Iterative Learning Algorithm based on Observer and Linear Quadratic Performance Function

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ABSTRACT

In this paper we propose an iterative learning algorithm based on observer and linear quadratic performance function. We calculate the initial control value for the iteractive learning algorithm based on the estimation of the states, which guarantees the efficient asymptotic tracking of any desired trajectories. Furthermore, with Linear quadratic optimal control theory, we obtain the optimized control value for the interactive progress by minimizing the performance function. Finally, we simulate the performance our ILC algorithm and it shows that this new method can provide the initial control value for the uncertain linear time-invariant systems, as well as decrease the tracking errors asymptotically in the interactive progress.

Keywords - iterative learning control(ILC), observer, linear quadratic performance function, initial control value

1. INTRODUCTION

Iterative learning control(ILC) is a branch of intelligent control which has a strict mathematical description. For repeated tracking control tasks, ILC improves the performance of a system via continual learning is one of the efficient control algorithms. In 1984, Arimoto et al.[1] proposed the first order D-type ILC to train industrial robotic system via the derivative of output error. Then, improvement algorithms and optimization algorithms were researched by many scholars at home and abroad[2-5]. In these new algorithms, quite a number of them are based on states. [6]proposed an optimal ILC scheme with current feedback for linear non-minimum phase plants based on an optimality criterion. Variable trajectory tracking control problems for a class of uncertain systems were studied in [7] which based on states to obtain control algorithms. [6,7] research results were based on the measurable of all system states. But in practical applications, not all of the system states can measure. So, to research new ILC algorithm based on measurable states has important significance. In [8], the observer-based iterative learnng control with evolutionary programming algorithm is proposed for MIMO nonlinear systems. Moreover, in order to speed up the convergence of the ILC, evolutionary programming is applied to search for the optimal learning gain to reduce the training time. An observer based adaptive ILC scheme is developed for a class of nonlinear system with unknown time-varying parameters and unknown time-varying delays in [9]. The learning law of unknown constant parameter is differential-difference-type, and the learning law of unknown time-varying parameter is difference-type. On the basis of existing research results, a new ILC algorithm based on the state observer and two quadratic performance index function is proposed to improve the tracking performance of desired trajectory in this paper. First, the ideal initial control variable is obtained based on state estimator. Then, the ILC algorithm design process based on two quadratic performance index function is given.

2. ILC IDEAL INITIAL CONTROL BASED ON STATE OBSERVER

State observer is a dynamic link which can reconstruction original system based on input and output. It gives estimated value of the real state. Consider the following class of linear time-invariant system which can observe:

$$\begin{cases} \dot{\boldsymbol{x}}(t)=\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{u}(t) \\ \boldsymbol{y}(t)=\boldsymbol{C}\boldsymbol{x}(t) \end{cases} \tag{1}$$

where $\boldsymbol{x}(t)\in\mathbf{R}^n$ 、 $\boldsymbol{u}(t)\in\mathbf{R}^m$ 、 $\boldsymbol{y}(t)\in\mathbf{R}^l$ are the state vector, the control input and the system output. $\boldsymbol{x}(t_0)=\boldsymbol{x}_0$ is initial state of the system. $\boldsymbol{A}\in\mathbf{R}^{n\times n}$ 、 $\boldsymbol{B}\in\mathbf{R}^{n\times m}$ 、 $\boldsymbol{C}\in\mathbf{R}^{l\times n}$ are constant matrixs with certain dimension.

