Design of PID Controller for Flexible Link Manipulator

S. Mallikarjunaiah*, S. Narayana Reddy**
*(Department of Electrical and Electronics Engineering, CR Engineering College, Tirupati, A.P., India)
** (Department of Electrical and Electronics Engineering, SV University, Tirupati, A.P., India)

ABSTRACT

This paper focuses on the end-point control of a single flexible link which rotates in the horizontal plane. The dynamic model is derived using a Lagrangian assumed modes method based on Euler–Bernoulli beam theory. Initially the system is modeled as a voltage-input model, and different controllers were applied to control the system. The position and trajectory control is performed by PID control methods for this system. The purpose of this study is to keep the rotate angle of the link at desired position and to eliminate the oscillation angle of end effectors. The results were produced for mode 1 and mode 2 operation. The control blocks required for this system are performed on MATLAB – SIMULINK. The simulated results of the system based on PID controller are quite satisfactory.

Keywords - Flexible Link Manipulator, PID controller, High Frequency modes

I. INTRODUCTION

An important advanced robotic system is the flexible-link robot arm. The desire to improve robot performance has led to the design of lighter flexible links. A light-flexible link robot arm has many advantages compared to conventional rigid link robots such as lower power consumption, higher payload-to-robot weight ratio, lower manufacturing cost, and easier transportation to name a few. Because of the elasticity of the flexible robot arm, the controller algorithms are different from that of the traditional rigid robot arm.

A flexible link robot arm is a distributed parameter system of infinite order, Due to elastic properties of flexible manipulators, the development of a mathematical description and subsequent model-based control of the system is a complicated task. This is made difficult by the presence of a large (infinite) number of modes of vibration in the system. The modes become significant in two ways: firstly, because the oscillations themselves prolong the settling time and secondly, because attempts to actively control some modes result in instability of the other modes. This non-linear behavior of the structure at high speeds, firstly, degrades end-point accuracy and secondly complicates controller development.

The flexible link systems play an important role in the industrial applications, mainly due to the use of lightweight materials in large space structures. The main difficulty associated with the flexible link manipulator is the vibrations at the end effectors. Various control strategies are available in the literature to minimize the vibrations. A brief description of these methods is given below.

It is known that the flexible system is a highly nonlinear and heavily coupled system, ddifferent assumed modes method [1-3] is used for the dynamic modeling and implemented via a commercially available symbolic manipulation program. Systematic method is developed to symbolically derive the full nonlinear dynamic equations.

High performance manipulators with dynamic behavior in which the flexibility is an essential aspect are addressed [4]. The mathematical representations commonly used in modeling flexible arms and arms with flexible drives are examined. Dynamic deformations of the flexible arm were represented in a simple and compact form with use of the virtual coordinate systems in [5]. Using the assumed-modes approach it is possible to find the transfer function between the torque input and the net tip deflection [6]. It is shown that when the number of modes is increased for more accurate modeling, the relative degree of the transfer function becomes ill-defined.

In [8] the first method designs a stable pre-filter using the extended bandwidth zero phase error tracking control method. The second feed forward method adds delay to the inverse model and then uses common filter design techniques to approximate this delayed frequency response.

PID type composite controller for controlling flexible arms modeled by the singular perturbation approach [9], and investigates a tuning method based on the proposed controller structure. For the slow sub-controller, a PD plus disturbance observer is used, which eventually takes on PID form, and for the fast sub-controller, modal feedback PID control is utilized. By using the Tchebyshev representation of a discrete-time transfer function and some new results on root counting with respect to the unit circle[10], were shown that how the digital PID stabilizing gains can be determined by solving sets of linear inequalities in two unknowns for a fixed value of the third parameter. The application of the $H\infty$ and PID control synthesis method is used to develop the tip position control of a flexible-link manipulator [11]. A modified PID control (MPID) is proposed [12] which depend only on vibration feedback to improve the response of the flexible arm without the massive need for measurements.

