

Control of Rotary Flexible Link with Linear Quadratic Regulator

Dr.G.V.Marutheswar*, C.Chandrakala**

*Professor, Department of Electrical Engineering, SV University, Tirupati

** student, Department of Electrical Engineering, SV University, Tirupati

ABSTRACT

This paper deals with the position control of robot arm, Rotary Flexible Link constitutes main function of positioning the arm. The precision with which the arm is controlled depends on the response of motor and on different parameters of flexible link. This paper concentrates on development of Linear Quadratic Regulator (LQR) for flexible link operated by a dc servomotor. The comparison of performance of real time system with simulation is done by simultaneously observing measured and simulated angular position of flexible link. It is found that the response is improved with LQR.

Keywords - MATLAB/Simulink, WinCon software, LQR, real-time experiment

1. INTRODUCTION

Rotary flexible link's plays a very important role in the industrial life. For problems that are more complex in nature, it requires control and automation. Though many techniques are employed for development of sophisticated control systems, LQR or state- feedback controller is one of the most successful technologies and at present is often not precisely known models [1].

The Quanser SRV02 rotary servo plant, pictured in Figure 1, consists of a DC motor that is encased in a solid aluminum frame and equipped with a planetary gearbox [3]. That is, the motor has its own internal gearbox that drives external gears. The basic SRV02 units come with a potentiometer sensor that can be used to measure angular position of the load gear. The SRV02 device can also be fitted with an encoder to obtain a digital position measurement. The mechanical modeling of the flexible link is described the mathematical derivation of the system [9].

Rotating the flexible link at 15° gain with 1Hz get the Real time and simulated responses of Theta and Alpha. The main aim is to reduce the vibrations or disturbances of the link's position.

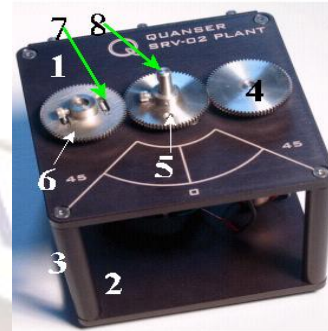


Fig 1: SRV02 Front view

Table 1: component names

s.no	Component
1	Top plate
2	Bottom plate
3	Posts
4	Motor pinion gear: 72-teeth (low-gear)
5	Load gear: 72-teeth (low-gear)
6	Potentiometer anti-backlash gear
7	Anti-backlash springs
8	Load shaft (i.e. output shaft)

2. Modeling of Rotary Flexible Link

The equations of motion involving a rotary flexible link, involves modeling the rotational base and the flexible link as rigid bodies [2]. As a simplification to the partial differential equation describing the motion of a flexible link, a lumped single degree of freedom approximation is used [4]. We first start the derivation of the dynamic model by computing various rotational moment of inertia terms.

$$\alpha = \frac{D}{L} \quad (1)$$

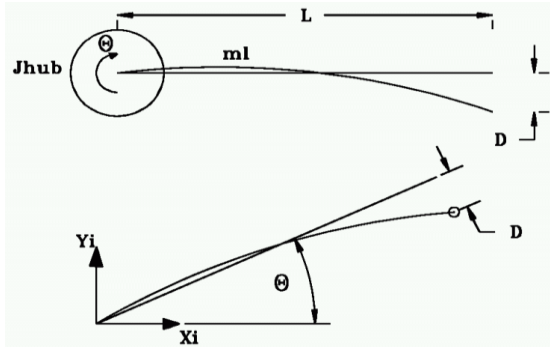


Fig 2: Schematic of Flexible link Module

In controlling the tip of the link, it is sufficient to use a simplified model that will adequately describe the motion of endpoint. In Figure (2)