Using system input $\boldsymbol{u}(t)$ and output $\boldsymbol{y}(t)$ to reconstruction the system input. The dynamic process can be described as:

$$\dot{\hat{\boldsymbol{x}}}(t)=(\boldsymbol{A}-\boldsymbol{G}\boldsymbol{C})\hat{\boldsymbol{x}}(t)+\boldsymbol{B}\boldsymbol{u}(t)+\boldsymbol{G}\boldsymbol{C}\boldsymbol{x}(t) \tag{2}$$

where $\hat{\boldsymbol{x}}(t)\in\mathbf{R}^n$ is observer state variables, namely state estimation value, $\tilde{\boldsymbol{x}}(t)=\boldsymbol{x}(t)-\hat{\boldsymbol{x}}(t)$ is the state estimation error, $\boldsymbol{G}\in\mathbf{R}^{n\times l}$ is feedback gain matrix that to be determined of state observer. $(\boldsymbol{A}-\boldsymbol{G}\boldsymbol{C})$ is asymptotic stability when matrix $\boldsymbol{G}$ is appropriate. The state observer state $\hat{\boldsymbol{x}}(t)$ will inevitably tends to the state $\boldsymbol{x}(t)$ of the original system when $t \to \infty$. Obviously, we can obtain the following formula:

$$\begin{aligned}\dot{\tilde{\boldsymbol{x}}}(t) &= \dot{\boldsymbol{x}}(t) - \dot{\hat{\boldsymbol{x}}}(t) \\ &= \boldsymbol{A}\boldsymbol{x}(t) - \boldsymbol{A}\hat{\boldsymbol{x}}(t) - \boldsymbol{G}\boldsymbol{C}(\boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)) \\ &= (\boldsymbol{A} - \boldsymbol{G}\boldsymbol{C})\tilde{\boldsymbol{x}}(t)\end{aligned} \quad (3)$$

Combine formula (1) with (3), we can obtain the closed loop system about state and state estimation error as follows:

$$\begin{cases}\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \\ \dot{\tilde{\boldsymbol{x}}}(t) = (\boldsymbol{A} - \boldsymbol{G}\boldsymbol{C})\tilde{\boldsymbol{x}}(t)\end{cases} \quad (4)$$

Consider the following state feedback controller $\boldsymbol{u}(t) = \boldsymbol{K}\dot{\boldsymbol{x}}(t)$ to stabilize system, where $\boldsymbol{K} \in \mathbf{R}^{m\times n}$ is control gain matrix. Then, we can define the system performance index function as:

$$J(\boldsymbol{Q}, \boldsymbol{R}, \boldsymbol{S}) = \int_0^\infty \left[\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{x}(t) + \boldsymbol{u}^{\mathrm{T}}(t)\boldsymbol{R}\boldsymbol{u}(t) + (\boldsymbol{G}\boldsymbol{C}\tilde{\boldsymbol{x}}(t))^{\mathrm{T}}\boldsymbol{R}\boldsymbol{S}\boldsymbol{C}\tilde{\boldsymbol{x}}(t)\right]dt \quad (5)$$

where $\boldsymbol{Q} \in \mathbf{R}^{n\times n}$, $\boldsymbol{R} \in \mathbf{R}^{m\times m}$, $\boldsymbol{S} \in \mathbf{R}^{l\times l}$ are positive definite matrices.

Taking the following Lyapunov function:

$$V(\boldsymbol{x}(t), \tilde{\boldsymbol{x}}(t)) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_1\boldsymbol{x}(t) + \tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{P}_2\tilde{\boldsymbol{x}}(t) \quad (6)$$

one easily obtains:

$$\begin{aligned}\dot{V}(\boldsymbol{x}(t), \tilde{\boldsymbol{x}}(t)) &= \dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{P}_1\boldsymbol{x}(t) + \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_1\dot{\boldsymbol{x}}(t) \\ &+ \dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(t)\boldsymbol{P}_2\tilde{\boldsymbol{x}}(t) + \tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{P}_2\dot{\tilde{\boldsymbol{x}}}(t) \\ &= \boldsymbol{x}^{\mathrm{T}}(t)\left[\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}_1 + \boldsymbol{P}_1\boldsymbol{A}\right]\boldsymbol{x}(t) + (\boldsymbol{B}\boldsymbol{u})^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{x}(t) + \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_1\boldsymbol{B}\boldsymbol{u} \\ &+ \tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\left[\left(\boldsymbol{A}^{\mathrm{T}} - (\boldsymbol{G}\boldsymbol{C})^{\mathrm{T}}\right)\boldsymbol{P}_2 + \boldsymbol{P}_2(\boldsymbol{A} - \boldsymbol{G}\boldsymbol{C})\right]\tilde{\boldsymbol{x}}(t)\end{aligned} \quad (7)$$

