A Novel Approach for the fuzzy optimal solution of Fuzzy Transportation Problem

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ABSTRACT

In this paper we propose a new method for the Fuzzy optimal solution to the transportation problem with Fuzzy parameters. We develop Fuzzy version of Vogels and MODI algorithms for finding Fuzzy basic feasible and fuzzy optimal solution of fuzzy transportation problems without converting them to classical transportation problems. The proposed method is easy to understand and to apply for finding Fuzzy optimal solution of Fuzzy transportation problem occurring in real world situation. To illustrate the proposed method, numerical examples are provided and the results obtained are discussed.

Keywords - Fuzzy sets, Fuzzy numbers, Fuzzy transportation problem, Fuzzy ranking, Fuzzy arithmetic

I. INTRODUCTION

The transportation problem is a special type of linear programming problem which deals with the distribution of single product (raw or finished) from various sources of supply to various destination of demand in such a way that the total transportation cost is minimized. There are effective algorithms for solving the transportation problems when all the decision parameters, i. e the supply available at each source, the demand required at each destination as well as the unit transportation costs are given in a precise way. But in real life, there are many diverse situations due to uncertainty in one or more decision parameters and hence they may not be expressed in a precise way. This is due to measurement inaccuracy, lack of evidence, computational errors, high information cost, whether conditions etc. Hence we cannot apply the traditional classical methods to solve the transportation problems successfully. Therefore the use of Fuzzy transportation problems is more appropriate to model and solve the real world problems. A fuzzy transportation problem is a transportation problem in which the transportation costs, supply and demand are fuzzy quantities.

Bellman and Zadeh [3] proposed the concept of decision making in Fuzzy environment. After this pioneering work, several authors such as Shiang-Tai Liu and Chiang Kao[23], Chanas et al [4], Pandian et.al [19], Liu and Kao [17] etc proposed different methods for the solution of Fuzzy transportation problems. Chanas and Kuchta [4] proposed the concept of the optimal solution for the Transportation with Fuzzy coefficient expressed as Fuzzy numbers. Chanas, Kolodziejckzy, Machaj[5] presented a Fuzzy linear programming model for solving Transportation problem. Liu and Kao [17] described a method to solve a Fuzzy Transportation problem based on extension principle. Lin introduced a genetic algorithm to solve Transportation with Fuzzy objective functions. Nagoor Gani and Abdul Razak [12] obtained a fuzzy solution for a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers. A. Nagoor Gani, Edward Samuel and Anuradha [9] used Arshamkhan's Algorithm to solve a Fuzzy Transportation problem. Pandian and Natarajan [19] proposed a Fuzzy zero point method for finding a Fuzzy optimal solution for Fuzzy transportation problem where all parameters are trapezoidal fuzzy numbers.

In general, most of the existing techniques provide only crisp solutions for the fuzzy transportation problem. In this paper we propose a simple method, for the solution of fuzzy transportation problems without converting them in to classical transportation problems. The rest of this paper is organized as:

In section II, we recall the basic concepts of Fuzzy numbers and related results. In section III, we define Fuzzy transportation problem and prove the related theorems. In section IV, we propose Fuzzy Version of Vogels Approximation Algorithm (FVAM) and fuzzy version of MODI method (FMODI). In section V, numerical examples are provided to illustrate the methods proposed in this paper for the fuzzy optimal solution of fuzzy transportation problems without converting them to classical transportation problems and the results obtained are discussed.

II. PRELIMINARIES

The aim of this section is to present some notations, notions and results which are of useful in our further consideration.

2.1 Fuzzy numbers

A fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be a fuzzy number if its membership function $\mu_{\tilde{A}} : R \to [0,1]$ has the following characteristics

(i) $\tilde{A}$ is normal. It means that there exists an $x \in R$ such that $\mu_{\tilde{A}}(x) = 1$

(ii) $\tilde{A}$ is convex. It means that for every $x_1, x_2 \in R$, $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}, \ \lambda \in [0,1]$

(iii) $\mu_{\tilde{A}}$ is upper semi-continuous.

