

A New Mathematical Model For Finding MTBF Levels In The Active And Delivery Modes By Using The Hormone Release Of Vasopressin

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ABSTRACT

The most common approach for development, testing programs include some corrective actions during testing and some delayed fixes incorporated at the end of test. This paper presents an Extended model that addresses this practical situation given in the application part of sec 3. In this application the testing mode is considered as active mode and the delayed fixes mode as at the delivery. The result concludes that MTBF is lesser in the action mode and then increases with the delivery mode.

1. INTRODUCTION

Vasopressin concentrations are markedly elevated at birth in the umbilical cord plasma of human infants [5, 10, 15]. Hormone concentrations are particularly high in the newborn after vaginal delivery or cesarean section preceded by labor. The stimulus to vasopressin release has been attributed to the stress and hypoxia that may accompany delivery, but in the human there are no clear correlations [15]. More specifically, Hadeed [10] have suggested that cranial compression during passage through the cervical canal may be the impetus for fetal vasopressin hyper secretion, whereas others have linked vasopressin secretion with the hypothalamic mechanisms initiating or maintaining parturition [4].

The potential ramifications of enhanced vasopressin secretion in the fetus have been more extensively explored in the chronically instrumented fetal lamb model. Alexander [1] reported generally undetectable values for fetal vasopressin (below the level of sensitivity of the bioassay) until the last few days of gestation; thereafter, hormone concentrations rose markedly. In contradistinction, we found an increased vasopressin secretion only after the onset of spontaneous uterine contractions [18]. A number of stressful conditions including fetal hypoxia [3], hemorrhage [2], umbilical cord occlusion [9], and hyperosmolar challenge are accompanied by increases in fetal plasma vasopressin. The potential for any one of these to stimulate the parturient rise in vasopressin is apparent. Iwamoto [13] have speculated that a role for vasopressin in the fetus may be in its effect on

the redistribution of cardiac output and as such plays an important part in the fetal cardiovascular response to stress.

The present study was designed to assess the relationship between fetal vasopressin release and the progression of parturition as well as the contribution of specific stimuli to vasopressin secretion during labor. There are practical problems in carrying out experiments of this nature during spontaneous term labor because of the difficulties in predicting the timing of the parturient process. Therefore, we elected to carry out our investigations utilizing the more consistent and predictable model of premature parturition induced by infusion of adrenocorticotropin (ACTH) to the fetus. Liggins [14] has shown that ACTH-induced labor is, accompanied by many of the same hormonal changes that accompany spontaneous labor.

2. NOTATION

λ	– Scale Parameter
β	– Shape Parameter
t	– Test time
T	– Total test time
MTBF	– Mean time between failures
X_i	– The i -th successive failure time
N	– Total number of failures
λ_A	– Type A modes failure intensity
λ_B	– Type B modes failure intensity
λ_P	– Projected failure intensity
λ_{CA}	– Achieved failure intensity
λ_{BD}	– Type BD modes failure intensity
λ_{EM}	– Extended model failure intensity
M_{CA}	– Achieved failure intensity of MTBF
M_{EM}	– Extended model MTBF

3. APPLICATION

Vasopressin concentrations in fetal plasma during various phases of labor are presented in Figure (i). The mean value before the institution of infusion was 1.85 ± 0.3 pg/ml ($N = 9$) and was similar to that found during the postoperative recovery period, 1.3 ± 0.3 pg/ml ($N = 25$). In fetuses infused with ACTH, the mean vasopressin concentration was 2.0 ± 1.4 pg/ml before labor ($N =$

51); in prodromal and early labor, 5.3 ± 3.4 pg/ml (N = 25; range, 1.0 to 30 pg/ml); in active labor, 39.6 ± 27.5 pg/ml (N = 28; range, 1.0 to 200 pg/ml); and during the expulsive phase, 173.3 ± 152.9 pg/ml (N = 20; range, 5.0 to 1090 pg/ml). The mean plasma hormone concentration at delivery in nine animals was 584.2 ± 433 pg/ml (range, 15 to 4000 pg/ml), and at 30 min after birth, it was 359.8 ± 90.0 pg/ml (range, 8 to 2400 pg/ml). Using techniques of analysis of variance with a fixed random design, the mean values of fetal vasopressin before and during

the early phase of experimental induction of labor were not significantly different from those in prodromal or early labor; however, hormone concentrations increased significantly during active labor (P < 0.05), expulsive labor (P < 0.01), and at delivery (P < 0.01). There was a significant increment in hormone concentration between active labor and delivery (P < 0.05), but not between values obtained during the expulsive phase of labor and those obtained at delivery.