Taking Hamilton function as:

$$\begin{aligned}H(\boldsymbol{x}, \tilde{\boldsymbol{x}}, \boldsymbol{u}, t) &= \dot{V}(\boldsymbol{x}(t), \tilde{\boldsymbol{x}}(t)) + \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{x}(t) \\ &+ \boldsymbol{u}^{\mathrm{T}}(t)\boldsymbol{R}\boldsymbol{u}(t) + (\boldsymbol{G}\boldsymbol{C}\tilde{\boldsymbol{x}}(t))^{\mathrm{T}}\boldsymbol{S}\boldsymbol{G}\boldsymbol{C}\tilde{\boldsymbol{x}}(t) \\ &= \boldsymbol{x}^{\mathrm{T}}(t)\left[\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}_1 + \boldsymbol{P}_1\boldsymbol{A} + \boldsymbol{Q}\right]\boldsymbol{x}(t) \\ &+ (\boldsymbol{B}\boldsymbol{u})^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{x}(t) + \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_1\boldsymbol{B}\boldsymbol{u} + \tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\left[\left(\boldsymbol{A}^{\mathrm{T}} - (\boldsymbol{G}\boldsymbol{C})^{\mathrm{T}}\right)\right. \\ &\left.\boldsymbol{P}_2 + \boldsymbol{P}_2(\boldsymbol{A} - \boldsymbol{G}\boldsymbol{C}) + (\boldsymbol{G}\boldsymbol{C})^{\mathrm{T}}\boldsymbol{S}\boldsymbol{G}\boldsymbol{C}\right]\tilde{\boldsymbol{x}}(t) + \boldsymbol{u}^{\mathrm{T}}(t)\boldsymbol{R}\boldsymbol{u}(t)\end{aligned} \quad (8)$$

The variational of $\boldsymbol{x}(t)$、$\boldsymbol{u}(t)$ relative to the optimal values $\boldsymbol{x}^*(t)$、$\boldsymbol{u}_0^*(t)$ are $\delta\boldsymbol{x}(t)$、$\delta\boldsymbol{u}(t)$. Then , the necessary condition of augmented functional J_a taken minimal value is $\delta J_a = 0$ for any $\delta\boldsymbol{x}(t)$ and $\delta\boldsymbol{u}(t)$. Namely, satisfing the following formula:

$$\frac{\partial H}{\partial \boldsymbol{u}} = \boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{x}(t) + \boldsymbol{R}\boldsymbol{u}_0^*(t) = 0 \quad (9)$$

Then, ILC optimal initial control value $\boldsymbol{u}_0^*(t)$ can obtain as:

$$\boldsymbol{u}_0^*(t) = -\boldsymbol{R}^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{x}(t) \quad (10)$$

Replace the system state $\boldsymbol{x}(t)$ in formula（10）with observer state variables $\hat{\boldsymbol{x}}(t)$, ILC optimal initial control value based on state observer can obtain as:

$$\hat{\boldsymbol{u}}_0^*(t) = -\boldsymbol{R}^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_1\hat{\boldsymbol{x}}(t) \quad (11)$$

Obviously, there exist errors when replace（10）with（11）. But it satisfies $\hat{\boldsymbol{u}}_0^*(t) \approx \boldsymbol{u}_0^*(t)$ when time is long enough. Set $\bar{\boldsymbol{x}}(t) = -\boldsymbol{G}\boldsymbol{C}\tilde{\boldsymbol{x}}(t)$, it can deduce the following result:

$$\frac{\partial H}{\partial \bar{\boldsymbol{x}}} = \boldsymbol{S}\bar{\boldsymbol{x}}(t) + \boldsymbol{P}_2\tilde{\boldsymbol{x}}(t) = 0 \quad (12)$$

Furthermore, the unknown feedback gain matrix $\boldsymbol{G}$ of state observer can get as $\boldsymbol{G} = \boldsymbol{S}^{-1}\boldsymbol{P}_2\boldsymbol{C}^{-1}$.