(iv) supp ($\tilde{A}$) is bounded in R.

2.2. Triangular fuzzy numbers

A fuzzy number $\tilde{A}$ in R is said to be a triangular fuzzy number if its membership function $\mu_{\tilde{A}} : R \to [0,1]$ has the following characteristics.

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ 1 & x = a_2 \\ \dfrac{a_3-x}{a_3-a_2} & a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

We denote this triangular fuzzy number by $\tilde{A} = (a_1, a_2, a_3)$. We use F(R) to denote the set of all triangular fuzzy numbers. Also if $m = a_2$ represents the modal value or midpoint, $\alpha = a_2 - a_1$ represents the left spread and $\beta = a_3 - a_2$ represents the right spread of the triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$, then the triangular fuzzy number $\tilde{A}$ can be represented by the triplet $\tilde{A} = (m, \alpha, \beta)$. That is $\tilde{A} = (a_1, a_2, a_3) = (m, \alpha, \beta)$.

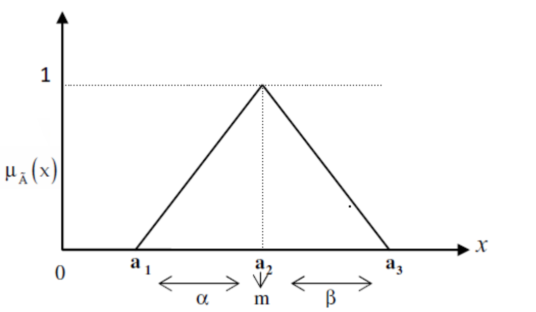

Fig 1. Triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3) = (m, \alpha, \beta)$

2.3. Ranking of triangular fuzzy numbers

Several approaches for the ranking of fuzzy numbers have been proposed in the literature. An efficient approach for comparing the fuzzy numbers is by the use of a ranking function based on their graded means. That is, for every $\tilde{A} = (a_1, a_2, a_3) \in F(R)$, the ranking function $\Re : F(R) \to R$ by graded mean is defined as

$$\Re(\tilde{A}) = \left(\frac{a_1 + 4a_2 + a_3}{6}\right)$$

For any two fuzzy triangular numbers $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ in F(R), we have the following comparison

(i). $\tilde{A} \prec \tilde{B}$ if and only if $\Re(\tilde{A}) < \Re(\tilde{B})$.

(ii). $\tilde{A} \succ \tilde{B}$ if and only if $\Re(\tilde{A}) > \Re(\tilde{B})$.

(iii). $\tilde{A} \approx \tilde{B}$ if and only if $\Re(\tilde{A}) = \Re(\tilde{B})$.

(iv). $\tilde{A} - \tilde{B} \approx \tilde{0}$ if and only if $\Re(\tilde{A}) - \Re(\tilde{B}) = 0$.

A triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ in $F(R)$ is said to be positive if $\Re(\tilde{A}) > 0$ and denoted by $\tilde{A} \succ \tilde{0}$. Also if $\Re(\tilde{A}) > 0$, then $\tilde{A} \succ \tilde{0}$ and if $\Re(\tilde{A}) = 0$, then $\tilde{A} \approx \tilde{0}$. If $\Re(\tilde{A}) = \Re(\tilde{B})$, then the triangular numbers $\tilde{A}$ and $\tilde{B}$ are said to be equivalent and is denoted by $\tilde{A} \approx \tilde{B}$.

2.4. Arithmetic operations on triangular fuzzy numbers

Ming Ma et al. [9] have proposed a new fuzzy arithmetic based upon both location index and fuzziness index functions. The location index number is taken in the ordinary arithmetic, whereas the fuzziness index functions are considered to follow the lattice rule which is least upper bound in the lattice L. That is for a, b∈L we define $a \vee b = \max\{a, b\}$ and $a \wedge b = \min\{a, b\}$.