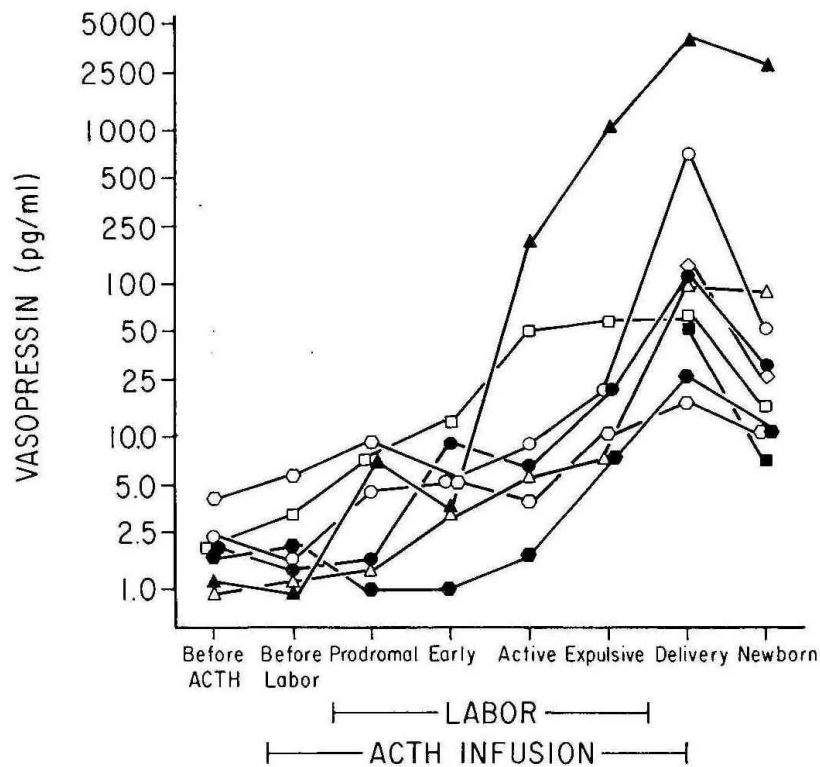


Fig (i): Plasma vasopressin (pg/ml) in fetal and newborn lambs before, during, and after premature parturition induced by infusion of ACTH to the fetus. The values before and during labor represent the mean of from two to individual vasopressin determinations. Fetus 167, (○); fetus 835, (●); fetus 116, (△); fetus 195, (◐); fetus 396, (◑); fetus 884, (◒); fetus 482, (◓); non-ACTH- infused twin of 396 (◔); fetus 884 (◕).

The fetal plasma vasopressin concentrations at delivery were strongly correlated with the hormone values obtained during the active and expulsive phases of labor with regression coefficients $r = 0.955$ and 0.985 (P < 0.001). A similar correlation was not obtained between hormone levels before or during prodromal and early labor with those at delivery.

4. DISCUSSION

The resting plasma concentrations of vasopressin in these experiments in both the ewe and her fetus are in accord with our previous studies [18] and those of other investigators [1, 13, 16, 17] using either bioassay or radioimmunoassay techniques for vasopressin determination. These investigations report resting plasma concentrations of the hormone consistently less than 3.5 mu/ml or about 10 pg/ml. The vasopressin levels before labor fit nicely within the lower portion of this range with mean fetal levels of 2.0 ± 1.4 pg/ml and maternal levels of 2.1 ± 0.8 pg/ml. The mean

hormone values observed in this study are lower than have been previously reported. The lower limit of sensitivity of our assay is 0.25 pg/ml [11] and allows for a precise estimate of resting vasopressin concentration.

5. MATHEMATICAL MODEL

The most widely used traditional reliability growth tracking model and reliability growth Projection model address reliability growth based on failure modes surfaced during the test. With the Projection model all corrective actions are delayed until the end of test. This Paper Presents an Extended Model that addresses this practical situation and allows for pre emptive corrective actions. The tradition reliability growth models [6,7,8], provide assessments when the failure modes corrected are surface during the testing . The focus of this paper is a combination of these approaches for the practical situations where some corrective actions are incorporated during the test and some corrective actions are delayed until the end of the test.

Suppose a development testing program begins at time 0 and is conducted until time T and stopped. Let N be the total number of failures recorded and let $0 < X_1 < X_2 < \dots < X_N < T$ denote the N successive failure times on a cumulative time scale. We assume that the Non-Homogeneous Poisson Process (NHPP) assumption applies to this set of data. Under the basic model the maximum likelihood estimates (MLEs) for λ and β (numerator of MLE for β adjusted from N to N-1 to obtain unbiased estimate) are

$$\hat{\lambda} = \frac{N}{T^\beta}, \quad \hat{\beta} = \frac{N-1}{\sum_{i=1}^N X_i^{\beta-1}} \quad \text{----- (1)}$$

The achieved or demonstrated failure intensity and Mean time between failures are estimated by

$$\hat{\lambda}_{CA} = \hat{\lambda} \beta T^{\beta-1}, \quad \text{----- (2)}$$

$$\hat{M}_{CA} = [\hat{\lambda}_{CA}]^{-1} \quad \text{----- (3)}$$

It is important to note that this model does not assume that all failures in the data set receive a corrective action. Based on the strategy some failures may receive a corrective action and some may not.