3. ILC ALGORITHM BASED ON QUADRATIC PERFORMANCE INDEX

As the optimal initial control $\hat{\boldsymbol{u}}_0^*(t)$ is obtained, the ILC algorithm based on quadratic performance index can describe as follows. According to (1), the error model of iteration domain can be derived as：

$$\begin{cases}\dot{\bar{\boldsymbol{x}}}_k(t) = \boldsymbol{A}\bar{\boldsymbol{x}}_k(t) + \boldsymbol{B}\bar{\boldsymbol{u}}_k(t) \\ \boldsymbol{e}_k(t) = \boldsymbol{C}\bar{\boldsymbol{x}}_k(t)\end{cases} \quad (13)$$

where $\bar{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_{k+1} - \hat{\boldsymbol{x}}_k$, $\bar{\boldsymbol{u}}_k = \boldsymbol{u}_{k+1} - \boldsymbol{u}_k$, $\boldsymbol{e}_k = \boldsymbol{y}_k - \boldsymbol{y}_d$. The linear quadratic performance function is defined as follows:

$$J_1(\boldsymbol{Q}_1, \boldsymbol{R}_1) = \int_t^\infty \left[\boldsymbol{e}_k^{\mathrm{T}}(t)\boldsymbol{Q}_1\boldsymbol{e}_k(t) + \bar{\boldsymbol{u}}_k^{\mathrm{T}}(t)\boldsymbol{R}_1\bar{\boldsymbol{u}}_k(t)\right]dt \quad (14)$$

Applying Lagrange multiplier vector $\boldsymbol{\lambda}_1(t)$ constructs generalized functional as follows:

$$J_a(\boldsymbol{Q}_1, \boldsymbol{R}_1) = \int_t^\infty \left\{\boldsymbol{e}_k^T(t)\boldsymbol{Q}_1\boldsymbol{e}_k(t) + \bar{\boldsymbol{u}}_k^T(t)\boldsymbol{R}_1\bar{\boldsymbol{u}}_k(t) + \boldsymbol{\lambda}_1^{\mathrm{T}}(t)\left[\boldsymbol{A}\bar{\boldsymbol{x}}_k(t) + \boldsymbol{B}\bar{\boldsymbol{u}}_k(t) - \dot{\bar{\boldsymbol{x}}}_k(t)\right]\right\}dt \quad (15)$$

The Hamilton function is defined as:

$$H(\bar{\boldsymbol{x}}_k, \bar{\boldsymbol{u}}_k, \boldsymbol{\lambda}_1, t) = \boldsymbol{e}_k^{\mathrm{T}}(t)\boldsymbol{Q}_1\boldsymbol{e}_k(t) + \bar{\boldsymbol{u}}_k^{\mathrm{T}}(t)\boldsymbol{R}_1\bar{\boldsymbol{u}}_k(t) + \boldsymbol{\lambda}_1^{\mathrm{T}}(t)\left[\boldsymbol{A}\bar{\boldsymbol{x}}_k(t) + \boldsymbol{B}\bar{\boldsymbol{u}}_k(t)\right] \quad (16)$$

