

## Extraction Of Radar Signal Using Auto-Correlation Functions

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### ABSTRACT

This research seeks to present post-integration processing in order to improve the sensitivity of electronic support measure (ESM) receivers. Correlation methods take advantage of the periodic character of radar signals. In such cases, autocorrelation and cross-correlation improve the detection of signals with high-repetition frequency. Furthermore, since the extraction of radar parameters is necessary to identify received signals, we study three types of estimators: straightforward method, interpolation method and maximum likelihood one. Simulation studies with realistic models and real signals are carried out to validate performances of such processing. With a view to implanting correlation functions, some architectures are studied. The choice of method is of interest since we need a lot of samples to be integrated. To conclude, as radar ESM receiver requires most information on received signals, the enhancement of the sensitivity using correlation method is of great interest.

**Keywords:** Electronic support measure, autocorrelation, estimators, sensitivity, signal-to-noise ratio, cross-correlation, simulation, recursive structure

### Introduction

Electronic support measure systems perform the function of electromagnetic area surveillance in order to determine the identity and direction of arrival of surrounding radar emitters. Automatic radar ESM systems are passive receivers which receive emissions from other platforms, measure the parameters of each detected pulse and thus sort the emissions to enable the determination of the radar parameters and identification of each emitter.

The contribution of our study is to improve the receiver sensitivity using post-integration processing such as autocorrelation and cross-correlation methods.

### 1.1 Radar ESM System

In practice, the amplitude envelope received by the ESM system is a series of pulses modulated by the radar frequency carrier.

After detection of radar pulses intercepted by the antennas, the ESM receiver takes digital measurements of frequency within the pulses, bearing, amplitude and time of arrival of the pulse. The processor de-interleaves pulses and assigns samples to a pulse train which represents the radar signal. Using a database, the processor identifies the emission. Then information is presented to the defense systems.

A basic microwave receiver consists of a front-end filter followed by a detector and a video low-pass filter with  $B_v$  bandwidth.

### 1.2 Need for Sensitivity

Those receivers always need to improve the probability of detection of radar signals. Using digital processing, we can extract signal from noise, keeping in mind that such a system must provide radar parameters for identification.

Sensitivity is defined in terms of the signal-to-noise ratio at the output of the video filter. This paper deals with correlation processing. Section 2 discusses the sensitivity gain obtained at the output of autocorrelation processing, detection processing of those methods and exploitation of the results. Special attention is paid in section 3 to the application of cross-correlation. Some simulations are presented in section 4 to validate the performances. Section 5 describes the complexity to implanting such processing since many samples should be integrated.

## 2. Autocorrelation

Since the radar signal is periodic, autocorrelation processing seems to be of interest, because it keeps the repetitive character with the same pulse repetition frequency. After quadratic detection, the received signal is a pulse train in additive Gaussian noise.

### 2.1 Estimators

Given a finite observation interval  $T_i$ , the autocorrelation is estimated by:

$$\hat{\Gamma}_1(k) = \frac{1}{N-k} \sum_{i=0}^{N-k-1} X(i)X(i+k) \quad 0 \leq k \leq N-1 \quad (1)$$

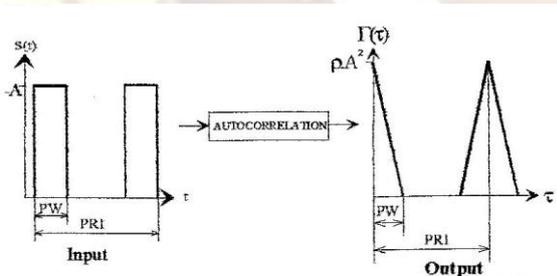
$$\hat{\Gamma}_2(k) = \frac{1}{N} \sum_{i=0}^{N-k-1} X(i)X(i+k) \quad (2)$$

$$\hat{\Gamma}_3(k) = \frac{1}{N_{corr}} \sum_{i=0}^{N_{corr}-1} X(i)X(i+k) \quad 0 \leq k \leq N_{corr}-1 \quad (3)$$

where N is the number of samples on the  $T_i$  interval.  $T_i$  has to be sufficiently long for the estimated correlation to be accurate. Both estimators  $\hat{\Gamma}_1$  (unbiased) and  $\hat{\Gamma}_2$  (biased) use all the samples, whereas the third one works with  $N_{corr}$  samples out of  $2N_{corr}$  ( $N_{corr}$  is a constant value,  $N_{corr} = N/2$ ). The third estimator carries out the correlation function on a 'sliding temporal window' but its performances decrease by 3dB in comparison with the two previous estimators. Despite this, the  $\hat{\Gamma}_3$  estimator gives the first result when  $N_{corr}$  samples have been integrated, whereas we have to wait for N samples with the first and second functions. This note achieves a compromise between SNR gain and extraction of samples in a prominent position.