For arbitrary triangular fuzzy numbers $\tilde{A} = (a_1, a_2, a_3) = (m_1, \alpha_1, \beta_1)$, $\tilde{B} = (b_1, b_2, b_3) = (m_2, \alpha_2, \beta_2)$ and $* = \{+, -, \times, \div\}$, the arithmetic operations on the triangular fuzzy numbers are defined by

$$\begin{aligned} \tilde{A} * \tilde{B} &= (m_1, \alpha_1, \beta_1) * (m_2, \alpha_2, \beta_2) \\ &= (m_1 * m_2,\ \max\{\alpha_1, \alpha_2\},\ \max\{\beta_1, \beta_2\} \\ &= (m_1 * m_2,\ \alpha_1 \vee \alpha_2,\ \beta_1 \vee \beta_2) \end{aligned}$$

In particular for any two triangular fuzzy numbers $\tilde{A}=(a_1,a_2,a_3)=(m_1,\alpha_1,\beta_1)$, $\tilde{B}=(b_1,b_2,b_3)=(m_2,\alpha_2,\beta_2)$, we define:

(i). Addition

$$\begin{aligned}\tilde{A}+\tilde{B}&=(a_1,a_2,a_3)+(b_1,b_2,b_3)\\&=(m_1,\alpha_1,\beta_1)+(m_2,\alpha_2,\beta_2)\\&=(m_1+m_2,\max(\alpha_1,\alpha_2),\max(\beta_1,\beta_2))\end{aligned}$$

(ii). Subtraction

$$\begin{aligned}\tilde{A}-\tilde{B}&=(a_1,a_2,a_3)-(b_1,b_2,b_3)\\&=(m_1,\alpha_1,\beta_1)-(m_2,\alpha_2,\beta_2)\\&=(m_1-m_2,\max(\alpha_1,\alpha_2),\max(\beta_1,\beta_2))\end{aligned}$$

(iii).Multiplication

$$\begin{aligned}\tilde{A}\tilde{B}&=(a_1,a_2,a_3)(b_1,b_2,b_3)\\&=(m_1,\alpha_1,\beta_1)(m_2,\alpha_2,\beta_2)\\&=(m_1m_2,\max(\alpha_1,\alpha_2),\max(\beta_1,\beta_2))\end{aligned}$$

(iv). Division

$$\frac{\tilde{A}}{\tilde{B}}=\frac{(a_1,a_2,a_3)}{(b_1,b_2,b_3)}=\left(\frac{m_1}{m_2},\max(\alpha_2,\alpha_1),\max(\beta_1,\beta_2)\right),$$

if, $\tilde{B}=(b_1,b_2,b_3)$ is non zero fuzzy number.

III. MAIN RESULTS

Consider a fuzzy transportation with m sources and n destinations with triangular fuzzy numbers. Let $\tilde{a}_i \succeq \tilde{0}$ be the fuzzy availability at source i and $\tilde{b}_j$, $(\tilde{b}_j \succeq \tilde{0})$ be the fuzzy requirement at destinations j. Let $\tilde{c}_{ij}$ $(\tilde{c}_{ij} \succeq \tilde{0})$ be the unit fuzzy transportation cost from source i to destination j. Let $\tilde{x}_{ij}$ denote the number of fuzzy units to be transformed from source i to destination j. Now the problem is to find a feasible way of transporting the available amount at each source to satisfy the demand at each destination so that the total transportation cost is minimized.

3.1. Mathematical formulation of fuzzy transportation problem

The mathematical model of fuzzy transportation problem is as follows

$$\text{Minimize } \tilde{Z}\approx\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}\tilde{x}_{ij}$$

$$\begin{aligned}\text{subject to}\quad &\sum_{j=1}^{n}\tilde{x}_{ij}\approx\tilde{a}_i,\ i=1,2,3,\dots,m\\&\sum_{i=1}^{m}\tilde{x}_{ij}\approx\tilde{b}_j,\ j=1,2,3,\dots,n\qquad(3.1)\\&\sum_{i=1}^{m}\tilde{a}_i\approx\sum_{j=1}^{n}\tilde{b}_j,\ i=1,2,3,\dots,m;\\&j=1,2,3,\dots,n \text{ and } \tilde{x}_{ij}\succeq\tilde{0} \text{ for all i and j.}\end{aligned}$$

where $\tilde{c}_{ij}$ is the fuzzy unit transportation cost from i^{th} source to the j^{th} destination. The objective is to minimize the total fuzzy transportation cost, in this paper the fuzzy transportation problem is solved by fuzzy version of Vogel's and MODI method. This fuzzy transportation problem is explicitly represented by the following fuzzy transportation table.