The equation from the fig (2) of rotary spring is

$$J_{Link} \ddot{\alpha} = -K_{Stiff} \alpha \quad (2)$$

$$\ddot{\alpha} = -\omega_c^2 \alpha \quad (3)$$

Combining equation (2) and (3) For a single degree of freedom system, the natural frequency (ω_c) is related with torsional stiffness and rotational inertia in the following manner

$$K_{Stiff} = \omega_c^2 J_{Link} \quad (4)$$

Link is modeled as a rod rotating about its end point with moment of inertia

$$J_{Link} = \frac{ML^2}{3} \quad (5)$$

2.1 Deriving the System Dynamic Equations

Developed a linear model for the link, the system dynamic equation can be obtained from Euler-Lagrange formulation. We obtain the potential and kinetic energies in our system as:

Potential Energy-The only Potential Energy in the system is in the spring:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \ddot{\theta} \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{K_{Stiff}}{J_{eq}} & \frac{-\eta_m \eta_g k_t k_m k_g^2 + B_{eq} R_m}{J_{eq} R_m} & 0 \\ 0 & \frac{-K_{Stiff} (J_{eq} + J_{Arm})}{J_{eq} J_{Arm}} & \frac{\eta_m \eta_g k_t k_m k_g^2 + B_{eq} R_m}{J_{eq} R_m} & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{\eta_m \eta_g k_t k_m k_g^2}{J_{eq} R_m} \\ \frac{-\eta_m \eta_g k_t k_m k_g^2}{J_{eq} R_m} \end{bmatrix} \quad (14)$$

3. Simulation of Flexible Link with LQR

Procedure for doing the experiment:

$$V = P.E._{Spring} = \frac{1}{2} K_{Stiff} \alpha^2 \quad (6)$$

Kinetic Energy-The Kinetic Energy arise from the moving hub and flexible arm:

$$T = K.E._{Hub} + K.E._{Link} = \frac{1}{2} J_{eq} \dot{\theta}^2 + \frac{1}{2} J_{Link} (\dot{\theta} + \dot{\alpha})^2 \quad (7)$$

Forming the Lagrangian:

$$L = T - V = \frac{1}{2} J_{eq} \dot{\theta}^2 + \frac{1}{2} J_{Link} (\dot{\theta} + \dot{\alpha})^2 - \frac{1}{2} K_{Stiff} \alpha^2 \quad (8)$$

The two generalized co-ordinates are θ and α

$$\frac{\delta}{\delta t} \left(\frac{\delta L}{\delta \dot{\theta}} \right) - \frac{\delta L}{\delta \theta} = T_{output} - B_{eq} \dot{\theta} \quad (9)$$

$$\frac{\delta}{\delta t} \left(\frac{\delta L}{\delta \dot{\alpha}} \right) - \frac{\delta L}{\delta \alpha} = 0 \quad (10)$$

solve equation (9) and (10) are left with:

$$J_{eq} \ddot{\theta} + J_{Arm} (\ddot{\theta} + \ddot{\alpha}) = T_{output} - B_{eq} \dot{\theta} \quad (11)$$

$$J_{Arm} (\ddot{\theta} + \ddot{\alpha}) + K_{Stiff} \alpha = 0 \quad (12)$$

Referring back to Experiment – position Control, we know that the output torque on the load from the motor is

$$T_{output} = \frac{\eta_m \eta_g k_t k_g (v_m - k_g k_m \dot{\theta})}{R_m} \quad (13)$$

Combining equation (11) (12) and (13), left with the state space representation of the complete system:

a.. The arm deflection signal (α) should be connected to encoder channel # 1 and the servomotor's position signal (θ) should be connected to encoder .

b. Analog Output channel #0 should be connected to the UPM (Amplifier) and from the amplifier to the input of the servomotor. This system has two inputs (θ & α) and one output (V_m).

