The Research of Power Quality Analysis Based on family of S-Transform

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ABSTRACT

Power quality (PQ) disturbance recognition is the foundation of power quality monitoring and analysis. The S- transform (ST) is an extension of the ideas of the continuous wavelet transform (CWT) or variable window of short time Fourier transform (STFT). It is based on a moving and scalable localizing Gaussian window. S-transform has better time frequency and localization property than traditional. With the excellent time—frequency resolution (TFR) characteristics of the S-transform, ST is an attractive candidate for the analysis and feature extraction of power quality disturbances under noisy condition also has the ability to detect the disturbance correctly but it involves high computational overhead which is of the order of *$O(N^2 \log N)$* . This paper overviewed the theory of basis S-transform and fast discrete S-transform (FDST) summarized their computational requirement in the area of power quality disturbance recognition.

The new Fast discrete S-transform algorithm, with a new frequency scaling and band pass filtering, Computational complexity is *$O(N \log N)$* in optimal conditions. So it becomes less time consuming and decreases cost overhead, tool for power signal disturbance assessment.

Keywords – **STFT, CWT, S-Transform, Discrete S-Transform, FDST.**

I. INTRODUCTION

Although the Fourier transform of the entire time series does contain information about the spectral components in time series, it cannot detect the time distribution of different frequency, so for a large class of practical applications, the Fourier transform is unsuitable. So the time-frequency analysis is proposed and applied in some special situations. The STFT is most often used. But the STFT cannot track the signal dynamics properly for non-stationary signal due to the limitations of fixed window width. The WT is good at extracting information from both time and frequency domains. However, the WT is sensitive to noise. The *S* transform was proposed by Stockwell and his coworkers in 1996. The properties of *S* transform are that it has a frequency dependent resolution of time-frequency domain and entirely refer to local phase information. For example, in the beginning of earthquake, the spectral components of the P-wave clearly have a strong dependence on time. So we need the generalized *S* transform to emphasize the time resolution in the beginning time and the frequency resolution in the later of beginning time. Based on different purposes, we can apply different window of *S* transform. For example, we will introduce the Gaussian window, the bi-Gaussian window, and the hyperbolic window. The comparison between the ST-based method and other methods such as the wavelet-transform-based method for power-quality disturbance recognition shows the method has good scalability and very low sensitivity to noise levels. All of these show FDST based methods has great potential for the future development of fully automated monitoring systems with online classification capabilities. The analysis direction and emphasis of studying about the power quality (PQ) disturbance recognition also put forward.

II. THE *S*- TRANSFORM

There are some different methods of achieving the *S* transform. We introduce the relationship between STFT and *S* transform. And the type of deriving the *S* transform from the "phase correction" of the CWT here, learned from [1]

2.1 The Continuous *S* Transform

2.1.1 Relationship between *S* Transform and STFT

The STFT of signal $h(t)$ is defined as

$$SFT(\tau, f) = \int_{-\infty}^{\infty} h(t) g(\tau - t) e^{-j2\pi ft} dt \tag{2.1}$$

where τ and f denote the time of spectral localization and Fourier frequency, respectively, and $g(t)$ denote a window function. The S transform can derive from (2.1) by replacing the window function $g(t)$ with the Gaussian function, shown as

$$g(t) = \frac{|f|}{\sqrt{2\pi}} e^{-\frac{t^2 f^2}{2}} \tag{2.2}$$

Then the S transform is defined as

$$S(\tau, f) = STFT(\tau, f) = \int_{-\infty}^{\infty} h(t) \frac{|f|}{\sqrt{2\pi}} e^{-\frac{(\tau - t^2) f^2}{2}} e^{-j2\pi ft} dt \quad (2.3)$$

So we can say that the S transform is a special case of STFT with Gaussian window function. If the window of S transform is wider in time domain, S transform can provide better frequency resolution for lower frequency. While the window is narrower, it can provide better time resolution for higher frequency.