Table 3.1 Fuzzy transportation table

	Destination					
Sources		1	2	…	n	Supply
	1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	…	$\tilde{c}_{1n}$	$\tilde{a}_1$
	2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	….	$\tilde{c}_{2n}$	$\tilde{a}_2$
	⋮	⋮	⋮	⋮	⋮	⋮
	m	$\tilde{c}_{m1}$	$\tilde{c}_{m2}$	….	$\tilde{c}_{mn}$	$\tilde{a}_n$
	Demand	$\tilde{b}_1$	$\tilde{b}_2$		$\tilde{b}_m$	

3.2. Basic Theorems

Theorem 3.1. (Existence of a fuzzy feasible solution)

The necessary and sufficient condition for the existence of a fuzzy feasible solution to the fuzzy transportation problem (3.1) is,

$$\sum_{i=1}^{m}\tilde{a}_i\approx\sum_{j=1}^{n}\tilde{b}_j \qquad (3.2)$$

(Total supply ≈ Ttotal demand)..

Proof: (Necessary condition)

Let there exist a fuzzy feasible solution to the fuzzy transportation problem

$$\text{Minimize } \tilde{Z}\approx\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}\tilde{x}_{ij}$$

$$\begin{aligned}\text{subject to}\quad &\sum_{j=1}^{n}\tilde{x}_{ij}\approx\tilde{a}_i, i=1,2,3,\dots,m.\\&\sum_{i=1}^{m}\tilde{x}_{ij}\approx\tilde{b}_j, j=1,2,3,\dots,n.\end{aligned}\qquad(3.3)$$

and $\tilde{x}_{ij}\succeq\tilde{0}$ for all i and j.

From $\sum_{j=1}^{n}\tilde{x}_{ij}\approx\tilde{a}_i,(i=1,2,3,\dots,m)$, we have

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{x}_{ij}\approx\sum_{i=1}^{m}\tilde{a}_i \qquad (3.4)$$

Also from $\sum_{i=1}^{m}\tilde{x}_{ij}\approx\tilde{b}_j$, $(j=1,2,3,\dots,n)$, we have

$$\sum_{j=1}^{n}\sum_{i=1}^{m}\tilde{x}_{ij}\approx\sum_{j=1}^{n}\tilde{b}_j \qquad (3.5)$$

From equations (3.4) and (3.5), we have

$$\sum_{i=1}^{m}\tilde{a}_i\approx\sum_{j=1}^{n}\tilde{b}_j$$

(Sufficient condition)

Since all $\tilde{a}_i$ and $\tilde{b}_j$ are positive, $\tilde{x}_{ij}$ must be all positive. Therefore equation (3.2) yields a feasible solution.

Theorem 3.2.

The dimension of the basis of a fuzzy transportation are (m+n-1)(m+n-1).That a fuzzy transportation problem has only (m+n-1) independent structural constraints and its basic feasible solution has only (m+n-1) positive components.

Proof: Consider a fuzzy transportation problem with m sources and n destinations,

$$\text{Minimize } \tilde{Z} \approx \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij}$$

$$\text{subject to } \sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_i,\ i = 1,2,3,\ldots,m.$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_j,\ j = 1,2,3,\ldots,n.$$

$$\text{and } \tilde{x}_{ij} \succeq \tilde{0} \text{ for all i and j.}$$

Let us assume that a fuzzy transportation has m rows (supply constraint equations) and n columns (demand constraint equations). Therefore there are totally (m+n) constraint equations.