1. Click on start then click on wincon server.
2. Click on start then click on wincon client.
3. Click on start then click on MATLAB then in MATLAB window click on open and open the file "SRV02_Exp5_Flexible Link" directory. This MATLAB script file will setup all the specific system named "Setup_SRV02_Exp5.m". under the parameters and will set the system state-space matrices A,B,C,D
4. Now run the file "Setp_SRV02_Exp5.m" by click on the "run" in the command window or pressing F5. Then calculated gains are displayed in the MATLAB window.
5. Open simulnk in matlab window, then under the same directory, open a Simulink model called "q_SRV02_Flexible_link.mdl". This model is a simulation of the Flexible link system with a feedback law $u = -kx$. The gain vector k is set be the LQR function. This model has the I/O connection blocks linking to the physical plant as well as a simulated block to compare real and simulated results.
6. Then, in the simulink model figure (3) initializes the set point amplitude as 20 by double clicking on it.

Figure (3) depicts the Rotary Flexible Link developed for this experiment. Notice that both the actual system and an exact simulation are running in parallel thus allowing comparing the actual and simulated results. Figure (3) appears when double

clicked on the SRV02-ROTARY FLEXIBLE LINK in Figure 3.

7. Before "**Building**" this controller, make sure that the state-feedback gain k is set according to final design parameters. Now proceed to "**build**" the controller through the WinCon menu.

8. After the code has compiled, start the controller through WinCon and open up two scopes, for theta (measure and simulated together).

3.1 Block diagram of Rotary Flexible Link State-Feedback Controller (LQR)

A square wave input of 1Hz with a gain of 15^0 was found to be sufficient to bring up the link to the upright position[6]-[7]. Adjusted the magnitude or the frequency to obtain a more satisfactory. To convert the degrees from the gain output to the radians using D to R converter shown in figure 2. These radians is connected to the set point to check out the error between theta (rad) and D to R output given to the k(1). The inputs comes from the encoder and strain gage subsystem like theta, alpha, theta dot and alpha dot to the gains k(1) k(2) k(3) and k(4) respectively.

These 4 inputs given to the multiplexer which gives an output is connected to the encoder and strain gage subsystem [8] output $V_m(v)$. The corresponding waveforms have been seen in the scope. Place as many scopes as to monitor the phase angle etc. Before given to the scope, convert the radians into degrees.

Encoder is used to measure the angular position of the servomotor. Theta and a strain gage sensor are used to measure the link deflection Alpha.

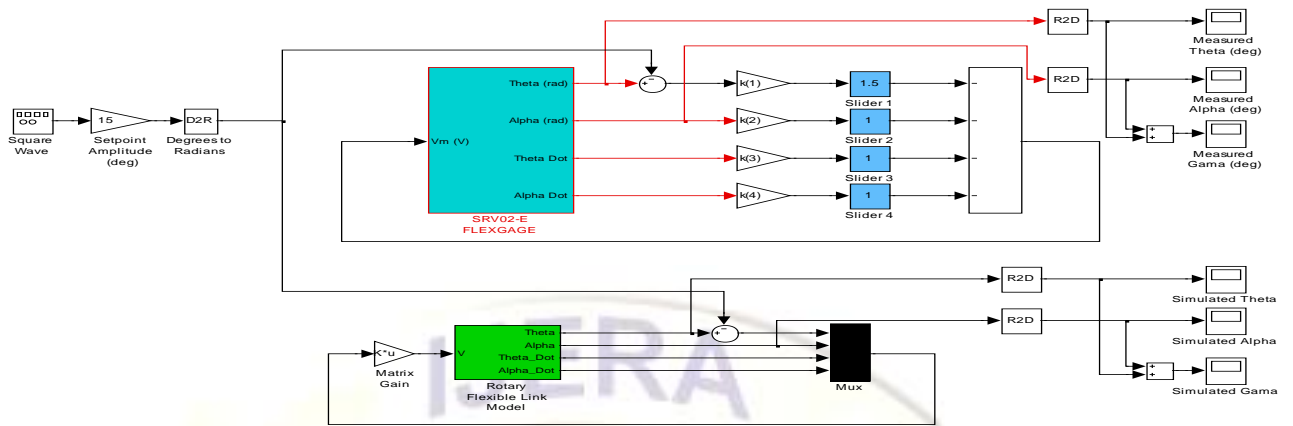


Fig 3: Block Diagram of Rotary Flexible Link State-Feedback Controller (LQR)

4. Results

To get the result after running the code, measured angles by clicking on the scope button in wincon and selected scope and press ok. This opens a real time plot. When rotated the link's position the trace will follow movement. The corresponding values are noted.