### 2.2 Sensitivity Gain

The output process of a radar signal without noise is as follows:



**Fig. 1:** Output signal where PRI = pulse repetition interval, PW = pulse-width

The signal is mixed with an additive Gaussian noise with zero mean,  $\sigma^2$  variance and  $B_v$  bandwidth. Signal and noise will not correlate then  $\Gamma_{SN}(\tau) = 0$ .

Furthermore, the observation interval should be much greater than PRI in order to decrease the error of the output signal since we integrate samples during a non whole number of PRI.

Given  $X(t) = S(t) + N(t)$ , the output process is:

$$\Gamma_X(\tau) = \Gamma_S(\tau) + \Gamma_N(\tau). \quad (4)$$

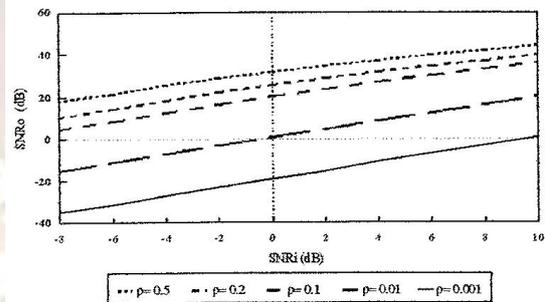
As we suppose that the noise correlation duration is very low in comparison with  $T_i$ , thus  $\Gamma_N(\tau)$  is very low compared with  $\Gamma_S(\tau)$ .

At a peak of correlation, the output SNR ( $SNR_o$ ) is:

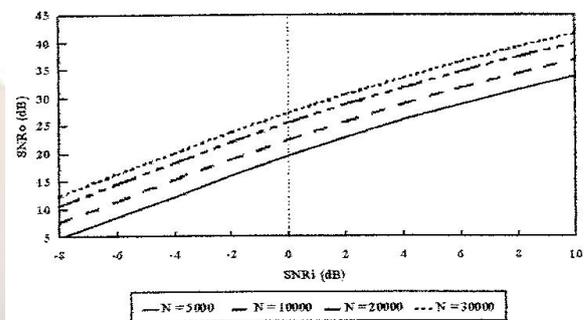
$$SNR_o = \frac{N_i p^2 SNR_i^2}{I + 2p SNR_i}$$

Where,  $SNR_i = A^2 / \sigma^2 - N_i = \pi \cdot B_v \cdot T_i = \pi \cdot B_v \cdot N \cdot T_s =$  Number of independent samples.

The SNR gain depends on the integration window (Fig.3), the duty cycle of the pulse train ( $p=PRI/PW$ ) (Fig.2). That is why such processing is of interest for radar signals with high pulse repetition frequency (HPRF signal). Furthermore, with a low  $SNR_i$ , the processing gain decreases quickly (Fig. 2).



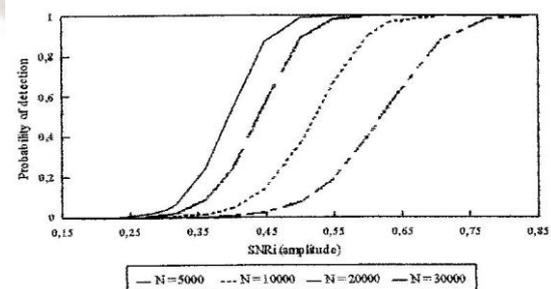
**Fig. 2:** Variation of  $SNR_o$  with  $SNR_i$  and  $\rho$  ( $N=20000$ ).



**Fig. 3:** Variation of  $SNR_o$  with  $SNR_i$  and  $N(\rho=0,2)$ .

### 2.3 Detection threshold

Fig.4 determines the detection of a peak of correlation.



**Fig. 4:** Probability of detection with  $SNR_i$  and  $N$  ( $P_{fa}=1e-5$ ).

**2.4 Parameter Estimation**

The aim of an ESM receiver is to identify emissions, so the system has to define its characteristics exactly. There are three parameters to estimate: amplitude, PW and PRI. We study three estimators

The sampling frequency ( $1/T_s$ ) of the autocorrelation function is the same as the signal one. Indeed, the autocorrelation function of the signal with a band since Fourier transform of the autocorrelation function of the signal with a band limited spectral density, has the same limited band since Fourier transform of the autocorrelation function is the power spectral density. In that case Shannon's theorem applied to the signal or its correlation function gives the same sampling rate.

**2.4.1 Straightforward Method**

Studying the output sample, we get:

PRI = interval between two peaks

PW = width is the halfway up of a triangle of correlation.