According to variational theorem, there exist optimal control value $\boldsymbol{u}_k^*(t)$ and optimal costate $\boldsymbol{\lambda}_1^*(t)$ satified following equation:

$$\begin{cases} \dfrac{\partial H\left(\bar{\boldsymbol{x}}_k^*, \bar{\boldsymbol{u}}_k^*, \boldsymbol{\lambda}_1^*, t\right)}{\partial \bar{\boldsymbol{u}}_k} = \bar{\boldsymbol{u}}_k^{*\mathrm{T}}(t)\boldsymbol{R}_1 + \boldsymbol{\lambda}_1^{*\mathrm{T}}(t)\boldsymbol{B} = 0 \\ \dot{\boldsymbol{\lambda}}_1^*(t) = -\dfrac{\partial H\left(\bar{\boldsymbol{x}}_k^*, \bar{\boldsymbol{u}}_k^*, \boldsymbol{\lambda}_1^*, t\right)}{\partial \bar{\boldsymbol{x}}_k} = -\left(\boldsymbol{e}_k^{\mathrm{T}}(t)\boldsymbol{Q}_1\boldsymbol{C} + \boldsymbol{\lambda}_1^{*\mathrm{T}}(t)\boldsymbol{A}\right) \end{cases} \tag{17}$$

Then, combining (11) to (17), the ILC algorithm based on state observer and quadratic performance index can obtain as follows:

$$\begin{cases} \hat{\boldsymbol{u}}_0^*(t) = -\boldsymbol{R}^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_1\hat{\boldsymbol{x}}(t) \\ \boldsymbol{u}_{k+1}^*(t) - \boldsymbol{u}_k^*(t) = -\left(\boldsymbol{B}\boldsymbol{R}_1^{-1}\right)^{\mathrm{T}}\boldsymbol{\lambda}_1^*(t) \\ \dot{\boldsymbol{\lambda}}_1^*(t) = -\boldsymbol{e}_k^{\mathrm{T}}(t)\boldsymbol{Q}_1\boldsymbol{C} - \boldsymbol{\lambda}_1^{*\mathrm{T}}(t)\boldsymbol{A} \end{cases} \tag{18}$$

4. SIMULATION EXPERIMENT

Experiment 1: Consider the dynamic model of an 2 rigid-link robot system as:

$$\boldsymbol{\tau} = \boldsymbol{H}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \boldsymbol{M}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + \boldsymbol{G}(\boldsymbol{\theta}) \tag{19}$$

Where $\boldsymbol{\theta}$、$\dot{\boldsymbol{\theta}}$、$\ddot{\boldsymbol{\theta}}$ are position vector, velocity vector, and acceleration vector, $\boldsymbol{\tau} = [M_1 \quad M_2]^{\mathrm{T}}$ are two-dimensional control torque, $\boldsymbol{H}(\boldsymbol{\theta})$ is inertia matrix, $\boldsymbol{M}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$ are two-dimensional coriolis forces, $\boldsymbol{G}(\boldsymbol{\theta})$ is gravity. The parameters are taken as follows:

$$\boldsymbol{H}(\boldsymbol{\theta}) = \begin{bmatrix} \left(\frac{1}{3}m_1 l^2 + \frac{4}{3}m_2 l^2 + m_2 l^2 \cos\theta_2\right) & \left(\frac{1}{2}m_2 l^2 \cos\theta_2 + \frac{1}{3}m_2 l^2\right) \\ \frac{1}{3}m_2 l^2 + \frac{1}{2}m_2 l^2 \cos\theta_2 & \frac{1}{3}m_2 l^2 \end{bmatrix} \tag{20}$$

$$\boldsymbol{M}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \begin{bmatrix} -m_2 l^2 \sin(\theta_2)\dot{\theta}_2 & \frac{1}{2}m_2 l^2 \sin(\theta_2)\dot{\theta}_2 \\ \frac{1}{2}m_2 l^2 \sin(\theta_2)\dot{\theta}_1 & 0 \end{bmatrix} \tag{21}$$