The requirement of controllers with faster response and higher accuracy introduces a challenge that the researchers have faced in different ways. The large mass and energy requirements of standard rigid link manipulators have led to a desire for flexible link manipulators characterized by low-mass links and actuators with low power requirements. This is particularly desirable in certain applications, such as space systems, where mass and energy requirements must be minimized for transport purposes. Flexible link dynamics are also found in certain mechanical pointing systems and in systems with links having high length-to-width ratios. These dynamics make the system outputs such as tip position more difficult to control. Therefore, before flexible link manipulators can be surrealistically implemented, it is necessary to study the nature of flexible link manipulators and determine effective methods for position control.

This paper deals with the modeling and PID control of single flexible link manipulator. The PID controller is a combination of PID controller and plant matrix controller that is adapted based on the output of PID controller. The paper aims at end-point control by using PID controllers, which overcomes the disadvantage of the conventional controllers such as more raise time and settling time.

II. Flexible Manipulator

The conventional approach to the design of an automatic control system often involves the construction of a mathematical model which best describes the dynamics behavior of the plant to be controlled, and the application of analytical techniques to this plant model to derive an appropriate control law. Usually, such a mathematical model consist of a set linear or non-linear differential equations, most of them are derived using some form of approximation and simplification. The traditional model-based control techniques break down, when a representative model is difficult to obtain due to uncertainty or sheer complexity. It is known that robot system is highly non-linear and heavily coupled system, and accurate mathematical model is difficult to obtain, thus it making difficult to control using conventional techniques. This paper presents the mathematical modeling of a single link flexible manipulator. The system is modeled by the Lagrange formulation and model expansion method.

III. Mathematical Modelling of Flexible Manipulator

The manipulator is illustrated in fig. 1, and is modeled as a pinned-free flexible beam with payload at one end. The beam can bend freely in the horizontal plane but is considered stiff with respect to vertical bending and torsion. The model is developed using the Lagrange formulation and model expansion method. The length of the manipulator is assumed to be constant, and deformation due to shear, rotary inertia and the effects of axial forces are neglected. The moment of inertia about the hub O is denoted by j_f and ρ is the linear mass density. The arm has length l , and the payload mass is given by M_e. The control torque T is applied at the hub of the manipulator by way of the rotary actuator. The angular displacement of the manipulator, moving in the xOy plane, is denoted by θ. The width of the arm is assumed to be much greater than it's the thickness, thus allowing it to vibrate dominantly in the horizontal direction. The shear deformation and rotary inertia effects are ignored.

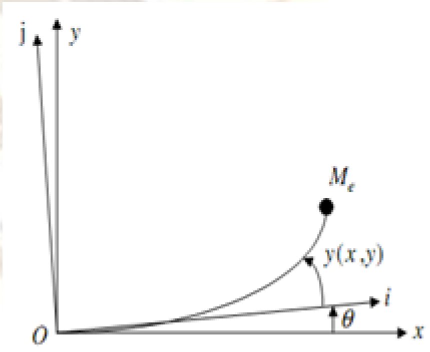

Fig. 1 Schematic representation of the flexible manipulator system

For an angular displacement θ and an elastic deflection y(x,t) the total displacement u(x,t) of a point, measured at a distance x from the hub can be described as a function of the above, measured from the direction of Ox.

$$(x, t) = u(x, t) + \theta (t)\, x \qquad (1)$$

The kinetic energy of the system can be written as

$$T = 0.5\, j_f \dot{\theta} + 0.5 \int_0^1 \left(\frac{\partial y}{\partial t} + x\theta\right)^2 \rho A\, dx + 0.5M \left(\frac{\partial y^2}{\partial x^2}\right)^2 \qquad (2)$$

In eqn. (2), the first term on the right hand side is due to the hub inertia, the second term is due to the rotation of the manipulator with respect to the origin, and the third term is due to the payload mass. The potential energy is related to the bending of manipulator. Since the width of the manipulator under consideration is assumed to be significantly larger than its thickness, the effects of the shear displacement can be neglected. In this way, the potential energy of the manipulator can be written as

$$V = 0.5\, EI \int_0^t \left(\frac{\partial y^2}{\partial x^2}\right)^2 \qquad (3)$$

where E is the modulus of the elasticity for the beam material, and I denote the second moment of area of the beam cross-section.