2.1.2 Relationship between S Transform and CWT

The continuous-time expression of the CWT is

$$W(\tau, d) = \int_{-\infty}^{\infty} h(t) \omega(t - \tau, d) dt \quad (2.4)$$

where t denotes time, h(t) denotes a function of time, τ denotes the time of spectral localization, d denotes the "width" of the wavelet w(t, d) and thus it controls the resolution, and w(t, d) denotes a scaled copy of the fundamental mother wavelet. Along with (2.4), there has a constraint of the mother wavelet w(t, d) that w(t, d) must have zero mean.

Then the S transform is defined as a CWT with a specific mother wavelet multiplied by the phase factor

$$S(\tau, f) = e^{-j2\pi ft} W(\tau, d) \quad (2.5)$$

where the mother wavelet is defined as

$$\omega(t, f) = \frac{|f|}{\sqrt{2\pi}} e^{-\frac{t^2 f^2}{2}} e^{-j2\pi ft} \quad (2.6)$$

Note that the factor d is the inverse of the frequency f.

However the mother wavelet in (2.6) does not satisfy the property of zero mean, (2.5) is not absolutely a CWT. In other words, the S transform is not equal to CWT, it is given by

$$S(\tau, f) = \int_{-\infty}^{\infty} h(t) \frac{|f|}{\sqrt{2\pi}} e^{-\frac{(\tau - t)^2 f^2}{2}} e^{-j2\pi ft} dt \quad (2.7)$$

If the S transform is a representation of the local spectrum, we can show that the relation between the S transform and Fourier transform as

$$\int_{-\infty}^{\infty} S(\tau, f) d\tau = H(f) \quad (2.8)$$

where H(f) is the Fourier transform of h(t). So the h(t) is

$$h(t) = \int_{-\infty}^{\infty} \{ \int_{-\infty}^{\infty} S(\tau, f) d\tau \} e^{j2\pi ft} df \quad (2.9)$$

This shows that the concept the S transform is different from the CWT.

The relation between the S transform and Fourier transform can be written as

$$S(\tau, f) = \int_{-\infty}^{\infty} H(\alpha + f) e^{\frac{2\pi^2 \alpha^2}{f^2}} e^{j2\pi\alpha\tau} d\alpha \qquad f \neq 0 \quad (2.10)$$

By taking the advantage of the efficiency of the Fast Fourier transform and the convolution theorem, the discrete analog of (2.10) can be used to compute the discrete S transform (we will describe it below). If not translating the cosinusoid basic functions, the S transform can localize the real and imaginary components of the spectrum independently.

2.2 The Instantaneous Frequency

We set the 1-D function of the variable τ and fixed parameter f1 as S(τ, f1) and called “voice”. Then the function can be written as

$$S(\tau, f_1) = A(\tau, f_1) e^{j\Phi(\tau, f_1)} \quad (2.11)$$

where A and Φ are the amplitude and phase. Because a voice isolates a particular frequency f1, we can use the phase Φ to determine the instantaneous frequency (IF):

$$IF(\tau, f_1) = \frac{1}{2\pi} \frac{\partial}{\partial t} \{ 2\pi f_1 \tau + \Phi(\tau, f_1) \} \quad (2.12)$$

The correctness of (2.11) can use a simple case of h(t) = cos(2πωt), where the function

$$\Phi(\tau, f) = 2\pi(\omega - f)\tau$$

2.3 The Discrete S Transform

Let h[kT], k=0, 1, …, N – 1 denote a discrete time series corresponding to h(t) with a time sampling interval of T. The discrete Fourier transform is shown as

$$H\left[\frac{n}{NT}\right]=\frac{1}{N}\sum_{k=0}^{N-1}h[kT]e^{\frac{j2\pi nk}{N}} \quad (2.13)$$

Using (2.10) and (2.13), the discrete time series h[kT]'s S transform is shown as

