Kinematic Synthesis of Variable Crank-rocker and Drag linkage planar type Five-Bar Mechanisms with Transmission Angle Control

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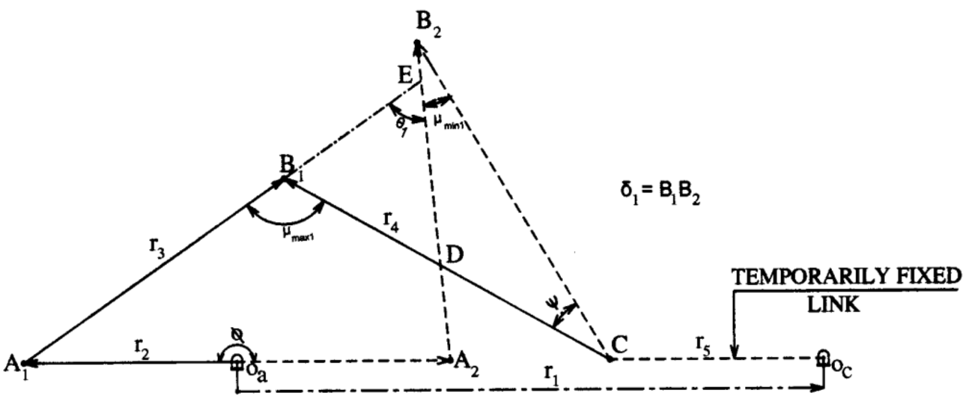
Abstract

Analytical method to synthesize a variable crank-rocker and drag-linkage planar type planar five-bar mechanism with transmission angle control is designed. The method is useful to reduce the solution space and thus the number of trials and the time required for synthesis. In this paper the synthesis of five-bar mechanism motion, for two separated positions are considered. The portion of the five-bar linkage in Phase-I and in Phase-II is assumed to be a crank rocker type four-bar mechanism and the portion of the five-bar linkage in Phase-III and in Phase-IV is assumed to be a Drag-Linkage type four-bar mechanism.

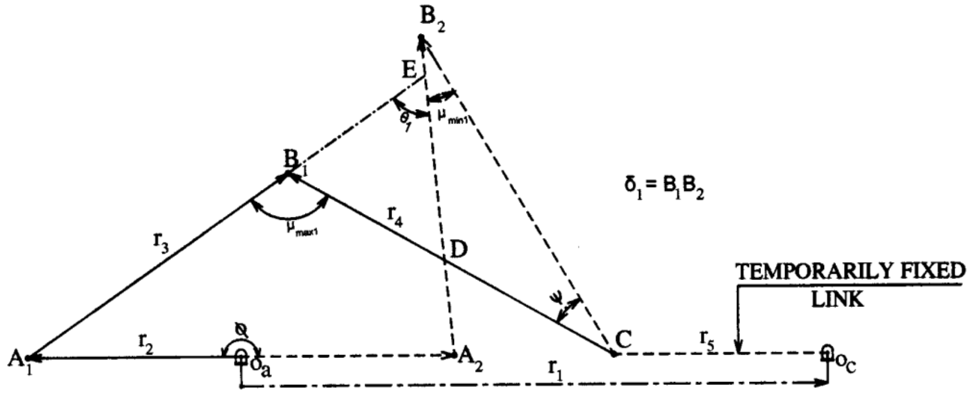
Introduction

A planar five-bar mechanism of variable crank-rocker and drag-linkage type mechanisms are operating in two-two phases. In each phase a link adjacent to the permanently fixed link of a five-bar linkage is fixed temporarily and the resulting linkage acts like a crank-rocker type four-bar mechanism. There are many factors to be considered for the effective motion transmission by a mechanism. The transmission angle control is one of the important criteria. This criterion is used to reduce the solution space with no iterations and thus the time required for kinematic synthesis is also reduced for the design of planar five-bar mechanism with variable crank-rocker and drag linkage type mechanisms. The problem is to develop an analytical procedure to determine the link lengths of a five-bar mechanism with variable crank rocker and drag linkage types.

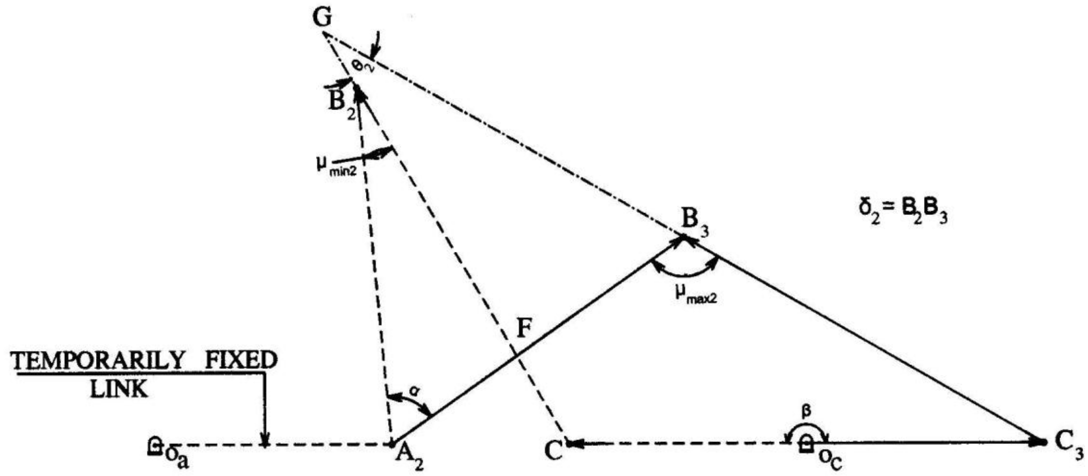
The objective is to simplify the synthesis procedure by reducing two degree of freedom five-bar mechanism into single degree of freedom four-bar mechanism in two-two phases. The mechanism may be designed for one task in I- phase and for different task in II- phase. By temporarily fixing one of the two input crank type links of a five-bar mechanism, then the five-bar linkage reduces to a four-bar linkage. Thus the problem of synthesizing a five-bar mechanism becomes a four-bar linkage synthesis. Similarly the mechanism may be designed for one task in III- phase and for different task in IV- phase. By temporarily fixing one of the two input crank type links of a five-bar mechanism, then the five-bar linkage reduces to a four-bar linkage. Thus the problem of synthesizing a five-bar mechanism becomes a four-bar linkage synthesis.