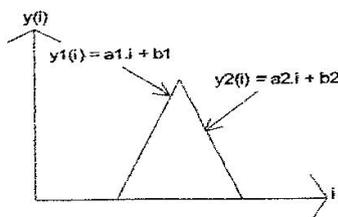
aplitude =  $\sqrt{\max p}$  (max is the peak value).

**2.4.2 Interpolation method**

At each triangle of correlation, we have to estimate the two slopes in the mean-square criterion:

$$PW = \frac{a_2 b_1 - a_1 b_2}{2a_1 a_2} \text{ and amplitude} =$$

$$\sqrt{\frac{2a_1 a_2 \text{ PRI}}{a_2 - a_1}}$$



**Fig. 5:** Slopes of the triangle interpolation.

**2.4.3 Maximum likelihood method**

$$P\hat{W}_{MV} = \max T_s \frac{\sum_{i=J-k_{PW}}^{J-1} (i-j)^2 + \sum_{i=J}^{J+k_{PW}} (i-j)^2}{\sum_{i=J-k_{PW}}^{J-1} (y_i - \max)(i-j) - \sum_{i=J}^{J+k_{PW}} (y_i - \max)(i-j)}$$

$$M\hat{A}X_{MV} = \frac{\sum_{i=J-k_{PW}}^{J-1} \left[ \frac{T_s}{PW} (i-j) + 1 \right] y_i + \sum_{i=J}^{J+k_{PW}} \left[ \frac{T_s}{PW} (i-j) - 1 \right] y_i}{\sum_{i=J-k_{PW}}^{J-1} \left[ \frac{T_s}{PW} (i-j) + 1 \right]^2 - \sum_{i=J}^{J+k_{PW}} \left[ \frac{T_s}{PW} (i-j) - 1 \right]^2}$$

where  $JT_s$  is the time of arrival of the peak,  $y_i$  the signal value at  $iT_s$ ,  $k_{LI}$  the number of samples used in a slope. We check that the pulse-width estimator and the peak value one are both efficient in the maximum likelihood criterion.

**Note:** We could estimate the PRI parameter with the maximum likelihood criterion using the Fourier transform. We will have an efficient estimator. Despite this, to implant the method we need an additional transform of the output process, and consequently more calculus.

**2.4.4 Simulation**

A computer simulation was conducted to compare the performances of the different estimators. The first method is the easiest one to implant, whereas the last estimator has the best accuracy thanks to the maximum likelihood principle as can be seen in Table 1.

SNR <sub>i</sub> (db)	Straight est. (%)		Interpol est. (%)		ML est.(%)	
	PW	peak	PW	peak	PW	peak
10	0	0,5	0,4	0,6	0,3	0,5
0	5,7	3,8	3,3	2,4	2,3	2,5
-3,6	13,8	7,2	10,7	6,5	5,5	6,6
-4,6	13,9	7,1	10,1	6,3	5,6	5,5

**Table 1** PW and peak correlation accuracy

The parameter accuracy is defined as follows:

$$\mathcal{E}_{c/o} = \frac{E[\text{parameter}]}{\sqrt{\text{variance}_{\text{parameter}}}} \cdot 100$$

**3. Cross-correlation**

The purpose of such a method is to recognize an a prior know signal  $Y(k)$  in the received noisy signal. The cross-correlation estimators are the same as those of autocorrelation where:

$$\hat{\Gamma}_1(k) = \frac{1}{N-k} \sum_{i=0}^{N-k-1} Y(i)X(i+k) \quad (6)$$

**3.1 Sensitivity Gain**

With same working hypothesis as 2.2, we get:

$$SNR_o = N_i \rho SNR_i \quad (7)$$

The gain is a linear function of the observation interval and of the duty cycle. Thus, we obtain a constant improvement (Fig. 6).

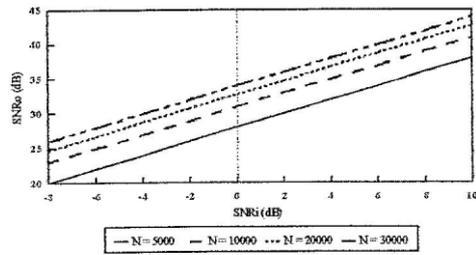


Fig. 6 Variation of  $SNR_o$  with  $SNR_i$  and  $N$  ( $\rho=0,2$ )

### 3.2 Detection Probability.

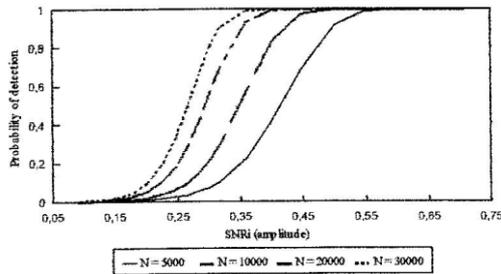


Fig. 7 Detection probability with  $SNR_i$  and  $N$  ( $P_{fa}=10e-5$ ).

