$ wgrα-cl(A) if and only if $V\cap A$=φ for every wgrα-open set V containing x.

Proof
Suppose there exist a wgrα-open set V containing x such that $V\cap A$ =φ. Since $A\subset X-V$,wgrα-cl(A) $\subset X-V$ and this implies $x\notin$wgrα-cl(A),a contradiction. Conversely, suppose that $x\notin$wgrα-cl(A),then there exist a wgrα-closed set F containing A such that $x\notin$ F.Then $x\in X-F$ and X−F is wgrα-open. Also $(X-F)\cap A$=φ, a contradiction.

Theorem: 3.5
If $A\subset X$ is wgrα-closed, then wgrα-cl(A)=A.

Proof
Straight forward.

Remark: 3.6
Converse of the above theorem need not be true as seen from the following example.

Example: 3.7
Let X={a,b,c,d},τ = {φ,X,{a},{b},{a,b}, {a,b,c} },then WGRαC(X)= {φ,X,{c}, {d}, {a,b},{c,d},{a,b,c},{a,b,d}{a,c,d},{b,c,d}}. Now wgrα-cl({a,c})={a,c},but {a,c} is not wgrα-closed subset in X.

Theorem: 3.8
If f:(X,τ)→(Y,σ) is wgrα-continuous, then f(wgrα-cl(A))$\subset$cl(f(A)) for every subset A of X.

Proof

Let $A\subset X$. Since f is wgrα-continuous and $A\subset f^{-1}(cl(f(A)))$,we obtain wgrα-cl(A)$\subset f^{-1}(cl(f(A)))$.Hence f(wgrα-cl(A))$\subset$cl(f(A)).

Remark: 3.9

Converse of the above theorem need not be true as seen in the following example.

Example: 3.10

Let X={a,b,c,d},τ = {φ,X,{b},{c},{b,c},{b,c,d}},σ={φ,Y,{a},{a,c,d}}.If A={a}, then f(wgrα-cl(A)) $\subset$cl(f(A),but f is not wgrα-continuous.

Definition: 3.11

For a topological space X,

(i)$\tau^*_{wgr\alpha}$={U$\subset$X:wgrα-cl(X−U)=X−U}

(ii) $\tau^*_{rg\alpha}$={ U$\subset$X:rgα-cl(X−U)=X−U}

(iii) τ^*_{rwg}={ U$\subset$X:rwg-cl(X−U)=X−U}

(iv) τ^*_{α}={U$\subset$X:αcl(X−U)=X−U}

Theorem: 3.12

Let f: (X,τ)→(Y,σ) be a function .The following statements are equivalent.

(i)For each $x\in X$ and each open set V containing f(x), there exists a wgrα-open set U containing x such that f (U) $\subset$V.

(ii) For every subset A of X, f(wgrα-cl(A))$\subset$cl(f(A)).

(iii)Suppose $\tau^*_{wgr\alpha}$ is a topology,the function f : $(X,\tau^*_{wgr\alpha})\to(Y,\sigma)$ is continuous.

Proof

(i)$\Rightarrow$(ii) Let y$\in$f(wgrα-cl(A)) and let V be any open neighborhood of Y. Then there exists a $x\in X$ and a wgrα-open set U such that f(x) =y, $x\in U$, $x\in$ wgrα-cl(A) and f(U)$\subset$V.By lemma 3.4,U$\cap$A$\neq\phi$ and hence f(A)$\cap$V$\neq\phi$. Therefore, y=f(x)$\in$ cl(f(A)).

(ii)$\Rightarrow$(iii) Let B be closed in (Y,σ).By hypothesis,f(wgrα-cl(f^{-1}(B)))$\subset$cl(f(f^{-1}(B)))$\subset$ cl(B)=B,which implies wgrα-cl(f^{-1}(B))$\subset f^{-1}$(B).Hence f^{-1}(B) is closed in $(X,\tau^*_{wgr\alpha})$.

(ii)$\Rightarrow$(i) Let $x\in X$ and V be any open set in (Y,σ) containing f(x). Let A=f^{-1} (Y–V). Since f(wgrα-cl(A))$\subset$cl(f(A))$\subset$Y−V,then wgrα-cl(A)=A. Since $x\notin$wgrα-cl(A), there exists a wgrα-open set U containing x such that U$\cap$A=ϕ and hence f(U)$\subset$ f(X−A)$\subset$V.