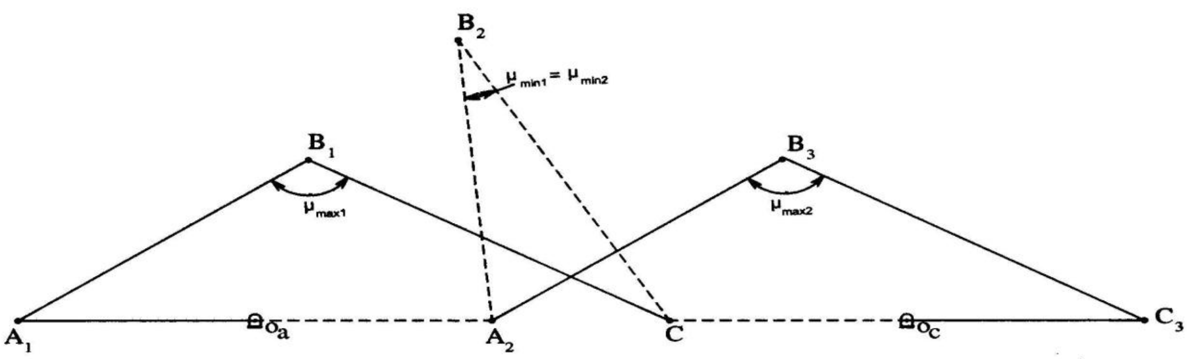
Phase - I : Five -bar mechanism with variable Crank - rocker mechanism position 1 : $o_a A_1 B_1 C$; Position 2 : $0_a A_2 B_2 C$



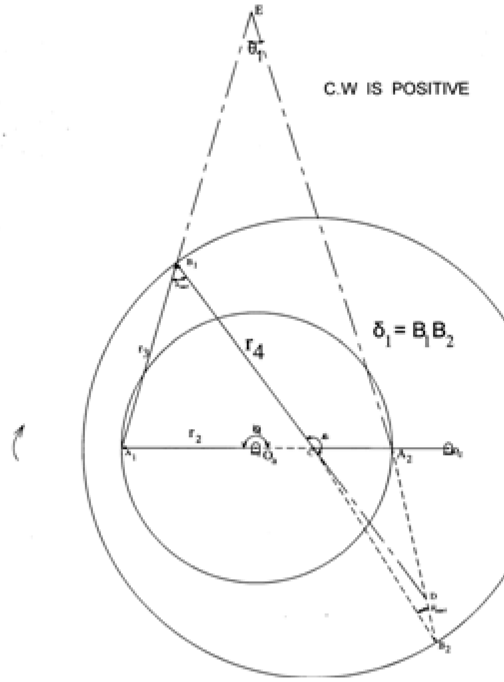
Phase - I : Five -bar mechanism with variable Crank - rocker mechanism
position 1 : $o_a A_1 B_1 C$; Position 2 : $0_a A_2B_2C$



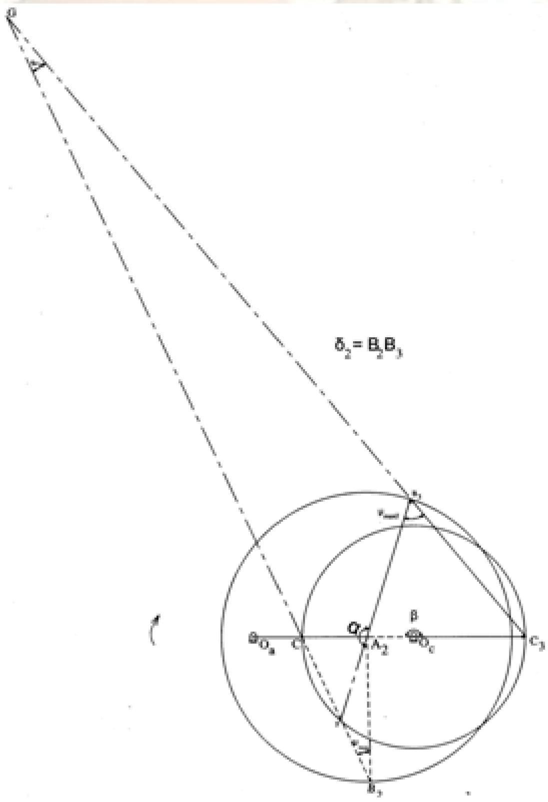
Phase - II :Five -bar mechanism with variable Crank - rocker mechanism
position 1 : $A_2 B_2 C o_c$; Position 2 : $A_2 B_3 C_3 o_c$



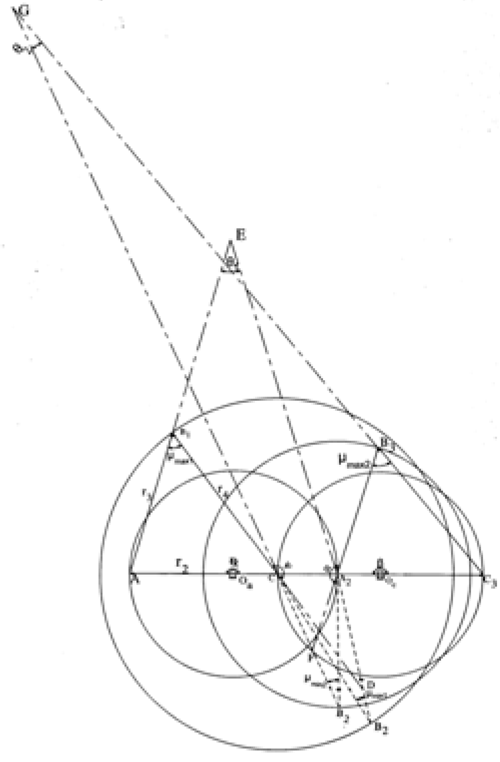
Phase - I and Phase - II combined
(Porsion A_2B_2C is common and $\mu_{min1} = \mu_{min2}$)