Fig. 7 shows that the more samples we integrate, the easier the detection of a signal with additive noise is.

### 3.3 Autocorrelation and cross-correlation comparison

The use of cross-correlation function provides sensitivity gain, but special attention is given to such applications. Indeed, the operator of the ESM system has no information on the received signal. That is why the aim of this method is to search for a given radar in the surrounding area in order to perform the warning function.

### 4 Simulation description and results

Simulation studies with realistic models and real signals were conducted to corroborate the theoretical performance predictions. The scenario presented here (Fig. 8-9-10) is a mixture of three signals.

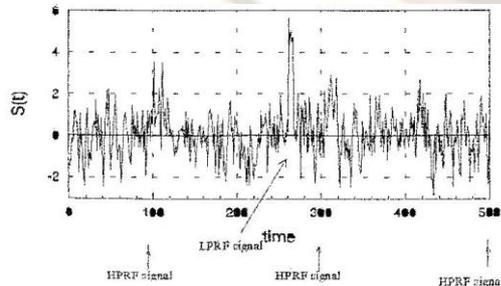


Fig. 8 Input signal with S1: HPRF signal with  $SNR_i = 0dB$ ;  $\rho=0,1$  - S2: LPRF (low PRF) signal with  $SNR_i=13dB$ ;  $\rho=5,1.1e-4$  - S3: LPRF signal with  $SNR_i=13dB$ ;  $\rho=6.1e-4$  -  $\sigma=1$  -  $N = 2000$ .

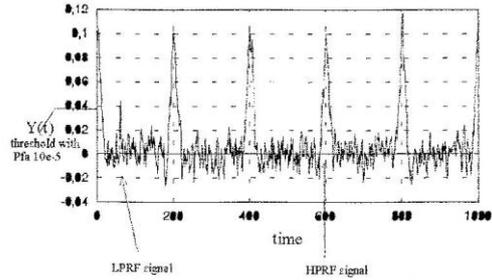


Fig. 9. Autocorrelation output signal.

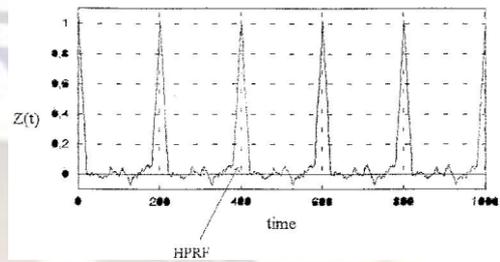


Fig. 10: Cross-correlation output signal.

### 5 Implanting Correlation Function

#### 5.1 Classical Structure

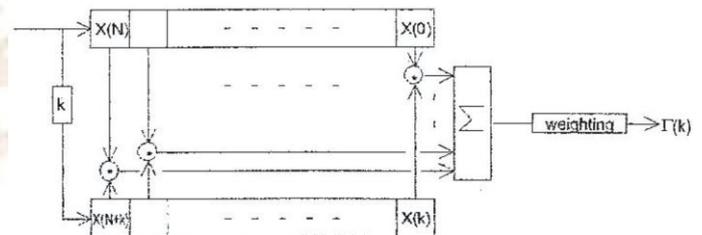


Fig. 11 Classical structure

#### 5.2 Recursive structure

Each correlation sample is the result of a recursive calculus (Eq. 8-9)

$$\Gamma(k) = \frac{R_k(N)}{N} \quad (8)$$

$$\text{Where, } R_k(N) = R_k(N-1) + X(N)X(N+k) \quad (9)$$

The recursive method uses one multiplier-accumulator cell (Fig.12) per correlation point. The correlator is designed as a macro-cell.

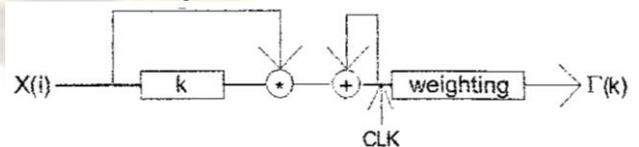


Fig. 12 Recursive structure.

#### 5.3 Fast correlation' Structure

Using a Fourier transform, we compare correlation and convolution (Eq. 10):

$$\Gamma_s(\tau) = S(t) * S^*(t) \xrightarrow{FT} \gamma_s(\nu) = \gamma_s(\nu)\gamma_s(\nu) \quad (10)$$

The advantage of such method is the use of a Fast Fourier transform algorithm.

## 6. CONCLUSION

To conclude, as the function of a radar ESM receiver is surveillance to determine radar activity, the system requires most information on received signals. So the enhancement of sensitivity achieved by correlation methods is of interest. Those functions contribute to the increased detection of HPRF signals with low SNR. This processing also carries out parameter estimation of radar signals. Future ESM system could incorporate such functions.

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