The non-conservative work for the input torque T can be written as

$$W = T\theta \qquad (4)$$

The Lagrangian for the system is formulated as

$$L = T - V \qquad (5)$$

To obtain the equation of the motion of the manipulator, Hamilton extended principle is used as described in eqn. (6)

$$\int_{t1}^{t2} (\partial L + \partial W)\, dt = 0 \qquad (6)$$

where t1 and t2 are two arbitrary times and ∂w represents virtual work. Manipulating Eqn. (1) - (6) yields the equation of motion of the manipulator as

$$EI \frac{\partial^4 y(x,t)}{\partial x^4} + \rho A \left(x\ddot{\theta} + \frac{\partial^2 y(x,t)}{\partial t^2} \right) = 0 \qquad (7)$$

The dynamic equation of the manipulator is described as

$$J_s \ddot{\theta} + M_e L \frac{\partial^2 y(x,t)}{\partial t^2} + \rho A \int_0^l \frac{\partial^2 y(x,t)}{\partial t^2} dx = T \qquad (8)$$

Where $J_s = J_f M_s L^2 + \frac{1}{3} \rho A L^3$

The corresponding boundary and initial conditions are given by

$$y(x,t)_{at\, x=0} = 0; \quad \left. \frac{dy(x,t)}{dt} \right|_{at\, x=0} = 0$$

$$EI \left. \frac{\partial^2 y(x,t)}{\partial x^2} \right|_{at\, x=0} = 0$$

$$EI \frac{\partial^3 y(x,t)}{\partial x^3} = M_e \left. \frac{\partial^3 y(x,t)}{\partial x^3} \right|_{at\, x=1}; \; y(x,0) = 0; \; \frac{dy(x,0)}{dx} = 0$$

(9)

Using the assumed mode method, the solution of the dynamic equation of motion of the manipulator can be obtained as linear combination of the product of the admissible function $\phi_i(x)$ and the time-dependent generalized coordinates $q_i(t)$, as follows, where

$$y(x,t) = \sum_{i=0}^{n} \phi_i(x) q_i(t) \qquad (10)$$

The admissible function ϕ_i(x) also called the mode shape, is purely a function of the displacement along the length of the manipulator and q_i(t) is purely a function of the time and includes an arbitrary, multiplicative constant. The parameter values for flexible manipulator are given in Table 1.

Table 1 Parametric values for the flexible manipulator

Physical parameter	Symbol	Value
Length	L	0.61 m
Section area	A	3×10^{-5} m^2
Density	P	7.8×10^3 kg/m^3
Young modulus	E	200×10^9 N/m^2
Second moment of area	I	2.5×10^{-12} m^4
Payload	M	31.7×10^{-3} kg
Moment of inertia of hub	J	4.3×10^{-3} kg-m^2

Substitution of eqn. (10) into eqn. (7) by apply boundary and initial conditions of eqn. (9), the following ordinary differential equations can be derived

$$J_s \ddot{\theta} + \sum_{i=0}^{n} a_i \frac{d^2 q}{dt^2} = T \qquad (11)$$

$$\frac{d^2 q_i}{dt^2} + w_i q_i + b_i \ddot{\theta} = 0 \qquad (12)$$

Where

$$w_i^2 = \frac{\beta_i^4 EI}{\rho A}$$

$$a_i = \frac{2\rho A}{\beta_i^2}$$

$$b_i = a_i \frac{(\sin\beta_i L + \sinh\beta_i L)[(\sin\beta_i L + \sinh\beta_i L) - \beta_i L(1 + \cos\beta_i L.\cosh\beta_i L)]}{\rho A(\sin\beta_i L + \sinh\beta_i L)^2 - 3M_e(\cos\beta_i L.\sin\beta_i L - \sinh\beta_i L.\cosh\beta_i L)^2}$$

β_i denotes the frequency of vibration, which can be determined from the above using the boundary conditions. The frequencies for the first and second modes of vibration and the corresponding terms β_i, a_i, b_i are shown in table 2.