Phase III: Five - Bar Mechanism with variable Drag-Linkage Mechanism
Porsion 1: o_a A_1 B_1 C; Position 2: b_a A_2 B_2 C
O_c C temporarily fixed link



Phase IV: Five - Bar Mechanism with variable Drag-Linkage Mechanism
Porsion 1: A_2 B_2 C o_c; Position 2: b_a A_2 B_3 C_3 o_c
O_a A_2 temporarily fixed link



Phase - III and Phase – IV combined
Porsion A_2B_2C is common and $\mu_{min1} = \mu_{min2}$

Variable crank-rocker and drag-linkage mechanisms :

A planar five-bar mechanism o_aABCo_c is shown in phase-I to phase-IV. In phase-I and phase-III , the link Co_c is temporarily fixed, C is temporarily fixed pivot. So the link becomes a four-bar mechanism $o_aA_1B_1C$. Now o_aA_1 is the input link, B_1C is the output link and A_1B_1 is the coupler. By definition $\angle A_1B_1C$ is maximum transmission angle (μ_{max1}) and $\angle A_2B_2C$ is minimum transmission angle(μ_{min1}). $o_aA_1B_1C$ and $o_aA_2B_2C$ are the two positions of phase-I and two positions of phase-III. Consider the angular motion of the coupler link AB between the two positions A_1B_1 and A_2B_2 is θ_1. Once the mechanism $o_aA_1B_1C$ has reached the prescribed position $o_aA_2B_2C$, the link Co_c is released to move and the link o_aA_2 is fixed temporarily, now A_2 is temporarily fixed pivot. Thus switching on to the phase-II and phase-IV.

In phase-II and phase-IV, $A_2B_2Co_c$ is again a four bar mechanism. Link Co_c is input link, link A_2B_2 is output link and B_2C is the coupler. By definition $\angle A_2B_2C$ is the minimum transmission angle (μ_{min2}) and $\angle A_2B_3C_3$ is the maximum transmission angle (μ_{max2}). $A_2B_2Co_c$ and $A_2B_3C_3o_c$ are the two positions of phase-II and phase-IV. The angular motion of the coupler link BC between the two positions B_2C and B_2C_3 is θ_2 . o_a and o_c are the permanently fixed pivots of a five-bar mechanism. The portion A_2B_2C of linkage is common to position:2 of phase-I and phase-III, and common to position:1 of phase-II and phase-IV.

The two-two positions between which the motion is considered are the positions when the driving link angle ϕ in phase-I and phase-III and β in phase-II and phase-IV is equal to 0^0 and 180^0 measured from the reference axis. At these positions the transmission angle (μ) is either maximum or minimum. Hence, it is assumed that the mechanism operates between two-two positions where the maximum and minimum values of transmission angle occur. In the present paper, the condition such as equal deviation of transmission angle at two design positions. i.e. The range of transmission angle ($\Delta\mu$) for phase-I and phase-III must be equal to $\Delta\mu$ for phase-II and phase-IV,that means $\mu_{min1} = \mu_{min2}$ and $\mu_{max1} = \mu_{max2}$.

Kinematic Synthesis of variable crank-rocker mechanism:

To synthesis a planar five-bar mechanism with variable crank-rocker type mechanism is shown in phase-I and phase-II. The linkage operates in two phases.

- Now writing the dyad equation for phase-I synthesis (Loope closer equation)

$\mathbf{r_2} + \mathbf{r_3} + \boldsymbol{\delta_1} - \mathbf{r_3}\, e^{-i\theta 1} - \mathbf{r_2}\, e^{i\phi} = 0$ --------- (1) (consider c.w is +ve and $\mathbf{r_2}$ must be c.w motion)

$\mathbf{r_2}\,(e^{i\phi} - 1) + \mathbf{r_3}\,(e^{-i\theta 1} - 1) = \boldsymbol{\delta_1}$

But we know that $\mathbf{r_4} + B_1B_2 = \mathbf{r_4}\, e^{i\psi} \Rightarrow \boldsymbol{\delta_1} = \mathbf{r_4}\,(e^{i\psi} - 1)$

Therefore Loope closer equation for phase-I is

$\mathbf{r_2}\,(e^{i\phi} - 1) + \mathbf{r_3}\,(e^{-i\theta 1} - 1) = \mathbf{r_4}\,(e^{i\psi} - 1)$ --------- (2)

where θ_1 is the angle between A_1B_1 and A_2B_2 in phase-I.

$\theta_1 = \angle B_1ED$
$= 180^0 - \angle EB_1D - \angle EDB_1$
$= 180^0 - (180^0 - \mu_{max}) - (180^0 - \angle EDC)$ [$\because \mu_{max1} = \mu_{max2}$]
$= \mu_{max} - 180^0 + \angle EDC$
$= \mu_{max} - 180^0 + (180^0 - \psi - \mu_{min})$ [$\because \mu_{min1} = \mu_{min2}$]
$= \mu_{max} - \mu_{min} - \psi$
$= \Delta\mu - \psi$ ----------- (3)

where $\Delta\mu$ is the range of transmission angle.