$$\boldsymbol{G}(\boldsymbol{\theta}) = \begin{bmatrix} \left(-\frac{1}{2}m_1 g l\cos(\theta_1) - \frac{1}{2}m_2 g l\cos(\theta_1 + \theta_2) - m_2 g l\cos(\theta_1)\right) \\ -\frac{1}{2}m_2 g l\cos(\theta_1 + \theta_2) \end{bmatrix} \tag{22}$$

where $m_1 = m_2 = 3\,\mathrm{kg}$, $l = 0.5\,\mathrm{m}$. The state space realization form of (19) can taken as follows:

$$\begin{cases} \dot{\boldsymbol{x}}_1 = \boldsymbol{x}_2 \\ \dot{\boldsymbol{x}}_2 = \boldsymbol{H}^{-1}(\boldsymbol{x}_1)[\boldsymbol{\tau} - \boldsymbol{M}(\boldsymbol{x}_1, \boldsymbol{x}_2)\boldsymbol{x}_2 - \boldsymbol{G}(\boldsymbol{x}_1)] \end{cases} \tag{23}$$

where $\boldsymbol{x}_1 = \boldsymbol{\theta} = [\theta_1 \quad \theta_2]^{\mathrm{T}}$, $\boldsymbol{x}_2 = \dot{\boldsymbol{\theta}} = [\dot{\theta}_1 \quad \dot{\theta}_2]^{\mathrm{T}}$. The measurable output is $\boldsymbol{y} = \boldsymbol{x}_1$. It needs to track the following desired trajectory in $t \in [0,1]\mathrm{s}$:

$$\boldsymbol{y}_d = \begin{bmatrix} \theta_{1d} \\ \theta_{2d} \end{bmatrix} = \begin{bmatrix} -\frac{5}{3}t^3 + \frac{5}{2}t^2 \\ -\frac{1}{3}t^3 - \frac{3}{2}t^2 - \frac{5}{6} \end{bmatrix} \tag{24}$$

Iterative learning is conducted for the robot system model by Formula 18 via MATLAB to demonstrate the tracking accuracy and convergence rate of the algorithm presented in this article for the expected track $\boldsymbol{y}_d$. The initial value of the observer is set as $[0 \quad 0 \quad 0 \quad 0]^{\mathrm{T}}$, and the initial control variables are optimized on basis of the state variables of the observer, on which basis the tracking outputs of Joint 1 and Joint 2 on the expected tracks are collected. The simulation curves are shown in the following Figures. Fig. 1 shows the tracking acquisition of Joint 1 on the expected track θ_{1d} after twice of iterations, while Fig. 2 shows that of Joint 2 on the expected track θ_{2d} after twice of iterations also. As shown in Fig. 1 and Fig. 2, a considerably high tracking accuracy can be achieved by a few repetitions of iterative learning, for the system is enabled to perform fast tracking of any expected track by predetermining the ideal initial controlled variables, and the iteration error is minimized effectively by the learning algorithm on basis of quadratic performance index.