6. PROJECTION MODEL

Suppose a system is tested for time T. During the testing problem failure modes are identified, but all corrected actions are delayed and incorporated at the end of the test phase. These delayed corrective actions are usually incorporated as a group and the result is generally a distinct jump in the system reliability. The projection model [8], estimates this jump in reliability due to the delayed fixes. This is called a "Projection."

The Projection model places all failure into two groups, A and B. Type A failure modes

are all modes such that if seen during test no corrective action will be taken. This accounts for all modes for which management determines that it is not cost- effective to increase the reliability by a design change. Type B failure modes are all modes such that if seen during test a corrective action will be taken. This Type A and Type B determination helps define the reliability growth management strategy. The basic projection model assumes that the Type A failure modes has constant failure intensity λ_A , the i-th Type B failure mode follows the exponential distribution with failure rate λ_i , and the initial failure intensity for Type B failure modes is λ_B .

An effectiveness factor (EF) d_j is the fraction decrease in λ_j after a corrective action has been made for the j-th Type B mode. The failure rate for the i-th Type B failure mode after a corrective action is $(1 - d_j) \lambda_j$. In Practice, for application of the Projection Model, the EFs are assigned based on engineering assessments, test results, etc.

The system failure intensity is constant, say, λ_S , during the testing and then jumps to a lower value due to the incorporation of corrective actions. The intensity at the end of the test T, before delayed corrective actions are introduced into the system, is the achieved intensity. The reciprocal of the intensity is the achieved Mean time between failure (MTBF) M_S .

We estimate the achieved failure intensity λ_S by

$$\hat{\lambda}_S = \hat{\lambda}_A + \hat{\lambda}_B, \quad \hat{\lambda}_A = N_A/T, \quad \hat{\lambda}_B = N_B/T \quad \text{----- (4)}$$

The estimated achieved Mean time between failure M_S at time T, before the jump is \hat{M}_S . We estimate the jump next.

The estimated projected failure intensity [8], is

$$\hat{\lambda}_P = \hat{\lambda}_A + \sum_{j=1}^M (1 - d_j) \frac{N_j}{T} + \bar{d} \hat{h}(T) \quad \text{----- (5)}$$

Where $\bar{d} = \frac{\sum_{j=1}^M d_j}{M}$ is the average EF, and

$$\hat{h}(T) = \hat{\lambda} \beta T^{\beta-1}, \quad \text{----- (6)}$$

The Projection Model $\hat{\lambda}$ and $\hat{\beta}$ for use only the M first occurrence failure times of the seen and unique Type B failure modes.

7. Extended Reliability Growth Model

The assessment and management metric structure for corrective actions during and after a test. We define two types of B modes. Type BC failure modes are corrected during test. Type BD failure modes are delayed to the end of the test. Type A failure modes, as before, are those failure modes that will not receive a corrective action. These define the management strategy and can be changed. This BC and BD failure mode designated is an important aspect of the Extended Model.

Those failure modes that received a corrective action during the test (BC modes) and also those failure modes that will receive a corrective action at the end of the test (BD modes). During test the Type A and Type BD failure modes do not contribute to reliability growth. The corrective actions for the BC failure modes affect the increase in the system reliability during the test and this is the same for both the management strategy. After the incorporation of corrective actions for the Type BD failure modes, the reliability increases. Estimating the increased reliability with the objective of this paper.

Here, for the Extended Model we assume that the achieved Mean Time between Failure (MTBF), before delayed fixes, based the data should be exactly the same as the achieved MTBF for the data. If K is the total number of distinct BD modes then, in the intensity to be estimated is

$$\lambda_p = \lambda_s - \lambda_B + \sum_{i=1}^K (1 - d_i) \lambda_i + dh(T) \text{ ----- (7)}$$

To allow for BC modes in the extended model we replace λ_s by λ_{CA} in (1). Let λ_{BD} be the constant intensity for the Type BD modes, and let $h(t \setminus BD)$ be the first occurrence function for the Type BD modes.

The Extended model Projected failure intensity is

$$\lambda_{EM} = \lambda_{CA} - \lambda_{BD} + \sum_{i=1}^K (1 - d_i) \lambda_i + dh(T \setminus BD) \text{ (8)}$$

The Extended model Projected MTBF is $\hat{M}_{EM} = 1/\lambda_{EM}$. This is the MTBF after the incorporation of the delayed BD modes.