Table 2 Mode dependent parameters

Mode	β_i	ω_i^2	a_i	b_i
1	2.6178	1.0035×10^2	6.8292×10^{-2}	4.4961×10^{-1}
2	6.9626	5.0215×10^3	9.6539×10^{-3}	2.2248×10^{-1}

If X is assumed as state space variables, $X^T = [\theta, \dot{\theta}, q_1, \dot{q}_1, \ldots\ldots q_n, \dot{q}_n]$ and Y is assumed as the output, $Y^T = [\theta, q_1, \ldots\ldots q_n]$, eqn. (11) and eqn .(12) can be written as

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{a_{w_i}^2}{j_s - a_i b_i} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-j_s w_i^2}{j_s - a_i b_i} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{j_s - a_i b_i} \\ 0 \\ \frac{-b_i}{j_s - a_i b_i} \end{bmatrix} T \qquad (13)$$

and

$$Y = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \qquad (14)$$

thus a fourth order model is considered as a system.

IV. PROBLEM FORMULATION

Control strategies are designed for models of a flexible link manipulator that are linearized about a particular operating point, that is, for a given set of hub angle and elastic mode positions and velocities. The greater the variation of these positions and velocities from the operating point, the greater is the variation of the linearized model from the actual system. This variation becomes more pronounced for high-performance control systems since high performance implies rapid motion and therefore large departures of the hub angle and elastic mode positions and velocities from their nominal values. System performance and stability will degrade unless the situation is addressed. The objective is to control the angular displacement θ, and to minimize the vibrations at the final deflected position.

4.1 PID Controller

A proportional–integral–derivative controller (PID controller) is a generic loop feedback (controller) widely used in industrial control systems. A PID controller attempts to correct the error between a measured process variable and a desired set point by calculating and then instigating a corrective action that can adjust the process accordingly and rapidly, to keep the error minimal.

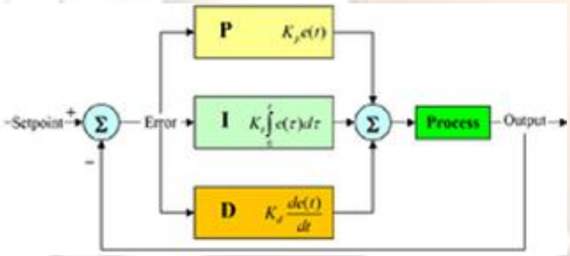

Fig. 2 Block diagram of PID Controller

The PID controller calculation involves three separate parameters; the proportional, the integral and derivative values as shown in fig. 2. The proportional value determines the reaction to the current error, the integral value determines the reaction based on the sum of recent errors, and the derivative value determines the reaction based on the rate at which the error has been changing. The weighted sum of these three actions is used to adjust the process via a control element such as the position of a control valve or the power supply of a heating element.

Some applications may require using only one or two modes to provide the appropriate system control. This is achieved by setting the gain of undesired control outputs to zero. A PID controller will be called a PI, PD, P or I controller in the absence of the respective control actions. PI controllers are particularly common, since derivative action is very sensitive to measurement noise, and the absence of an integral value may prevent the system from reaching its target value due to the control action.

4.2 Turning of PID Controller

If the PID controller parameters are chosen in correctly, the controlled process input can be unstable, i.e. its output diverges, with or without oscillation and is limited only by saturation or mechanical breakage. Tuning a control loop is adjustment of its control parameters to the optimum values for the desired control response.

There are several methods for tuning a PID loop. The most effective methods are generally involve the development of some form of process model, then choosing P, I and D based on the dynamic model parameters. Manual tuning methods can be relatively inefficient. The choice of the method will depend largely on whether or not the loop can be taken offline for tuning, and the response time of the system. If the system can be taken offline, the best tuning method often involve subjecting the system to a step change in input, measuring the output as a function of time, and using this response to determine the control parameters.

V. RESULTS AND ANALYSIS

By exerting this controller to the system, the step response and the closed loop response in both modes will be as shown in Fig. 3 to 8.