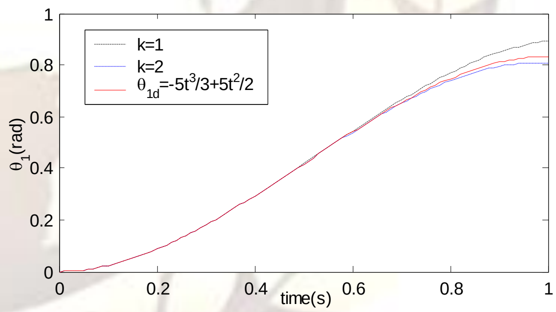

Figure 1: The output tracking curve of Joint 1 by proposed algorithm for k=1,2

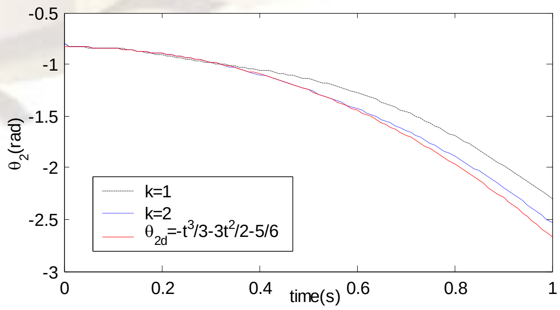

Figure 2: The output tracking curve of Joint 2 by proposed algorithm for k=1,2

Experiment 2: The conventional algorithm simulations are performed on such model of robot system, adopting the regular D-type open-loop learning law with the initial control variables set as 0, in order to further illustrate the effectiveness of the algorithm presented in this paper; both the initial state and the expected tracks are the same of the above mentioned simulations. The tracking profiles of Joint 1 and Joint 2 for the expected trajectory of θ_{1d} and θ_{2d} are presented in Fig. 3 and Fig. 4, which shows a noticeable deviation of the output trajectory with the expected trajectory emerged after twice of iterations, and that the tracking output after 15 iterations (k=15) is equally accurate as that when K=2. Although the tracking errors are shrinking as more repetitions of iterations are performed, the convergence rates are also affected to some extent. Thus the algorithm brought up in this article has shown its prospect to minimize the repetition of iteration and to increase the convergence rate.

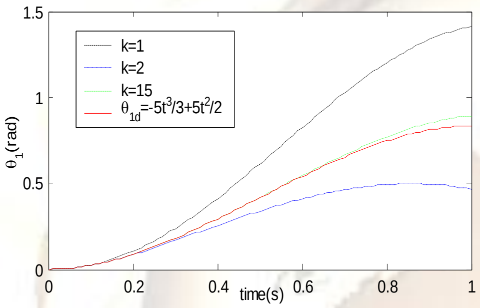

Figure 3: The output tracking curve of Joint 1 by contrast experiment for k=1,2,15

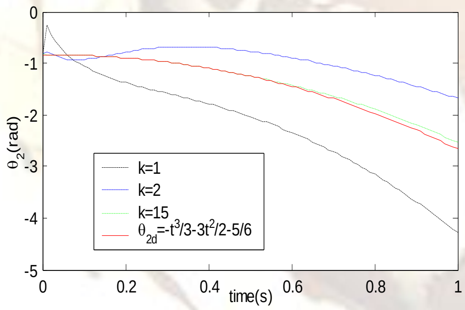

Figure 4: The output tracking curve of Joint 2 by contrast experiment for k=1,2,15

The comparison of tracking performance by basic algorithm and proposed algorithm for k=1,2 and k=1,2,15 are given in Table 1. Seeing from the simulation results, to track expected trajectory by proposed algorithm achieves better performance than basic algorithm with smaller maximum error and Average error. Taking time and precision into consideration, the proposed algorithm here is of more practical value.

5. CONCLUSION

In this paper, the author analyzed the iterative-learning algorithm on basis of the status observer data and the quadratic performance indexes on the observable linear time-invariant system with part of its status variables available for measurement. The repetition of iteration is minimized by predetermining the nearly-ideal initial controlled variables in estimated status after reconfiguring the system input, instead of choosing the initial variables randomly. On that basis, the author offers the new iterative-learning control algorithm using the new quadratic performance indexes, which enables quick tracking on any expected track. The effectiveness of such algorithm has been demonstrated by simulations.

Table 1: Comparison of tracking performance by different algorithm

Joint	Algorithm	iteration times	maximum error	Average error
Joint 1	Basic algorithm	1	0.5755	0.2309
		2	0.3676	0.1204
		15	0.0533	0.0042
	Proposed algorithm	1	0.0586	0.0107
		2	0.0260	0.0039
Joint 2	Basic algorithm	1	1.6240	0.7893
		2	0.9881	0.4464
		15	0.2257	0.0382
	Proposed algorithm	1	0.3687	0.1309
		2	0.1300	0.0305

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