Referring to phase-I, $\phi = 180^0$

$\therefore$ Dyad equation for phase-I is

$-2\,\mathbf{r_2} + \mathbf{r_3}\,(e^{i(\psi-\Delta\mu)} - 1) = \mathbf{r_4}\,(e^{i\psi} - 1)$ ------------ (4)

$\therefore$ The displacement vector $\boldsymbol{\delta_1}$ or $\mathbf{r_4}$, ψ and $\Delta\mu$ are prescribed, $\mathbf{r_2}$ is the free choice then unknowns $\mathbf{r_3}$ and $\mathbf{r_4}$ can be determined.

Now writing the Dyad equation for phase-II synthesis

$o_cC + CB_2 + B_2B_3 + B_3C_3 + C_3\, o_c = 0$

i.e $\mathbf{r_5} + \mathbf{r_4}\, e^{i\psi} + \boldsymbol{\delta_2} - \mathbf{r_4}\, e^{i\psi}\, e^{-i\theta 2} - \mathbf{r_5}\, e^{-i\beta} = 0$ ----------- (5) (consider c.c.w is -ve and $\mathbf{r_5}$ must be c.c.w)

$\mathbf{r_5}(e^{-i\beta} - 1) + \mathbf{r_4}\, e^{i\psi}\,(e^{-i\theta 2} - 1) = \boldsymbol{\delta_2}$

but we know $A_2B_2 + B_2B_3 = A_2B_3$

$\Rightarrow \mathbf{r_3}\, e^{-i\theta 1} + \boldsymbol{\delta_2} = \mathbf{r_3}\, e^{-i\theta 1}\, e^{i\alpha}$

$\Rightarrow \boldsymbol{\delta_2} = \mathbf{r_3}\, e^{-i\theta 1}\,(e^{i\alpha} - 1)$

$\therefore$ Loop closer equation for phase – II synthesis is

$\mathbf{r_5}(e^{-i\beta} - 1) + \mathbf{r_4}\, e^{i\psi}\,(e^{-i\theta 2} - 1) = \mathbf{r_3}\, e^{-i\theta 1}\,(e^{i\alpha} - 1)$ --------- (6)

where θ_2 is the angle between C_3B_3 and CB_2 in phase-II.

$\theta_2 = \angle B_3GF$
$= 180^0 - \angle GFB_3 - \angle GB_3F$
$= 180^0 - (180^0 - \angle GFA_2) - (180^0 - \mu_{max})$ [$\because \mu_{max1} = \mu_{max2}$]
$= \angle GFA_2 - 180^0 + \mu_{max}$

$= (180^0 - \alpha - \mu_{min}) - 180^0 + \mu_{max}$ $[\because \mu_{min1} = \mu_{min2}]$
$= \mu_{max} - \mu_{min} - \alpha$
$= \Delta\mu - \alpha$ -------------- (7)

Referring to phase-II, $\beta = 180^0$

$\therefore$ Dyad equation for phase-II is

$-2\mathbf{r_5} + \mathbf{r_4}\, e^{i\psi} (e^{i(\alpha-\Delta\mu)}-1) = \mathbf{r_3} e^{i(\psi-\Delta\mu)}(e^{i\alpha}-1)$ ------------ (8)

The displacement vector $\boldsymbol{\delta_2}$ or α are prescribed then unknown $\mathbf{r_5}$ can be determined.
Once $\mathbf{r_2}, \mathbf{r_3}, \mathbf{r_4}, \mathbf{r_5}$ are known, $\mathbf{r_1}$ the fixed link $o_a o_c$ can be determined.

Kinematic Synthesis of variable drag-linkage mechanism:

To synthesis a planar five-bar mechanism with variable Drag-linkage type mechanism is shown in phase-III and phase-IV. The linkage operates in two phases.

- Now writing the dyad equation for phase-I synthesis (Loope closer equation)

$\mathbf{r_2} + \mathbf{r_3} + \boldsymbol{\delta_1} - \mathbf{r_3}\, e^{-i\theta 1} - \mathbf{r_2}\, e^{i\phi} = 0$ --------- (9) (consider c.w angular rotation is +ve and $\mathbf{r_2}$ must be c.c.w motion)

$\mathbf{r_2}\,(e^{i\phi} - 1) + \mathbf{r_3}\,(e^{-i\theta 1} - 1) = \boldsymbol{\delta_1}$

But we know that $\mathbf{r_4} + B_1B_2 = \mathbf{r_4}\, e^{i\psi} \Rightarrow \boldsymbol{\delta_1} = \mathbf{r_4}\,(e^{i\psi} - 1)$

Therefore Loope closer equation for phase-III is

$\mathbf{r_2}\,(e^{i\phi} - 1) + \mathbf{r_3}\,(e^{-i\theta 1} - 1) = \mathbf{r_4}\,(e^{i\psi} - 1)$ --------- (10)

where θ_1 is the angle between A_1B_1 and A_2B_2 in phase-III.

$\theta_1 = \angle B_1ED$
$= 180^0 - \angle EB_1D - \angle EDB_1$
$= 180^0 - (180^0 - \mu_{max}) - (180^0 - \angle CDB_2)$ $[\because \mu_{max1} = \mu_{max2}]$
$= \mu_{max} - 180^0 + \angle CDB_2$
$= \mu_{max} - 180^0 + (180^0 - \angle B_2CD - \mu_{min})$ $[\because \mu_{min1} = \mu_{min2}]$
$= \mu_{max} - \mu_{min} - (\psi - 180^0)$
$= \Delta\mu - \psi + 180^0$ ----------- (11)

where $\Delta\mu$ is the range of transmission angle.

Referring to phase-III, $\phi = 180^0$

$\therefore$ Dyad equation for phase-III is

$-2\,\mathbf{r_2} + \mathbf{r_3}\,(e^{-i(180-\psi+\Delta\mu)}-1) = \mathbf{r_4}\,(e^{i\psi} -1)$ ------------ (12)

$\therefore$ The displacement vector $\boldsymbol{\delta_1}$ or $\mathbf{r_4}$, ψ and $\Delta\mu$ are prescribed, $\mathbf{r_2}$ is the free choice then unknowns $\mathbf{r_3}$ and $\mathbf{r_4}$ can be determined.