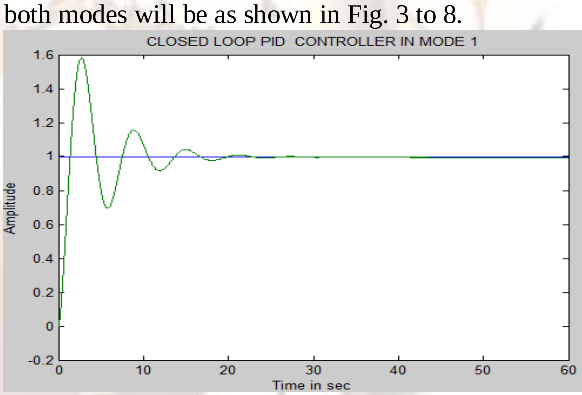

Fig. 3 Step response (alpha) of the system with PID controller for mode 1

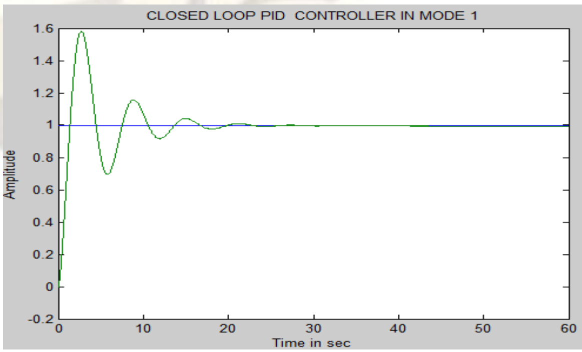

Fig. 4 Step response (theta) of the system with PID controller for mode 1

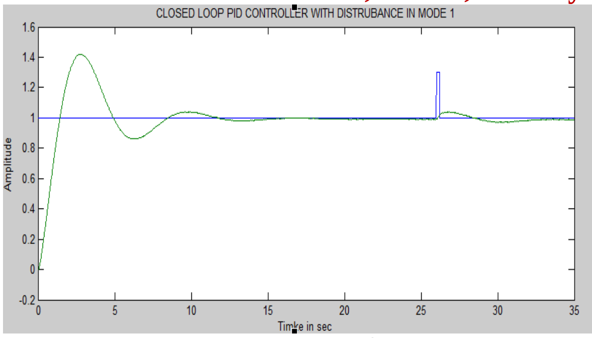

Fig. 5 Step response (Alpha) of PID controller with a disturbance for mode1

Fig. 3 and 4 are shown the amplitude (alpha) and angular velocity (Theta) of the step response of the system PID controller without addition of disturbance for mode 1 operation where as fig. 5 shows the step response(alpha) of the PID controller with addition of disturbance under mode 1 operation.