8. Estimation for the Extended Reliability Growth Model

The estimate of the projected failure intensity for the Extended Model is

$$\hat{\lambda}_{EM} = \hat{\lambda}_{CA} - \hat{\lambda}_{BD} + \sum_{j=1}^M (1 - d_j) \frac{N_j}{T} + \bar{d} \hat{h}(T \setminus BD) \text{ -----(9)}$$

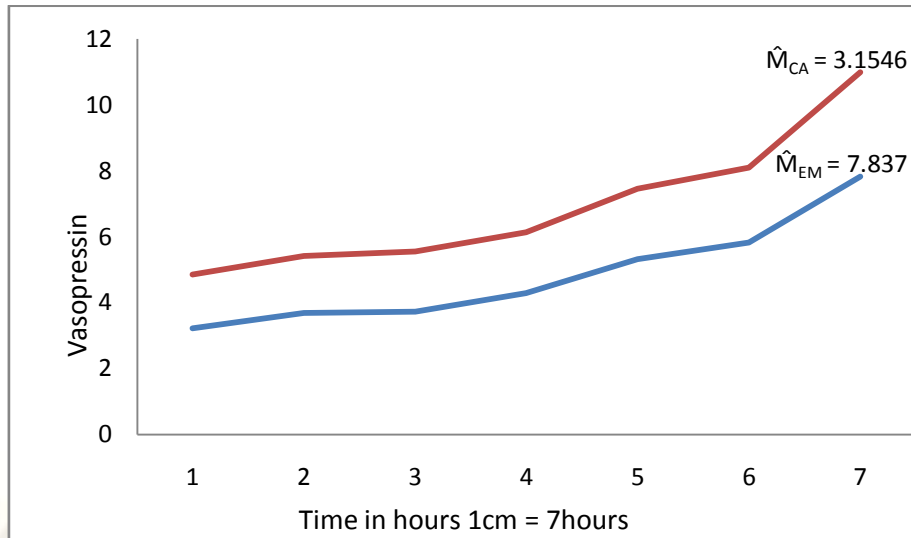
9. VASOPRESSIN CONCENTRATIONS

Fetal vasopressin concentrations rose significantly during the active and expulsive phases of ACTH-induced labor (Fig.i) in a similar fashion to that we have previously described during spontaneous labor. These data further reinforce the postulate that fetal vasopressin secretion increases in response to labor and not before labor. They do not support the concept that fetal vasopressin is integrally related to the onset of parturition, as others have suggested [4].

Vasopressin hyper secretion into the fetal lamb circulation is not just a momentary phenomenon occurring at delivery, but also proceeds during active labor. The significant correlations between elevated vasopressin concentrations in lamb plasma at delivery with the hormone concentrations obtained in the latter stages of labor may be important in interpreting data obtained from human umbilical cord blood. High concentrations of the hormone are found in human fetal plasma at vaginal delivery [5, 10, 15]. These observations of vasopressin hyper secretion at the moment of birth may similarly imply a more prolonged exposure of the human fetus to vasopressin at concentrations which may have extensive effects on the fetal cardiovascular system.

One of the factors responsible for vasopressin release in the latter stage of delivery may be mechanical compression of the fetus during uterine contractions and passage through the cervical canal. Hadeed [10] have attributed the hyper secretion of vasopressin during normal human labor to cerebral compression. We have observed a parallel phenomenon in the rat where body compression for 60 sec before decapitation results in an 100-fold increase in vasopressin levels [12]. It is speculated that compression causes impaired oxygenation and/or hemodynamic alterations which trigger vasopressin release.

10.RESULT



Here we have found that, according to fetus, 116, 167, 195, 835, 396, 884, 482 cases the corresponding \hat{M}_{EM} Values and \hat{M}_{CA} Values increases in the above curves. We can find the bounds of the above curves to check their Monotonocities.

11. Conclusion

The Extended Model Projected Mean Time Between Failure $\hat{M}_{EM} = 7.8370$. The achieved Mean Time Between Failure (MTBF) before the 8 delayed fixed is estimated by $\hat{M}_{CA} = 3.1546$. We therefore have based on the Extended Model estimates that the MTBF grew to 3.1546 as a result of corrective actions for BC failure modes during the test, and then jumped to 7.8370 as a result of the delayed corrected actions after the test for the BD failure modes. In the medical mode we estimate the MTBF grew to 3.1546 as a result of corrective actions for before active mode during the labor and then jumped to 7.8370 as a result of the delayed corrected actions after the labor for the after delivery mode.

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