Now writing the Dyad equation for phase-IV synthesis

$o_cC + CB_2 + B_2B_3 + B_3C_3 + C_3\, o_c = 0$

i.e $\mathbf{r_5} + \mathbf{r_4}\, e^{i\psi} + \boldsymbol{\delta_2} - \mathbf{r_4}\, e^{i\psi}\, e^{-i\theta 2} - \mathbf{r_5}\, e^{i\beta} = 0$ ----------- (13) (consider c.c.w angular rotation is -ve and

$\mathbf{r_5}$ must be c.c.w)

$\mathbf{r_5}(e^{i\beta} - 1) + \mathbf{r_4}\, e^{i\psi}\,(e^{-i\theta 2} - 1) = \boldsymbol{\delta_2}$

but we know $A_2B_2 + B_2B_3 = A_2B_3$

$\Rightarrow \mathbf{r_3}\, e^{-i\theta 1} + \boldsymbol{\delta_2} = \mathbf{r_3}\, e^{-i\theta 1}\, e^{i\alpha}$

$\Rightarrow \boldsymbol{\delta_2} = \mathbf{r_3}\, e^{-i\theta 1}\,(e^{i\alpha} - 1)$

∴ Loop closer equation for phase – IV synthesis is

$$\mathbf{r_5}(e^{-i\beta} - 1) + \mathbf{r_4}\, e^{i\psi}\,(e^{-i\theta 2} - 1) = \mathbf{r_3}\, e^{-i\theta 1}\,(e^{i\alpha} - 1) \text{ --------- } (14)$$

where θ_2 is the angle between C_3B_3 and CB_2 in phase-IV.

$$\begin{aligned}\theta_2 &= \angle B_3GF \\ &= 180^0 - \angle GFB_3 - \angle GB_3F \\ &= 180^0 - (180^0 - \angle B_2FA_2) - (180^0 - \mu_{max}) \qquad [\because \mu_{max1} = \mu_{max2}] \\ &= \angle B_2FA_2 - 180^0 + \mu_{max} \\ &= (180^0 - \angle B_2A_2F - \mu_{min}) - 180^0 + \mu_{max} \qquad [\because \mu_{min1} = \mu_{min2}] \\ &= \mu_{max} - \mu_{min} - \angle B_2A_2F \\ &= \Delta\mu - (\alpha - 180^0) \\ &= \Delta\mu - \alpha + 180^0 \text{ -------------- } (15)\end{aligned}$$

Referring to phase-IV, $\beta = 180^0$

∴ Dyad equation for phase-IV is

$$-2\mathbf{r_5} + \mathbf{r_4}\, e^{i\psi}\,(e^{-i(\Delta\mu - \alpha + 180)} - 1) = \mathbf{r_3} e^{-i(\Delta\mu - \psi + 180)}\,(e^{i\alpha} - 1) \text{ ------------ } (16)$$

The displacement vector $\boldsymbol{\delta_2}$ or α are prescribed then unknown $\mathbf{r_5}$ can be determined.
Once $\mathbf{r_2}, \mathbf{r_3}, \mathbf{r_4}, \mathbf{r_5}$ are known, $\mathbf{r_1}$ the fixed link o_ao_c can be determined.

Case Steady-1 : Synthesize a planar five-bar mechanism with variable crank-rocker type shown in phase-I and in phase-II. Given that $\Delta\mu=85^0$, $\psi=35^0$ c.w and $\alpha=50^0$ c.w

from equation (4)

$$-2\,\mathbf{r_2} + \mathbf{r_3}\,(e^{i(\psi-\Delta\mu)} - 1) = \mathbf{r_4}(e^{i\psi} - 1)$$

$$\Rightarrow \quad -2\,\mathbf{r_2} + \mathbf{r_3}(e^{i(35-85)} - 1) = \mathbf{r_4}(e^{i35} - 1)$$

$$\Rightarrow \quad -2\,\mathbf{r_2} + \mathbf{r_3}(e^{-i50} - 1) = \mathbf{r_4}(e^{i35} - 1) \text{ ----------- } (17)$$

From equation (8)

$$-2\,\mathbf{r_5} + \mathbf{r_4}\, e^{i\psi}(e^{i(\alpha-\Delta\mu)} - 1) = \mathbf{r_3}\, e^{i(\psi-\Delta\mu)}\,(e^{i\alpha} - 1)$$

$$\Rightarrow \quad -2\,\mathbf{r_5} + \mathbf{r_4}\, e^{i35}(e^{i(50-85)} - 1) = \mathbf{r_3}\, e^{i(35-85)}\,(e^{i50} - 1)$$

$$\Rightarrow \quad -2\,\mathbf{r_5} + \mathbf{r_4}\, e^{i35}(e^{-i35} - 1) = \mathbf{r_3}\, e^{-i50}\,(e^{i50} - 1)$$

$$\Rightarrow \quad -2\,\mathbf{r_5} + \mathbf{r_4}\,(1 - e^{i35}) = \mathbf{r_3}\,(1 - e^{-i50}) \text{ ------------- } (18)$$