Suppose we use $k=[.5,.9,.005,.01,.02]$ Fig 4, Fig 5

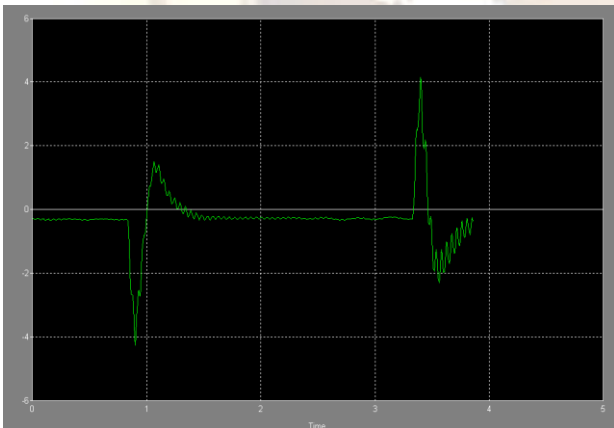


Fig 4: Real time Response of alpha with more disturbance

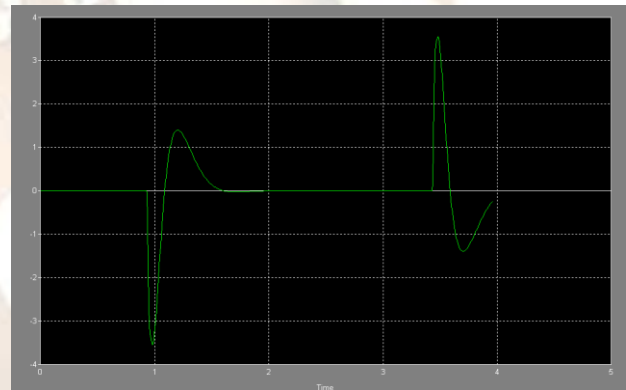


Fig 5: Simulated Response of Alpha

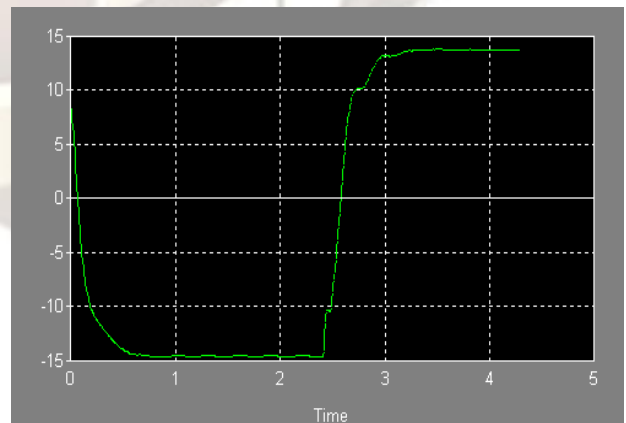


Fig 6: Realtime response of Theta

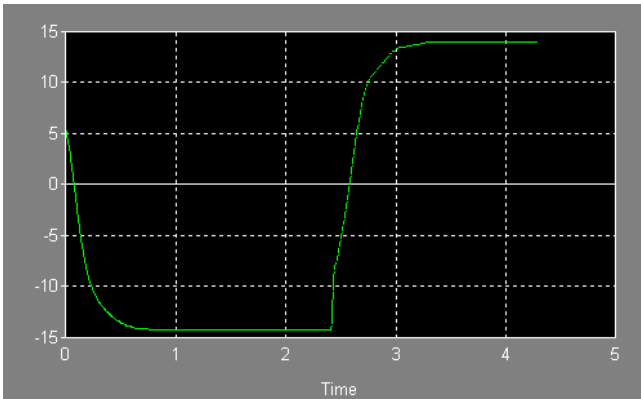


Fig 7: Simulated response of Theta

Using another state feedback vector, we obtain: $k=[.05,-.15,.009,.01,.02]$ following Alpha real and simulated responses Fig 7, Fig 8