(iii)$\Rightarrow$(ii) Let A be any subset of X .Since cl(f(A)) is closed in (Y,σ) and f :$(X,\tau^*_{wgr\alpha})\to(Y,\sigma)$ is continuous,f^{-1} (cl(f(A)) is closed in $(X,\tau^*_{wgr\alpha})$ and hence wgrα-cl(f^{-1}(cl(f(A))= f^{-1}(cl(f(A)).Also since A$\subset f^{-1}$(f(A))$\subset f^{-1}$(cl(f(A))).By lemma 3.1,we have wgrα-cl(A)$\subset$wgrα-cl(f^{-1}(cl(f(A))= f^{-1} (cl(f(A))).So f(wgrα-cl(A))$\subset$ cl(f(A)).

Theorem: 3.13

If WGRαO(X,τ) is a topology, then $\tau^*_{wgr\alpha}$ is a topology.

Proof

Clearly, ϕ, X $\in\tau^*_{wgr\alpha}$ .Let {A_i : i$\in\nabla$}$\in\tau^*_{wgr\alpha}$ .Then wgrα-cl(X-($\bigcup A_i$))= wgrα-cl($\bigcap$(X−A_i)=$\bigcap$(X−A_i)= X-($\bigcup A_i$).Hence, $\bigcup A_i\in\tau^*_{wgr\alpha}$.Let A,B $\in\tau^*_{wgr\alpha}$. Now wgrα-cl((X-A)$\bigcup$(X−B))= wgrα-cl(X-A)$\bigcup$ wgrα-cl(X-B)=(X−A)$\bigcup$(X−B). Hence, A$\bigcap$B$\in\tau^*_{wgr\alpha}$. Thus $\tau^*_{wgr\alpha}$ is a topology.

Theorem: 3.14

(a) Every wgrα-closed is α-closed if and only if $\tau^*_{wgr\alpha}=\tau^*_{\alpha}$.

(b) Every wgrα-closed is closed if and only if $\tau^*_{wgr\alpha}$=τ.

(c) Every wgrα-closed is rgα-closed if and only if $\tau^*_{wgr\alpha}=\tau^*_{rg\alpha}$.

(d) Every rwg-closed is wgrα-closed if and only if $\tau^*_{wgr\alpha}=\tau^*_{rwg}$.

Proof

(a) A$\in\tau^*_{wgr\alpha}$, Then wgrα-cl(X−A)=X−A.By hypothesis, αcl(X−A)=wgrα-cl(X− A) =X−A.This implies A$\in\tau^*_{\alpha}$. Conversely, let A be wgrα-closed, then wgrα-cl(A)=A. Hence X−A$\in\tau^*_{wgr\alpha}$ implies X−A $\in\tau^*_{\alpha}$. Therefore A is α-closed.

(b)A$\in\tau^*_{wgr\alpha}$. Then wgrα-cl(X−A)=X−A.By hypothesis,X−A=cl(X−A).This implies A$\in$τ. Conversely, let A be wgrα-closed, then wgrα-cl(A)=A. Hence X−A $\in\tau^*_{wgr\alpha}$ implies X−A$\in$τ .Therefore A is closed.

(c) A$\in\tau^*_{wgr\alpha}$,then wgrα-cl(X−A)=X−A.By hypothesis, rgα-cl(X−A) = wgrα-cl(X− A) =X−A.This implies A$\in\tau^*_{rg\alpha}$. Conversely, let A be wgrα-closed, then wgrα-cl(A)=A. Hence X−A$\in\tau^*_{wgr\alpha}$ implies X−A $\in\tau^*_{rg\alpha}$. Therefore A is rgα-closed.

(d) A$\in\tau^*_{rwg}$, Then rwg-cl(X−A)=X−A.By hypothesis, wgrα-cl(X−A) = rwg-cl(X− A) = X−A.This implies A$\in\tau^*_{wgr\alpha}$. Conversely, let A be rwg-closed, then rwg-cl(A)=A. Hence X−A$\in\tau^*_{rwg}$ implies X−A $\in\tau^*_{wgr\alpha}$. Therefore A is wgrα-closed.

4. wgrα-$T_{1\backslash 2}$ Spaces

Definition: 4.1

A space (X,τ) is called wgrα-$T_{1\backslash 2}$ space if every wgrα closed set is α-closed.

Definition: 4.2

A space (X,τ) is called $T_{wgr\alpha}$-space if every wgrα closed set is closed.

Theorem: 4.3

For a topological space(X,τ) the following conditions are equivalent

(i)X is wgrα-$T_{1\backslash 2}$ space.