This is due to the condition that $\sum_{i=1}^{m} \tilde{a}_i \approx \sum_{j=1}^{n} \tilde{b}_j$ which is the last requirement constraint. Therefore one of (m+n) constraints can always be derived from the remaining (m+n-1). Thus there exists only (m+n-1) independent constraints and its basic feasible solution has only (m+n-1) positive components.

Theorem 3.3

The values of the fuzzy basic feasible solution are all differences between the partial sum of $\tilde{a}_i$ and the partial sum of $\tilde{b}_j$.

That is $\tilde{x}_{ij} = \pm\sum_{i=1}^{m} \tilde{r}_i\tilde{a}_i \pm \sum_{j=1}^{n} \tilde{s}_j\tilde{b}_j$, where $\tilde{r}_i, \tilde{s}_j$ are either $\tilde{1} = (1,1,1)$ or $\tilde{0} = (0,0,0)$.

Theorem 3.4

The fuzzy transportation problem has a triangular basis.

Proof: We note that every equation has a basic variable, otherwise, the equation cannot be satisfied for $\tilde{a}_i \not\approx 0$, $\tilde{b}_j \not\approx 0$. Suppose every row and column equations has atleast two basic variables, since there are m rows and n columns, the total number of basic variables in row equations and column equations will be atleast 2m and 2n respectively. Suppose if the total number of basic variables is D, then obviously $D \geq 2m$, $D \geq 2n$.

Case (i). If $m < n$, then $m + n < n + n \Rightarrow m + n < 2n$
$\Rightarrow m + n < 2n < D \Rightarrow m + n < D$.

Case (ii). If $m > n$, then $m + n > m + n$
$\Rightarrow m + m > m + n \Rightarrow 2m > m + n$
$\Rightarrow m + n < 2m \Rightarrow m + n < 2m \leq D$
$\Rightarrow m + n < D$.

Case (iii). If $m = n$, then $m + n = m + n$
$\Rightarrow m + m = m + n \Rightarrow 2m = m + n$
$\Rightarrow m + n = 2m \Rightarrow m + n = 2m < D$
$\Rightarrow m + n < D$.

Thus in all cases $D \geq m + n$. But the number of basic variables in fuzzy transportation problem is (m+n-1) which is a contradiction.

Therefore there is atleast one equation, either row or column having only one basic variable. Let the r^{th} equation have only one basic variable. Let $\tilde{x}_{rs}$ be the only basic variable in the r^{th} row and s^{th} column ,then $\tilde{x}_{rs} \approx \tilde{a}_r$. Eliminate r^{th} row from the system of equation and substitute $\tilde{x}_{rs} \approx \tilde{a}_r$ in s^{th} column equation and replace $\tilde{b}_s$ by $\tilde{b}'_s \approx \tilde{b}_s - \tilde{a}_r$. After eliminating the r^{th} row , the system has (m+n-2) linearly independent constraints. Hence the number of basic variables is (m+n-2).

Repeat the process and conclude that in the reduced system of equations there is an equation which has only one basic variable. But if this is in s^{th} column equation, then it will have two basic variables.

This concludes that in our original system of equations, there is an equation which has atleast two basic variables. Continue the process to get the theorem.

IV. PROPOSED ALGORITHMS

4.1 Fuzzy Version of Vogels Approximation Algorithm (FVAM)

Step 1:

From the Fuzzy Transportation table, determine the penalty for each row or column. The penalties are calculated for each row column by substituting the lowest cost element in that row or column from the next cost element in the same row or column. Write down the penalties below and aside of the rows and columns respectively of the table.

Step 2:

Identify the column or row with largest fuzzy penalty. In case of tie, break the tie arbitrarily. Select a cell with minimum fuzzy cost in the selected column (or row, as the case may be) and assign the maximum units possible by considering the demand and supply position corresponding to the selected cell.

Step 3:
Delete the column/row for which the supply and demand requirements are met.