(making $f \rightarrow n/NT$ and $\tau \rightarrow jT$)

$$S\left[jT,\frac{n}{NT}\right]=\sum_{m=0}^{N-1}H\left[\frac{m+n}{NT}\right]e^{\frac{2\pi^2 m^2}{n^2}}e^{\frac{j2\pi mj}{N}}$$

$$n\neq 0 \quad (2.14)$$

where j, m, and n = 0, 1, ..., N-1. If n = 0 voice, it is equal to the constant defined as

$$S[jT,0]=\frac{1}{N}\sum_{m=0}^{N-1}h\left(\frac{m}{NT}\right) \quad (2.15)$$

This equation makes the constant average of the time series into the zero frequency voice, so it will ensure that the inverse is exact. The inverse of the discrete S transform is

$$h[kT]=\sum_{n=0}^{N-1}\{\frac{1}{N}\sum_{j=0}^{N-1}S[jT,\frac{n}{NT}]\}e^{\frac{j2\pi nk}{N}} \quad (2.16)$$

III. GENERALIZED S-TRANSFORM

3.1 The Generalized S Transform

The generalized S transform is defined as [3]

$$S(\tau,f,p)=\int_{-\infty}^{\infty}h(t)\omega(\tau-t,f,p)e^{-j2\pi ft}dt \quad (3.1)$$

where p denotes a set of parameters which determine the shape and properties of w and w denotes the S transform window shown as

$$\omega(t,f,p)=\frac{|f|}{\sqrt{2\pi}\,p}e^{\frac{t^2 f^2}{2p^2}} \quad (3.2)$$

given by As (2.10), the generalized S transform can also be obtained by the Fourier transform

$$S(\tau,f,p)=\int_{-\infty}^{\infty}H(\alpha+f)W(\alpha,f,p)e^{j2\pi\alpha\tau}d\alpha \quad (3.3)$$

The S transform window w has to satisfy four conditions. The four conditions are as below

$$\int_{-\infty}^{\infty}\Re\{\omega(\tau,f,p)\}d\tau=1, \quad (3.4)$$

$$\int_{-\infty}^{\infty}\Im\{\omega(\tau,f,p)\}d\tau=0, \quad (3.5)$$

$$\omega(\tau-t,f,p)=[\omega(\tau-t,-f,p)]^*, \quad (3.6)$$

$$\frac{\partial}{\partial t}\Phi(\tau-t,f,p)\Big|_{t=\tau}=0 \quad (3.7)$$

The first two conditions assure that when integrated over all τ, the S transform converges to the Fourier transform:

$$\int_{-\infty}^{\infty}S(\tau,f,p)d\tau=H(f). \quad (3.8)$$

The third condition can ensure the property of symmetry between the shapes of the S transform analyzing function at positive and negative frequencies.

3.2 The Gaussian Window

Before introducing the bi-Gaussian window, we first mention the Gaussian window. As we can see in (3.2), ω is a Gaussian. To difference Gaussian window from the bi-Gaussian, we use the subscript GS to represent (3.2)'s modification. ωGS is rewritten as [4]

$$\omega_{GS}(\tau-t,f,\{\gamma_{GS}\})=\frac{|f|}{\sqrt{2\pi}\gamma_{GS}}e^{\frac{-f^2(\tau-t)^2}{2\gamma_{GS}^2}} \quad (3.9)$$

Where γ_{GS} is the number of periods of Fourier sinusoid which are contained within one standard deviation of the Gaussian window. We show the Gaussian S transform of the time series for γ_{GS} = 1 in Fig. 3.1. The result of Fig. 3.1 is obtained by using the discrete S transform (2.14), shown as S_{GS}. In order to get S_{GS}, we have to obtain W_{GS} first. W_{GS} is shown as

$$W_{GS}(\alpha,f,\{\gamma_{GS}\})=e^{\frac{2\pi^2\alpha^2\gamma_{GS}^2}{f^2}} \quad (3.10)$$

From Fig. 3.1, there has a problem that the long front taper of the window let the correlation of event signatures with the time of event initiation be complex