(ii) Every singleton of X is either regular α-closed (or) α-open.

Proof

(i)$\Leftrightarrow$(ii)Let $x\in X$ and assume that {x} is not regular α-closed. Then clearly X−{x} is not regular α-open and X-{x} is trivially wgrα-closed set. By (i) it is α-closed and thus {x} is α-open. (ii)$\Leftrightarrow$(i) let A$\subset$X be wgrα-closed. Let $x\in$ cl(int(A)). To show $x\in A$.

Case (i)

The set {x} is regular α-closed. Then if $x\notin A$, then A$\subset$X-{x}.Since X is wgrα-closed and X-{x}is

regular-α-open, $cl(int(A)) \subset X$-{x}and hence $x \notin cl(int(A))$.This is a contradiction. Therefore, $x \in A$.
Case(ii)
The set {x} is α-open. Since $x \in cl(int(A))$,then $\{x\} \cap A \neq \phi$ implies $x \in A$. In both the cases $x \in A$. This shows that A is α-closed.

Theorem: 4.4
Let (X,τ) be any topological space and (Y,σ) be a $T_{wgr\alpha}$-space and $f:(X,\tau) \to (Y,\sigma)$ be a map.Then the following are equivalent.
(i) f is wgrα-irresolute.
(ii) f is wgrα-continuous.

Proof
(i)$\Rightarrow$(ii) Let U be a closed set in (Y,σ).Since f is wgrα-irresolute, $f^{-1}(U)$ is wgrα-closed in (X,τ).Thus f is wgrα-continuous.
(ii)$\Rightarrow$(i) Let F be a wgrα-closed set in (Y,σ).Since Y is a $T_{wgr\alpha}$-space,F is closed in Y.By hypothesis $f^{-1}(F)$ is wgrα-closed in(X,τ).Therefore f is wgrα-irresolute.

Theorem: 4.5
(i) $\alpha O(X,\tau) \subset WGR\alpha O(X,\tau)$.
(ii) A space is wgrα-$T_{1\backslash 2}$ space if and only if $\alpha O(X,\tau)=WGR\alpha O(X,\tau)$.

Proof
(i) Let A be α-open . Therefore X−A is α-closed. Every α-closed set is wgrα-closed. Therefore X−A is wgrα-closed, which implies A is wgrα-open.
(ii) Let X be wgrα-$T_{1\backslash 2}$ space. Let $A \in WGR\alpha O(X,\tau)$,X –A is wgr$\alpha$-closed implies X−A is α-closed. Hence $A \in \alpha O(X,\tau)$.Therefore $WGR\alpha O(X,\tau) \subset \alpha O(X,\tau)$ and $\alpha O(X,\tau) \subset WGR\alpha O(X,\tau)$.Which implies $\alpha O(X,\tau)= WGR\alpha O(X,\tau)$.Conversely, let $\alpha O(X,\tau)= WGR\alpha O(X,\tau)$. A is wgr$\alpha$-closed implies X−A is wgrα-open. By assumption X−A is α-open and hence A is α-closed.

Theorem: 4.6
Let (Y,σ) be a $T_{wgr\alpha}$-space,f: $(X,\tau) \to (Y,\sigma)$ and $g:(Y,\sigma) \to (Z,\eta)$ be two wgrα-continuous functions, then $g \circ f : (X,\tau) \to (Z,\eta)$ is also wgrα-continuous.

Proof
Let V be any closed set in (Z,η).Since $g^{-1}(V)$ is wgrα-closed in (Y,σ).wgrα-continuity of f implies that $f^{-1}(g^{-1}(V))=(g \circ f)^{-1}(V)$ is wgrα-closed. Hence $(g \circ f)$ is wgrα-continuous.

5. wgrα-closed maps and wgrα-open maps

Definition: 5.1
A map f: $X \to Y$ is said to be wgrα-open if the image of every open set set in X is wgrα-open in Y.

Definition: 5.2
A map f: $X \to Y$ is said to be prewgrα-closed if the image of every wgrα-closed set set in X is wgrα-closed in Y.

Theorem: 5.3
Every open map is wgrα-open, but not conversely.

Proof
Let f: $X \to Y$ is an open map and V be an open set in X. Then f (V) is open and hence wgrα-open in Y. Thus f is wgrα-open.