Step 4:
Continue steps 1 to 3 for the resulting fuzzy transportation table until the supply and demand of all sources and destinations have been met.

4.2 Fuzzy Version of MODI Algorithm (FMODI)

Step 1:
Given an initial fuzzy basic feasible solution of a fuzzy transportation problem in the form of allocated and unallocated cells of fuzzy transportation table. Assign the auxiliary variables $\tilde{u}_i$, i=1,2,3,…,m and $\tilde{v}_j$, j=1,2,3,…,n for rows and columns respectively. Compute the values of $\tilde{u}_i$ and $\tilde{v}_j$ using the relationship $\tilde{c}_{ij} = \tilde{u}_i + \tilde{v}_j$ for all i, j for all occupied cells. Assume either $\tilde{u}_i$ or $\tilde{v}_j$ as zero arbitrarily for the allocations in row/column.

Step 2:
For each unoccupied cell (i, j), compute the fuzzy opportunity cost $\tilde{\delta}_{ij}$ using $\tilde{\delta}_{ij} \approx \tilde{c}_{ij} - (\tilde{u}_i + \tilde{v}_j)$.

Step 3:
(i) If all $\tilde{\delta}_{ij} \succeq \tilde{0}$, then the current solution under the test is optimal
(ii) If at least one $\tilde{\delta}_{ij} \prec \tilde{0}$, then the current solution under the test is not optimal and proceeds to the next step.

Step 4:
Select an unoccupied cell (i, j) with most negative opportunity cost among all unoccupied cells.

Step 5:
Draw a closed path involving horizontal and vertical lines for the unoccupied cells starting and ending at the cell obtained in step 4 and having its other corners at some allocated cells. Assign +θ and – θ alternately starting with +θ for the selected unoccupied cells.

Step 6:
On the closed path, identify the corners with – θ. Select the smallest allocation among the corners with – θ which indicate the number of units that can be shifted to some other unoccupied cells. Add this quantity to those corners marked with +θ and subtract this quantity to those corners marked with – θ on the closed path and cheek whether the number of nonnegative allocations is (m+n-1) and repeat step 1 to step 7, till we reach $\tilde{\delta}_{ij} \succeq \tilde{0}$ for all i and j.

4.3 Unbalanced Fuzzy Transportation Problem.

Suppose the fuzzy transportation problem is unbalanced one, i. e , if $\left(\sum_{i=1}^{m} \tilde{a}_i \not\approx \sum_{j=1}^{n} \tilde{b}_j\right)$, then convert this into a balanced one by introducing a dummy source or dummy destination with zero fuzzy unit transportation costs. Solve the resulting balanced fuzzy transportation problem using above said algorithms.

V. NUMERICAL EXAMPLES

The following two numerical examples are taken from the paper "Simplex type algorithm for solving fuzzy transportation problem by Edward Samuel et.al [9].

Example 5.1

A company has two factories O_1, O_2 and two retail stores D_1, D_2. The production quantities per month at O_1, O_2 are (150, 201, 246) and (50, 99, 154) tons respectively. The demands per month for D_1 and D_2 are (100,150,200) and (100,150,200) tons respectively. The transportation cost per ton $\tilde{c}_{ij}$, i =1, 2; j=1, 2 are $\tilde{c}_{11}$=(15,19,29), $\tilde{c}_{12}$=(22,31,34), $\tilde{c}_{21}$=(8,10,12) and $\tilde{c}_{22}$=(30,39,54).

Solution: Transportation table of the given fuzzy transportation problem is

Table 5.1 Fuzzy Transportation Problem

	D_1	D_2	Supply
O_1	(15,19,29)	(22,31,34)	(150,201,246)
O_2	(8,10,12)	(30,39,54)	(50,99,154)
Demand	(100,150,200)	(100,150,200)	(200,300,400)

To apply the proposed algorithms and the fuzzy arithmetic, let us express all the triangular fuzzy numbers based upon both location index and fuzziness index functions. That is in the form of (m, α, β) given in table 5.2.