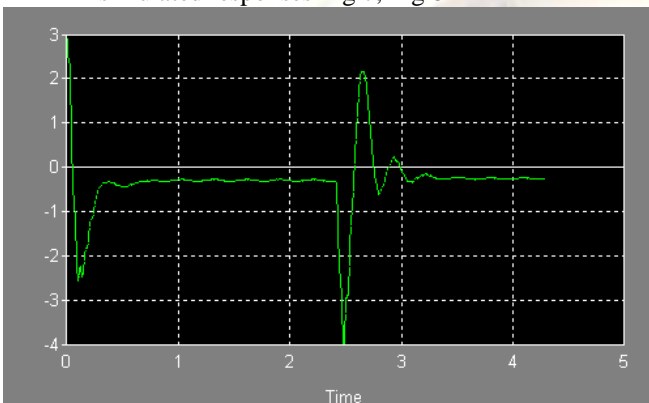


Fig 8: Real time Response of alpha with less disturbance

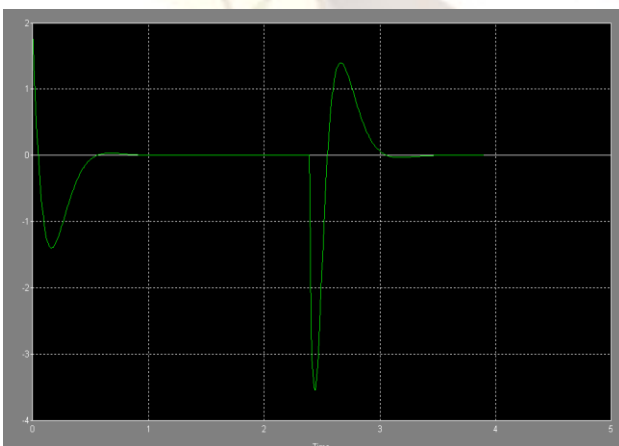


Fig 9: Simulated Response of alpha

Another real-time experiment tests the Disturbance rejection capability of the control system. As we can see from Figure 8 and Figure 9, we obtain very good reference track and vibration suppression Performances. Also, if the flexible link deflection

angle is perturbed, the control system acts fast and precise.

4. CONCLUSIONS

In this article is presented a Quanser Consulting Inc. laboratory experiment: the Flexible Link vibration suppression. Using Dc motor voltage we are able to control the deflection angle and vibration. The controller method is the LQR or State-feedback. The Lagrange methodology was used to derive the rigid flexible link set up equations of motion, considering the device was a simple spring mass model. The comparison of performance of real time system with simulation is done by simultaneously observing measured and simulated angular position of flexible link. It is found that the response is improved with LQR. The results are able to seen in the WinCon software server.

REFERENCES

- [1] A., Albassam, Fast maneuver control design for flexible structures using concentrated masses. *Journal of Sound and Vibration*, 273, (2004), 755-775.
- [2] R.Y. Chiang, M.G. Safonov: "Robust Control Toolbox", The Math Works, Inc., 1992
- [3] Rotary Flexible Link Experiment. [Online: www.quanser.com].
- [4] Flexible Link Experiment ROTFLEX. [Online: www.quanser.com].
- [5] WinCon web site. [Online: www.quanser.com].
- [6] Bobasu E, Modelling and simulation. Theory and applications, Craiova, 2002.
- [7] Ionete C., LQG/LTR controller design for flexible link quanser real-time experiment, International Symposium on System Theory SINTES11, Craiova, 2003.
- [8] W.J. Book, "Modeling, design and control of flexible link manipulator arms: a tutorial review", *Proceedings of IEEE Conference on Decision and Control*, pp. 500-506, 1990.
- [9] Camacho E. F and Bordons C., *Model Predictive Control*, Springer, London, 1999.