Example: 5.4
Let $X=\{a,b,c\}=Y,\tau=\{\phi,X,\{a\},\{b\},\{a,b\}\},\sigma=\{\phi,Y,\{a,b\}\}$ and let f be the identity map.Thus f is wgrα-open, but not open.

Theorem: 5.5
A map $f:X \to Y$ is wgrα-closed if and only if subset S of Y and for each open set U containing $f^{-1}(S)$ there is a wgrα-open set V of Y such that $S \subset V$ and $f^{-1}(V) \subset U$.

Proof
Suppose f is wgrα-closed.Let S be a subset of Y and U be an open set of X such that $f^{-1}(S) \subset U$,then $V=Y-f(X-U)$ is a wgrα-open set containing S such that $f^{-1}(V) \subset U$.Conversely,suppose that f is a closed set of X,then $f^{-1}(Y-f(F)) \subset X-F$ and X−F is open.By hypothesis,there is a wgrα-open set V of Y such that $Y-f(F) \subset V$ and $f^{-1}(V) \subset X-F$.Therefore, $F \subset X-f^{-1}(V)$.Hence $Y-V \subset f(F) \subset f(X-f^{-1}(V)) \subset Y-V$,which implies(F)=Y−V.Since Y−V is wgrα-closed,f(F) is wgrα-closed and thus f is wgrα-closed map.

Theorem: 5.6
If f: $(X,\tau) \to (Y,\sigma)$ is wgrα-closed and A is closed subset of X, then $f|A:A \to Y$ is wgrα-closed.

Proof
Let $B \subset A$ be closed in A,then B is closed in X,since A is closed in X.f(B) is wgrα-closed in Y as f is wgrα-closed map. But $f(B)=(f|A)(B)$.So$(f|A)(B)$ is wgrα-closed in Y. Therefore $f|A$ is wgrα-closed.

Theorem: 5.7
If f: $(X,\tau) \to (Y,\sigma)$ is wgrα-closed and A is closed subset of X, then $f|A:A \to Y$ is continuous and wgrα-closed.

Proof
Let F be a closed set of A .Then F is wgrα-closed set of X.By theorem 5.9,$(f|A)(F)=f(F)$ is wgrα-closed set of Y.$(f|A)$ is wgrα-closed and continuous.

Theorem: 5.8
If a map f: $X \to Y$ is closed and a map $g:Y \to Z$ is wgrα-closed,then $g \circ f:X \to Z$ is wgrα-closed.

Proof
Let H be a closed in X, then f(H) is closed and $(g \circ f)(H)=g(f(H))$ is wgrα-closed.

Theorem: 5.9
Every α-open,ω-open,gα-open,rgα-open ,swg-open maps are wgrα-open, but not conversely.

Proof
Straight forward.

Example: 5.10
Let $X=Y=\{a,b,c,d\},\tau = \{\phi,X,\{a\},\{a,b\},\{a,b,c\}\}$ and $\sigma = \{\phi,Y,\{a\},\{b\},\{a,b\}, \{a,b,c\}\}$. Define f is an identity map.Hence f is wgrα-open map , but not α-open,

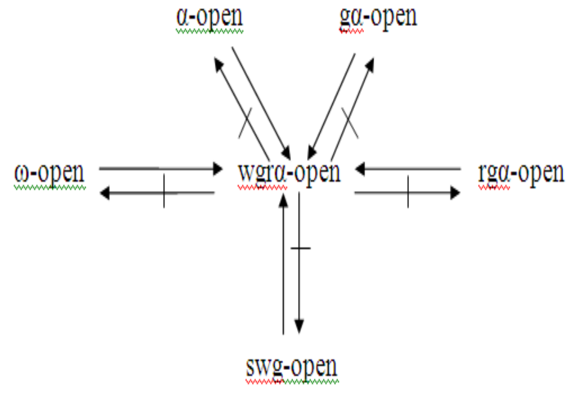
ω-open,gα-open,swg-open maps.

Example: 5.11

Let X=Y={a,b,c,d},τ ={ϕ,X,{a},{c},{a,b,c}}and σ = {ϕ,Y,{a},{c,d},{a,c,d}}. Define f is an identity map.Hence f is wgrα-open map , but not rgα-open.

Remark:5.12

The above discussions are summarized in the following diagram.



Remark: 5.13

Composition of two wgrα-closed maps need not be wgrα-closed map.

Example: 5.14

Let X=Y=Z={a,b,c},τ={φ,X,{a},{a,b}},σ={φ,Y,{c},{a, c}}and η={φ,X,{a},{b}, {a,b}}.Define f: X→Y is an identity map. f and g are wgrα-closed, but g∘f is not wgrα-closed.