Let $\mathbf{r_2} = -2.0 + 0.0i$

$\mathbf{r_3} = 2.8 + 1.2i$

Substitute $\mathbf{r_2}$ and $\mathbf{r_3}$ in equation (17) then

$$-2.0(-2.0) + (2.8+1.2i)(e^{-i50}-1) = \mathbf{r_4}(e^{i35}-1)$$

$$\Rightarrow \quad 4.0 + (2.8+1.2i)(0.6428-0.766i-1.0) = \mathbf{r_4}(0.8192+0.5736i-1)$$

$$\Rightarrow \quad 4.0 + (2.8+1.2i)(-0.3572-0.766i) = \mathbf{r_4}(-0.1808+0.5736i)$$

$$\Rightarrow \quad 4.0 + (-1.0002-2.1448i-0.4286i + 0.9192) = \mathbf{r_4}(-0.1808+0.5736i)$$

$$\Rightarrow \quad 3.919-2.5734i = -\mathbf{r_4}(0.1808 - 0.5736i)$$

$$\Rightarrow \quad \mathbf{r_4} = \frac{-3.919+2.5734i}{0.1808-0.5736i} \text{ X } \frac{0.1808+0.5736i}{0.1808+0.5736i}$$

$$\Rightarrow \quad \mathbf{r_4} = \frac{-0.7086-2.2479i+0.4653i-1.4761}{0.0327+0.3290}$$

$$\Rightarrow \quad \mathbf{r_4} = -6.04 - 4.9284i$$

Substituting $\mathbf{r_3}$ and $\mathbf{r_4}$ in equation (18) then

$$-2\mathbf{r_5} - (6.04 + 4.9284i)(1-e^{i35}) = (2.8 + 1.2i)(1-e^{-i50})$$

$$\Rightarrow \quad -2\mathbf{r_5} - (6.04 +4.9284i)(1-0.8192-0.5736i) = (2.8+1.2i)(1-0.6428+0.766i)$$

$$\Rightarrow \quad -2\mathbf{r_5} - (6.04 +4.9284i)(0.1808-0.5736i) = (2.8+1.2i)(0.3572+0.766i)$$

$$\Rightarrow \quad -2\mathbf{r_5} = (1.0002 + 0.4286i + 2.1448i - 0.9192) + (1.092 + 0.8911i - 3.4645i + 2.8269)$$

$$\Rightarrow \quad -2\mathbf{r_5} = 4.0+0i$$

$$\Rightarrow \quad \mathbf{r_5} = -2.0 + 0i$$

$$\mathbf{r_1} = \mathbf{r_2} + \mathbf{r_3} - \mathbf{r_4} - \mathbf{r_5}$$

$$\Rightarrow \quad \mathbf{r_1} = (-2.0 +0i) + (2.8 + 1.2i) - [-(6.04 + 4.9284i)] - (-2.0+0i)$$

$$\Rightarrow \quad \mathbf{r_1} = -2.0 + 2.8 + 1.2i + 6.04 + 4.9284i + 2.0$$

$$\Rightarrow \quad \mathbf{r_1} = 8.84 + 6.1284i$$

Case steady-2 : Synthesize a planar five-bar mechanism with variable crank-rocker type shown in phase-I and in phase-II. Given that the displacement $\delta_1 = 3.0 + 1.5i$, $\delta_2 = 2.5-2.0i$, $\Delta\mu=85^0$, $\psi=35^0$ c.w and $\alpha=50^0$ c.w.

We know that $\delta_1 = \mathbf{r_4}(e^{i\psi}-1)$

$$\delta_2 = \mathbf{r_3} e^{-i\theta 1}(e^{i\alpha}-1) = \mathbf{r_3} e^{-i(\Delta\mu-\psi)}(e^{i\alpha}-1)$$

$$\delta_2 = \mathbf{r_3} e^{i(\psi-\Delta\mu)}(e^{i\alpha}-1)$$

$\mathbf{r_4} = \delta_1/(e^{i\psi} -1) = (3.0 +1.5i)/(e^{i35} -1) = \dfrac{3.0+1.5i}{-0.1808+0.5736i}$ X $\dfrac{-0.1808-0.5736i}{-0.1808-0.5736i}$

$\mathbf{r_4} = \dfrac{-0.5424-1.7208i-0.2712i+0.8604}{0.0327+0.329}$

$\mathbf{r_4} = 0.8792 - 5.5073i$

$\delta_2 = \mathbf{r_3}\, e^{i(\psi-\Delta\mu)} (e^{i\alpha} -1)$

$\mathbf{r_3} = (2.5\text{-}2.0i)/(e^{i(35-85)} . (e^{i50}\text{-}1)) = (2.5\text{-}2.0i)/(1.0\text{-}e^{-i50})$

$\mathbf{r_3} = \dfrac{2.5-2.0i}{0.3572+0.766i}$ X $\dfrac{0.3572-0.766i}{0.3572-0.766i}$

$\mathbf{r_3} = \dfrac{0.893-1.915i-0.7144i-1.532}{0.7143}$

$\mathbf{r_3} = \dfrac{0.893-1.915i-07144i-1.532}{0.7143}$

$\mathbf{r_3} = -0.8946\text{-}3.6811i$

Substitute $\mathbf{r_3}$, $\mathbf{r_4}$ in equation (4) then

$-2\mathbf{r_2} + \mathbf{r_3}(e^{i(\psi-\Delta\mu)}\text{-}1) = \mathbf{r_4}\, (e^{i\psi}\text{-}1)$

$\Rightarrow$ $-2\mathbf{r_2} - (0.8946 + 3.6811i)\, (e^{i(35-85)}\text{-}1) = (0.8792\text{-}5.5073i)\, (e^{i35}\text{-}1)$

$\Rightarrow$ $-2\mathbf{r_2} = (0.8792 - 5.5073i)\, (-0.1808+0.5736i) + (0.8946+3.6811i)\, (-0.3572\text{-}0.766i)$

$\Rightarrow$ $-2\mathbf{r_2} = (-0.159 + 0.5043i + 1.0i + 3.159) + (-0.3196\text{-}0.6853i\text{-}1.3149i+2.8197)$

$\Rightarrow$ $-2\mathbf{r_2} = 5.5 - 0.496i$

$\Rightarrow$ $\mathbf{r_2} = -\ 2.75 + 0.248i$

Substitute $\mathbf{r_3}$, $\mathbf{r_4}$ in equation (8) then.