Table 5.2 Balanced fuzzy transportation problem in which all the triangular numbers are of the form (m,α,β)

	D_1	D_2	Supply
O_1	(19, 4-4r, 10-10r)	(31, 9-9r, 3-3r)	(201, 51-51r, 45-45r)
O_2	(10, 2-2r, 2-2r)	(39, 9-9r, 15-15r)	(99, 49-49r, 55-55r)
Demand	(150, 50-50r, 50-50r)	(150, 50-50r, 50-50r)	

By applying fuzzy version of Vogel's Approximation method (FVAM)), the initial fuzzy basic feasible solution is given in table 5.3.

Table 5.3 Initial fuzzy basic feasible solution

	D_1	D_2
O_1	(19, 4-4r, 10-10r) (51, 50-50r, 55-55r)	(31, 9-9r, 3-3r) (150, 50-50r, 50-50r)
O_2	(10, 2-2r, 2-2r) (99, 49-49r, 55-55r)	(39, 9-9r, 15-15r)

The corresponding initial fuzzy transportation cost is given by

$$\begin{aligned}\text{IFTC} &\approx (19,\ 4-4r,\ 10-10r)(51,\ 50-50r,\ 55-55r)\\ &+(31,\ 9-9r,\ 3-3r)(150,\ 50-50r,\ 50-50r)\\ &+(10,\ 2-2r,\ 2-2r)(99,\ 49-49r,\ 55-55r)\\ &\approx(6609, 50-50r, 55-55r).\end{aligned}$$

By applying fuzzy version of MODI method (FMODI), it can be seen that the current initial fuzzy basic feasible solution is optimal.

Hence the fuzzy optimal solution in terms of location index and fuzziness index is given by

$$\tilde{x}_{11}=(51, 50-50r, 55-55r);$$
$$\tilde{x}_{12}=(150, 50-50r, 50-50r);$$
$$\tilde{x}_{21}=(99, 49-49r, 55-55r),\ \text{where } 0 \le r \le 1.$$

The corresponding fuzzy optimal transportation cost is given by

$$\text{Minimize } \tilde{Z} \approx \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}\tilde{x}_{ij} \approx (6609, 50-50r, 55-55r),$$

where $0 \le r \le 1$ can be suitably chosen by the decision maker.

Case(i). When $r=0$, the fuzzy optimal transportation cost in terms of the form (m, α, β) is (6609, 50, 55). The corresponding fuzzy optimal transportation of the form (a_1,a_2,a_3) is (6559, 6609, 6664) and its defuzzyfied transportation cost is 609.8.

Case(ii). When $r = 0.5$, the fuzzy optimal transportation cost in terms of the form (m, α, β) is (6609, 25, 27.5). The corresponding fuzzy optimal transportation cost of the form (a_1,a_2,a_3) is (6584, 6609, 6636.5) and its defuzzyfied transportation cost is 6609.4

Case(iii). When $r=1$, the fuzzy optimal transportation cost in terms of the form (m, α, β) is (6609,0,0). The corresponding fuzzy optimal transportation cost of the form (a_1,a_2,a_3) is (0, 6609, 0) and its defuzzyfied transportation cost is 4406. Optimum fuzzy transportation cost is (1150, 6609, 12882). Defuzzified fuzzy transportation cost is 6745.

Example 5.2

Find a fuzzy optimum solution for the unbalanced fuzzy transportation problem given below.

Table 5.4. Unbalanced Fuzzy Transportation Problem

	B1	B2	Supply
A1	(1,3,5)	(3,5,13)	(300,399,504)
A2	(2,3,10)	(3,4,11)	(250,301,346)
A3	(5,6,13)	(1,3,5)	(300,399,504)
Demand	(400,448,508)	(300,351,396)	

Solution: Introduce a dummy destination B3 with (150,300,450) as its fuzzy demand and (0,0,0) as its fuzzy transportation costs per unit. Hence the balanced fuzzy transportation with a dummy destination becomes

Table 5.5 Balanced Fuzzy Transportation Problem

	B1	B2	B3	Supply
A1	(1,3,5)	(3,5,13)	(0,0,0)	(300,399,504)
A2	(2,3,10)	(3,4,11)	(0,0,0)	(250,301,346)
A3	(5,6,13)	(1,3,5)	(0,0,0)	(300,399,504)
Demand	(400,448,508)	(300,351,396)	(150,300,450)	

Now the balanced fuzzy transportation problem in which all the triangular numbers are of the form (m,α,β) is given in table 5.6.