Theorem: 5.15

If f: X →Y is a closed map and g: Y →Z is a wgrα closed map, then g∘f: X →Z is wgrα closed.

Proof

Let H be a closed set in X. Then f(H) is closed. (g∘f) (H=g(f(H)) is wgrα closed as g is wgrα closed map. Thus g∘f is wgrα closed.

Theorem: 5.16

Every prewgrα-closed map is a wgrα-closed map.

Remark: 5.17

Converse of the above theorem need not be true as seen in the following example.

Example: 5.18

Let X=Y=Z={a,b,c},τ ={φ,X,{a},{b},{a,b}},σ ={φ,Y,{a},{a,b}and η={φ,X,{a}, {b},{a,b}}.Define f: X→Y as a identity map.f is a wgrα-closed map,but f is not pre-wgrα-closed.

Theorem: 5.19

If f: X→Y is a bijective mapping , then the following statements are equivalent
(i) f is a wgrα-open map
(ii) f is a wgrα-closed map
(iii) f^{-1}:Y→X is wgrα-continuous

Proof

(i)⟹(ii)Let U be closed in X and f be a wgrα-open map. Then X−U is open in X. By assumption,we get f(X−U) is a wgrα-open set in Y.That is Y−f(X−U)=f(U) is wgrα-closed in Y.

(ii)⟹(iii) Let U be closed in X .By assumption, f(U) is wgrα-closed in Y.As f (U) =$(f^{-1})^{-1}$ (U),f^{-1} is wgrα-continuous.

(iii)⟹(i) Let U be open in X.By assumption $(f^{-1})^{-1}$(U) =f(U).That is,f(U) is wgrα-open in Y.Hence f is a wgrα-open map.

Theorem: 5.20

Suppose WGRαO(X,τ) is closed under arbitrary unions. Let f: X→Y be a mapping. Then the following statements are equivalent
(i)f is a wgrα-open .
(ii) For a subset A of (X,τ),f(int(A))⊂wgrα-int(f(A)).
(iii) For each x∈X and for each neighborhood U of x in (X,τ),there exists a wgrα-neighborhood W of f(x) in (Y,σ) such that W⊂f(U).

Proof

(i)⟹(ii) Suppose f is a wgrα-open mapping.Let A ⊂X.Since int(A) is open in (X,τ).f(int(A)) is wgrα-open in (Y,σ).Hence f(int(A))⊂f(A) and we have f(int(A))⊂wgrα-int(f(A)).

(ii)⟹(iii) Let U be any arbitrary neighborhood of x in (X,τ).Then there exists an open set G such that x ∈G⊂U. By assumption, f(G)=f(int(G))⊂wgrα-int(f(G)) and f(G)=wgrα-int(f(G)).Hence f(G) is wgrα-open in (Y,σ) and f(x)∈f(G)⊂f(U) and by taking W=f(G).(iii) holds.

(iii)⟹(i) Let U be an open set in (X,τ) such that x ∈U and f(x)=y. Then x∈U and for each y∈ f(U),there exists a wgrα-neighborhood W_y of y in (Y,σ) such that W_y⊂f(U).Since W_y is a neighborhood of y there exists a wgrα-open set V_y in (Y,σ) such that y∈V_y⊂W_y.Therefore f(U)=⋃{V_y:y∈f(U)}.Hence f(U) is a wgrα-open set of (Y,σ) and f is a wgrα-open mapping.

Definition: 5.21

A function f:X→Y is said to be strongly wgrα-irresolute if f^{-1}(V) is open in X for every wgrα-open set V of Y.

Theorem: 5.22

For any bijective f:X→Y, then the following statements are equivalent
(i) f^{-1}:Y→X is wgrα-irresolute .
(ii) f is a pre-wgrα-open map.
(iii) f is a pre-wgrα-closed map.

Proof

(i)⟹(ii) Let U be wgrα-open in X .By (i) $(f^{-1})^{-1}$(U)=f(U) is wgrα-open in Y. So f is a pre-wgrα-open map.

(ii)⟹(iii) Let V be wgrα-closed in X. By (ii) f(X−V)=Y−f(V) is wgrα-open in Y. That is f(V) is wgrα-closed in Y and so f is an pre-wgrα-closed map.