$-2\mathbf{r_5} + \mathbf{r_4}\, e^{i\psi}\, (e^{i(\alpha-\Delta\mu)}\text{-}1) = \mathbf{r_3}\, e^{i(\psi-\Delta\mu)}\, (e^{i\alpha}\text{-}1)$

$\Rightarrow$ $-2\mathbf{r_5} + (0.8792 - 5.5073i)\, e^{i35}\, (e^{i(50-85)} -1) = -(0.8946 + 3.6811i)\, e^{i(35-85)}\, (e^{i50}\text{-}1)$

$\Rightarrow$ $-2\mathbf{r_5} + (0.8792 - 5.5073i)\, (1\text{-}\ e^{i35}) = -(0.8946 + 3.6811i)\, (1\text{-}\ e^{-i50})$

$\Rightarrow$ $-2\mathbf{r_5} = (0.8792 - 5.5073i)\, (0.1808\text{-}0.5736i) + (0.8946 + 3.6811i)\, (0.3572 + 0.766i)$

$\mathbf{r_5} = -2.75 + 0.248i$

we know $\mathbf{r_1} = \mathbf{r_2} + \mathbf{r_3} + -\ \mathbf{r_4} - \mathbf{r_5}$

$= (-2.75 +0.248i) - (0.8946 + 3.6811i) - (0.8792 - 5.5073i) - (-2.75 + 0.248i)$

$= -0.8946 - 3.6811i - 0.8792 + 5.5073i$

= -1.7738 + 1.8262i

Case Steady-3: Synthesize a planar five-bar mechanism with variable drag-linkage type shown in phase-III and in phase-IV, Given that $\Delta\mu=60^0(\mu_{max} = 125^0$ & $\mu_{min}= 65^0$), $\psi=220^0$ c.w and $\alpha=205^0$ c.w

from equation (12)

$$-2\,\mathbf{r_2} + \mathbf{r_3}\,(e^{-i(180 - \psi + \Delta\mu)} - 1) = \mathbf{r_4}(e^{i\psi} - 1)$$

$$\Rightarrow \quad -2\,\mathbf{r_2} + \mathbf{r_3}(e^{-i(180-220+60)} - 1) = \mathbf{r_4}(e^{i220}-1)$$

$$\Rightarrow \quad -2\,\mathbf{r_2} + \mathbf{r_3}(e^{-i20} - 1) = \mathbf{r_4}(e^{i220}-1) \text{ ----------- (19)}$$

From equation (16)

$$-2\,\mathbf{r_5} + \mathbf{r_4}\,e^{i\psi}(e^{-i(\Delta\mu - \alpha + 180)}-1) = \mathbf{r_3}\,e^{-i(\Delta\mu - \psi + 180)}\,(e^{i\alpha}-1)$$

$$\Rightarrow \quad -2\,\mathbf{r_5} + \mathbf{r_4}\,e^{i220}\,(e^{-i(60-205+180)}-1) = \mathbf{r_3}\,e^{-i(60-220+180)}\,(e^{i205}-1)$$

$$\Rightarrow \quad -2\,\mathbf{r_5} + \mathbf{r_4}\,e^{i220}\,(e^{-i35}-1) = \mathbf{r_3}\,e^{-i20}\,(e^{i205}-1)$$

$$\Rightarrow \quad -2\,\mathbf{r_5} + \mathbf{r_4}\,(e^{i185} - e^{i220}) = \mathbf{r_3}\,(e^{i185} - e^{-i20}) \text{ ------------- (20)}$$

Let $\mathbf{r_2} = -3.0 + 0.0i$

$\mathbf{r_3} = 3.5 + 2.5i$

Substitute $\mathbf{r_2}$ and $\mathbf{r_3}$ in equation (19) then

$$-2.0(-3.0) + (3.5+2.5i)\,(e^{-i20}-1) = \mathbf{r_4}\,(e^{i220}-1)$$

$$\Rightarrow \quad 6.0 + (3.5+2.5i)\,(0.9397- 0.342i -1.0) = \mathbf{r_4}\,(-0.766 - 0.6428i-1)$$

$$\Rightarrow \quad 6.0 + (3.5+2.5i)\,(-0.0603-0.342i) = \mathbf{r_4}\,(-1.766 - 0.6428i)$$

$$\Rightarrow \quad 6.0 + (-0.211-0.1507i-1.197i + 0.855) = \mathbf{r_4}\,(-1.766 - 0.6428i)$$

$$\Rightarrow \quad 6.644 - 1.3477i = -\mathbf{r_4}\,(1.766 + 0.6428i)$$

$$\Rightarrow \quad \mathbf{r_4} = -(11.7333 - 2.38i - 4.2708i - 0.8663) / (3.1187 + 0.4132)$$

$$\Rightarrow \quad \mathbf{r_4} = -\,3.0768 + 1.8831i$$

Substituting $\mathbf{r_3}$ and $\mathbf{r_4}$ in equation (20) then

$$-2\mathbf{r_5} + (-3.0768 + 1.8831i)\,(e^{i185} - e^{i220}) = \mathbf{r_3}\,(e^{i185} - e^{-i20})$$

$$\Rightarrow -2\mathbf{r_5}+(-3.0768+1.8831i)(-0.9962-0.0872i+0.766 +0.6428i)= (3.5+2.5i)(-0.9962-0.0872i- 0.9397+0.342i)$$

$$\Rightarrow \quad -2\mathbf{r_5} +(-3.0768+1.8831i)\,(-0.2302 +0.5556i) = (3.5+2.5i)\,(-1.9358+0.2548i)$$

$$\Rightarrow \quad -2\mathbf{r_5} + (0.7083 - 1.7095i - 0.4335i - 1.0463) = (-6.7756 - 4.8397i + 0.8918i - 0.637\,)$$

$$\Rightarrow \quad -2\mathbf{r_5} = -7.0746-1.8049i$$

$\Rightarrow \quad \mathbf{r_5} = 3.5373 + 0.9025i$

Case steady-4 : Synthesize a planar five-bar mechanism with variable Drag-linkage type shown in phase-III and in phase-IV. Given that the displacement $\delta_1 = 4.0 - 5.5i$, $\delta_2 = 6.0+5.0i$, $\Delta\mu=65^0$(consider $\mu_{max} = 120^0$ & $\mu_{min}=55^0$) ,$\psi=220^0$ and $\alpha=200^0$ c.w