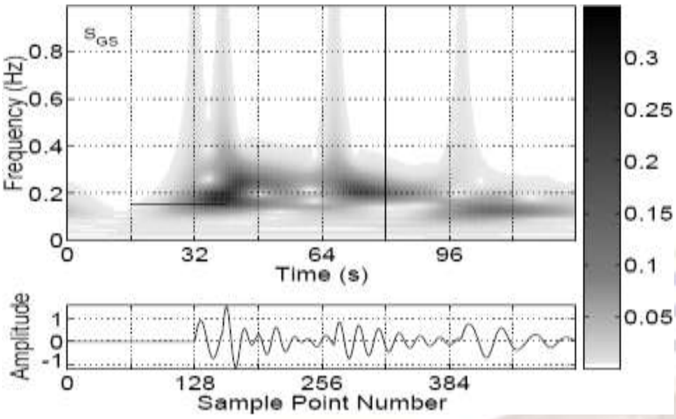

Fig. 3.1 The time series and the amplitude spectrum of Gaussian S transform of time series at $\gamma_{GS}=1$. From the event signatures, we can see the "holes", which is due to localized destructive interference between signal components. [4]

In order to improve the front time resolution of ω_{GS}, we can decrease the value of γ_{GS} for narrowing the window. However, a drawback is that if γ_{GS} is too small, the window may reserve too few cycles of the sinusoid. So the frequency resolution may be poor and may let the time-frequency spectrum be meaningless. There is an example in Fig. 3.2.

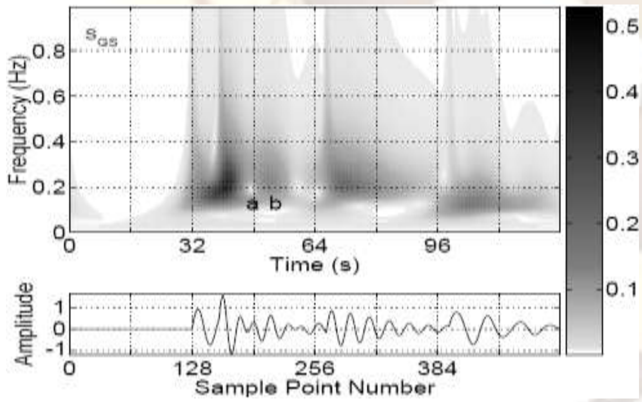

Fig. 3.2 The time series and the amplitude spectrum of Gaussian S transform of time series at γ_{GS} = 0.5. "a" is the position that has destructive localized interference between two events and "b" is a phantom fifth event between second and third real events. [4]

IV. THE FAST DISCRETE S-TRANSFORM

The next Chapter describes the Fast discrete S-Transform and the time-frequency analysis of the power signal disturbance using the modified S-Transform.

S-Transform is a powerful tool for power signal disturbance assessment it involves high computational overhead which is of the order of $O(N^2 \log N)$ using the entire data window for the signal. The computational complexity of S-transform involves long calculation time even for short data window and processing large volumes of power signal data it becomes time consuming and increases cost overhead. Thus to reduce the computational overhead of S-transform, several attempts for generalization and faster computation of the S-transform have been proposed using Generalized Fourier family transform (GFT) [11-13]. In this work, earlier proposed techniques are explored and a new Fast discrete S-transform algorithm, with a new frequency scaling and band pass filtering for primarily analyzing power signals is presented. Computational complexity of this new approach known as Fast S-Transform (DFST) is $O(N \log N)$ in optimal conditions.

V. CONCLUSION

We have shown the concept of the transform between the S transform and the STFT, WT. From the power quality analysis, the S transform exhibit the ability of identifying the power quality disturbance by noise or transient. This is the wavelet transform cannot achieve because its drawback of sensitive to noise. But the *S* transform still have two drawbacks, first one is in the DC term (frequency = 0), the *S* transform cannot analyze the variation of *S* transform on time. Second, in high frequency, the window will be too narrow, so the points we can practically apply will be too less. ST involves high computational overhead which is of the order of O(N2 log N).

A Fast Discrete S-transform which uses frequency scaling and band pass filtering reduces the computational overhead of implementing the S-transform significantly in the order of $\boldsymbol{O(N \log N)}$ and is thus very useful for the analysis of huge amount power quality data.

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