(iii)⟹(i) Let V be a wgrα-closed set in X. By (iii),f(V)=$(f^{-1})^{-1}$(V) is wgrα-closed in Y.Hence (i) holds.

Theorem: 5.23

Let f: $(X,\tau)\rightarrow(Y,\sigma)$ and $g:(Y,\sigma)\rightarrow(Z,\eta)$ be two mappings and let $g\circ f : (X,\tau) \rightarrow(Z,\eta)$ be wgrα-closed. Then
(i)If f is continuous and surjective, then g is wgrα-closed. (ii) If g is wgrα-irresolute and injective ,then f is wgrα-closed (iii) If f is wgrα-continuous,surjective and (X,τ) is a wgrα-space, then g is wgrα-closed. (iv) If g is strongly wgrα-irresolute and injective, then f is closed.

Proof

(i)If f is continuous, then for any closed set A of Y. $f^{-1}(A)$ is closed in X . Also $g\circ f$ is wgrα-closed implies $(g\circ f)(f^{-1}(A))=g(f(f^{-1}(A)))=g(A)$ is wgrα-closed in Z and g is wgrα-closed map.

(ii)Let A be a closed set in (X,τ).Then $(g\circ f)(A)$ is wgrα-closed in Z and$g^{-1}(g\circ f)$ (A) =f(A) is wgrα-closed in Y. Hence f is wgrα-closed.

(iii)Let A be a closed set of (Y,σ).Then $f^{-1}(A)$ is wgrα-closed set in (X,τ) and g(A) is wgrα-closed set in (Z,η).

(iv) Let A be closed in (X,τ), then $(g\circ f)(A)$ is wgrα-closed in (Z,η),g is strongly wgrα-irresolute implies $g^{-1}(g\circ f)(A)=f(A)$ is closed in (Y,σ) and f is a closed map.

References

[1] Ahmad Al-omari and Salmi Md Noorani, Regular Generalized ω-closed sets, International Journal of Mathematics and mathematical Sciences,Volume 2007, article(ID),16292.

[2] I.Arockiarani,Studies on Generalizations of Generalized Closed Sets and Maps in Topological Spaces,Ph.D Thesis,Bharathair University,Coimbatore, (1997).

[3] S.P.Arya and R.Gupta, On Strongly Continuous Mappings,Kyung pook Math.,J.14(1974),131-143.

[4] K.Balachandran,P.Sundaram and H. Maki , On Generalized Continuous Maps in Topological Spaces,Mem.I ac Sc.Kochi Univ . Math., 12(1991),5-13.

[5] P.Bhatttacharyya and B.K.Lahiri, Semi-generalized Closed Sets in Topology, Indian J. Math.,29(1987),376-382.

[6] R.Devi,K.Balachandran and H.Maki, On Genaralized α-continuous maps,Far. East J. Math. Sci.Special Volume,part 1(1997),1-15.

[7] W.Dunham,A New Closure Operator for Non-T_1 Topologies, Kyunpook Math.J., 22(1982),55-60.

[8] Y.Gnanambal, On Generalized Pre-regular closed sets in Topological Spaces, Indian J. Pure Appl.Math., 28(1997),351-360.

[9] C.Janaki,Studies On πgα-Closed Sets in Topology,Ph.D Thesis, Bharathair University 2009.

[10] N.Levine,Generalized closed sets in Topology, Rend.Circ. Mat.Palermo , 19 (1970),89-96.

[11] S.R.Malghan,Generalized closed Maps , J. Karnatak Univ. Sci.,27(1982),82-88.

[12] N.Nagaveni,Studies on Generalizations of Homeomorphisms in Topological Spaces,Ph.D Thesis,Bharathair University,Coimbatore,(1999).

[13] N.Palaniappan and K.C .Rao,Regular Generalized Closed Sets,Kyung-pook, Math.J.,33(1993),211-219.

[14] J.H.Park and J.K.Park .On πgp Continuous functions in topological Spaces, chaos,Solitons and fractals 20(2004),467-477.

[15] R.Parimelazhagan and N.Nagaveni, Weak Generalized Closed Map .World Academy of Science,Engineering and Technology 50 2009(1041-1045).

[16] Stone.M,Application of the theory of Boolean Rings to General Topology, Trans. Amer.Math.Soc.,41(1937),374-481.

[17] A.Vadivel and K. Vairamanickam, rgα-Closed Sets and rgα-Open Sets in Topological Spaces, Int. Journal of Math. Analysis, Vol. 3, 2009, no. 37, 1803-1819.