We know that $\delta_1 = \mathbf{r_4}\,(e^{i\psi}-1)$

$$\delta_2 = \mathbf{r_3}\, e^{-i\theta 1}\,(e^{i\alpha}-1) = \mathbf{r_3}\, e^{-i(\Delta\mu-\psi+180)}\,(e^{i\alpha}-1)$$

$$\delta_2 = \mathbf{r_3}\, e^{i(\psi - \Delta\mu -180)}\,(e^{i\alpha}-1)$$

$$\mathbf{r_4} = \delta_1/(e^{i\psi} -1) = (4.0 -5.5i)/(e^{i220} -1) = (4.0 -5.5i)/(-1.766 -0.6428i)$$

$$\mathbf{r_4} = (-7.064 + 9.713i +2.5712i +3.5354) / (3.1188 + 0.4132)$$

$$\mathbf{r_4} = -1.0 + 3.478i$$

$$\delta_2 = \mathbf{r_3}\, e^{i(\psi-\Delta\mu - 180)}\,(e^{i\alpha} -1)$$

$$\mathbf{r_3} = (6.0+5.0i)/(e^{i(220-65-180)} . (e^{i200}-1)) = (6.0+5.0i)/(e^{-i25}. (e^{i200} - 1))$$

$$\mathbf{r_3} = (6.0 + 5.0i) (0.9063 + 0.4226i) / (-1.9397 - 0.342i)$$

$$\mathbf{r_3} = (3.3248 + 7.0671i) / (-1.9397 - 0.342i)$$

$$\mathbf{r_3} = (-6.4491 - 13.708i + 1.1371i - 2.4169) / (3.7624 + 0.117)$$

$$\mathbf{r_3} = -2.2854 - 3.2404i$$

Substitute $\mathbf{r_3}$, $\mathbf{r_4}$ in equation (iv) then

$$-2\mathbf{r_2} + \mathbf{r_3}(e^{-i(180 - \psi + \Delta\mu)}-1) = \mathbf{r_4}\,(e^{i\psi}-1)$$

$\Rightarrow -2\mathbf{r_2} - (2.2854 + 3.2404i)\,(e^{-i(180-220+65)}-1) = (-1.0 + 3.478i)\,(e^{i220}-1)$
$\Rightarrow -2\mathbf{r_2}$ - $(2.2854 + 3.2404i)\,(-0.0937-0.4226i) = (-1.0+3.478i)\,(-1.766-0.6428i)$
$\Rightarrow -2\mathbf{r_2} = (1.766 - 6.1421i + 0.6428i + 2.2357) + (-0.2141- 0.3036i - 0.9658i + 1.3694)$
$\Rightarrow -2\mathbf{r_2} = 5.157 - 6.7687i$
$\Rightarrow \mathbf{r_2} = - 2.579 + 3.3844i$

Substitute $\mathbf{r_3}$, $\mathbf{r_4}$ in equation (viii) then.

$$-2\mathbf{r_5} + \mathbf{r_4}\, e^{i\psi}\,(e^{-i(\Delta\mu - \alpha +180)}-1) = \mathbf{r_3}\, e^{-i(\Delta\mu - \psi + 180)}\,(e^{i\alpha}-1)$$

$\Rightarrow -2\mathbf{r_5} + (-1.0 + 3.478i)\, e^{i220}\,(e^{-i(65-200+180)} -1) = (-2.2854 - 3.2404i)\, e^{-i(65-220+180)}\,(e^{i200}-1)$
$\Rightarrow -2\mathbf{r_5} + (-1.0 + 3.478i)\, e^{i220}\,(e^{-i45} - 1) = (-2.2854 - 3.2404i)\, e^{-i25}\,(e^{i200}-1)$
$\Rightarrow -2\mathbf{r_5} + (-1.0 + 3.478i)\,(e^{i175} - e^{i220}) = (-2.2854 - 3.2404i)\,(e^{i175}- e^{-i25})$

$\Rightarrow -2\mathbf{r_5}+(-1.0 + 3.478i)(-0.9962+0.0872i+0.766+0.6428i) =$
$(-2.2854-3.2404i)(-0.9962+0.0872i-0.9063+0.4226i)$

$\Rightarrow -2\mathbf{r_5}+(-1.0 + 3.478i)(-0.2302+0.73i) =(-2.2854-3.2404i)(-1.9025+0.5098i)$
$\Rightarrow -2\mathbf{r_5} = (4.348 + 6.1649i - 1.1651i + 1.652) - (0.2302 - 0.8006i - 0.73i - 2.5389)$
$\Rightarrow -2\mathbf{r_5} = 8.3087 + 6.5304i$
$\mathbf{r_5} = - 4.1544 - 3.2652i$

Conclusions

An analytical method of kinematic synthesis of five-bar mechanisms with variable crank-rocker and drag-linkage planar mechanisms in two-two phases is proposed. Variable crank-rocker and drag linkage planar of a five-bar mechanisms are designed for the motion between two finitely separated positions of minimum and maximum transmission angles. The transmission angle criterion of design leads to a synthesis of mechanism with transmission angle control and reduces the solution space. Some of the practical applications are circuit breaker mechanism, embossing mechanism, ON-OFF switch mechanism. .

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