Table 5.6 Balanced Fuzzy Transportation Problem in which all the triangular numbers are of the form (m,α,β)

	B_1	B_2	B_3	Supply
A_1	(3,2-2r,2-2r)	(5,2-2r,8-8r)	(0,0,0)	(399,99-99r,105-105r)
A_2	(3,1-r,7-7r)	(4,1-r,7-7r)	(0,0,0)	(301,51-51r,45-45r)
A_3	(6,1-r,7-7r)	(3,2-2r,2-2r)	(0,0,0)	(399,99-99r,105-105r)
Demand	(448,48-48r,60-60r)	(351,51-51r,45-45r)	(300,150-150r,150-150r)	

By applying fuzzy version of Vogels Approximation method, the initial fuzzy basic feasible solution is given by

Table 5.7 Initial fuzzy basic feasible solution

	B1	B2	B3
A1	(3,2-2r,2-2r) (399,99-99r,105-105r)	(5,2-2r,8-8r)	(0,0,0)
A2	(3,1-r,7-7r) (1,150-150r,150-50r)	(4,1-r,7-7r)	(0,0,0)
A3	(6,1-r,7-7r) (48,99-99r,105-105r)	(3,2-2r,2-2r) (351,51-51r,45-45r)	(0,0,0)

By applying fuzzy version of MODI method, it can be seen that the current initial fuzzy basic feasible solution is optimal. The fuzzy optimal solution in terms of location index and fuzziness index is given by

$\tilde{x}_{11} = (399,\ 99-99r,\ 105-105r);$

$\tilde{x}_{21} = (49, 150-150r,\ 150-150r);$

$\tilde{x}_{23} = (252, 150-150r,\ 150-150r);$

$\tilde{x}_{32} = (351, 51-51r, 45-45r)$ and

$\tilde{x}_{33} = (48, 99-99r, 105-105r)$, where $0 \le r \le 1$.

The corresponding fuzzy optimal transportation cost is given by

$$\text{Minimize } \tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij}$$

$$\approx (2357, 150-150r, 150-150r)$$

where *0 ≤ r ≤ 1* can be suitably chosen by the decision maker.

Case(i). When *r = 0*, the fuzzy optimal transportation cost in terms of the form (m, α, β) is (2397, 150,150). The corresponding fuzzy optimal transportation cost of the form $\tilde{A} = (a_1, a_2, a_3)$ is (2217, 2397, 2547). Its defuzzyfied transportation cost is 2392.

Case(ii). When *r = 0.5*, the fuzzy optimal transportation cost in terms of the form (m, α, β) is (2397, 75, 75). The corresponding fuzzy optimal transportation cost of the form $\tilde{A} = (a_1, a_2, a_3)$ is (2322, 2397, 2472). Its defuzzyfied transportation cost is 2397.

Case(iii). When r *=1*, the fuzzy optimal transportation cost in terms of the form (m, α, β) is (2397, 0, 0). The corresponding fuzzy optimal transportation cost of the form $\tilde{A} = (a_1, a_2, a_3)$ is (0, 2397, 0). The optimum fuzzy transportation cost = (-548, 2397, 7120). Defuzzied fuzzy transportation cost is 2693.

VI. CONCLUSION

In this paper, the transportation costs are considered as imprecise numbers described by triangular fuzzy numbers which are more realistic and general in nature. We proposed a fuzzy version of VAM and MODI algorithms to solve fuzzy transportation problem without converting them to classical transportation problems. Two numerical examples are solved using the proposed algorithms and obtained results are better than the existing results.

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