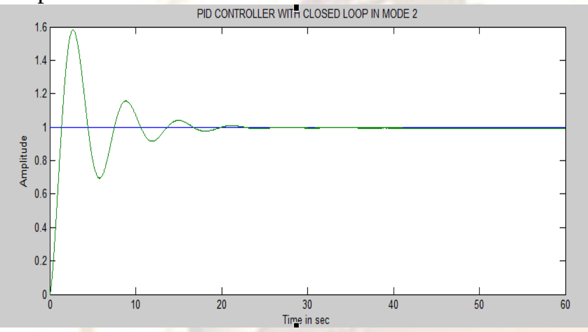

Fig. 6 Step response (theta) of PID controller with disturbance input

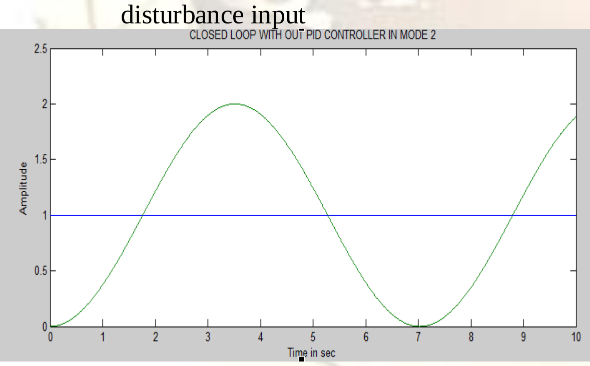

Fig. 7 Step response (Alpha) of the system with PID controller for mode 2

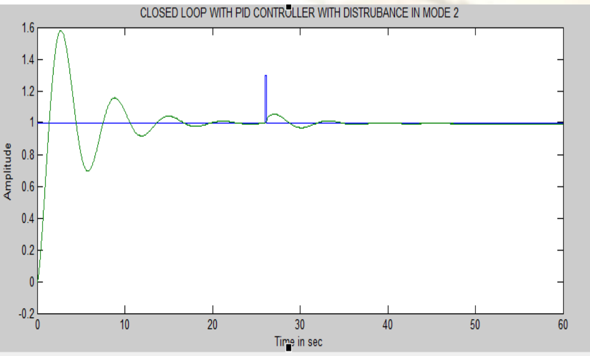

Fig. 8 Step response (Theta) of the system with PID controller for mode 2

Fig. 6 and 7 are shown the amplitude (alpha) and angular velocity (Theta) of the step response of the system PID controller without addition of disturbance for mode 2 operation where as fig. 8 shows the step response(alpha) of the PID controller with addition of disturbance under mode 2 operation. The operating modes are shown with different parameters in the table 3.

Table 3 Results of different parameters of PID Controller

Type of operation	**Settling time (Sec)**	**Peak over Shoot (Amps)**
Mode -1	15.2	1.40
Mode - 2	19.5	1.58

From the Fig. 3 to 8, it has observed that the PID controller doesn't create a suitable step response for the system. Intense the settling time is 15.2 seconds for mode-1 operation and 19.5 seconds for mode -2 operation. It shows the Peak over shoot is 1.4 Amps for mode-1 operation and 1.58 Amps for mode-2 operation.

VI. CONCLUSION

PID Control of a Single Flexible Link Manipulator (SFLM) has been investigated in this paper. In this paper initially simulations have been carried out using PID controller, the step response and the closed-loop response in both modes are shown in fig 3 to 8. It is clear that the PID controller doesn't create a suitable step response for the system. Intense transient oscillation and high over shoot are the shortcoming of such controller. Moreover the parameters of this controller are constant, no adaption with system dynamical changes. Since the amplitude of oscillation persists and in order improves the transient response of the system, a PID controller has been designed and simulated along with the system.

REFERENCES

[1] S. Centikunt, B. Siciliano, and W. J. Book, Symbolic modeling and dynamic analysis of flexible manipulators, *Proceedings on IEEE International Conference Systems, Man, and Cybernetics*, Oct. 1986, 798–803.

[2] G. G. Hastings and W. J. Book, A linear dynamic model for flexible robotic manipulators, *IEEE Control System Magazine*, 7, Apr. 1987, 61–64.

[3] P. B. Usoro, R. Nadira, and S. S. Mahil, “A finite element/Lagrange approach to modeling lightweight flexible manipulators,” *Journal of Dynamic Systems, Measurement, and Control*, 108, 1986, 198–205.

[4] B. Siciliano and W. J. Book, A singular perturbation approach to control of lightweight manipulators, *International Journal on Robotic Research*, 7(4), Aug. 1988, 79–90.

[4] W. J. Book, Modeling, design, and control of flexible manipulator arms: A tutorial review, *Proceedings on 29th IEEE Conference on Decision and Control*, Dec. 1990, pp. 500–506.

[5] H. Asada, Z.-D. Ma and H. Tokumaru, Inverse dynamics of flexible robot arms: modeling and computation for trajectory control, *Journal of Dynamic Systems, Measurement, and Control*, 112, June 1990, 177–185.

[6] D. Wang and M. Vidyagasar, Transfer functions for a single flexible link, *International Journal on Robotic Research*, 10(5), Oct. 1991, 540–549.

[7] M. W. Vandegrift, F. L. Lewis, and S. Zhu, Flexible-link robot arm control by a feedback linearization/singular perturbation approach, *Journal on Robotic Systems*, 11(7), 1994, 591–603.

[8] D. E. Torfs, R.Vuerinckx, J. Sewevers and J. Schoukens, Comparison of two feed forward design methods aiming at accurate trajectory racking of the end point of a flexible robot arm, *IEEE Transactions Control Systems Technology*, 6(1), Jan. 1998, 2-14.

[9] Joono Cheong, PID composite controller and its tuning for flexible link robots, *IEEE/RSJ International Conference on Intelligent Robots and Systems*, 3, 2002, 2122-2127.

[10] L. H. Keel, J. I. Rego, and S. P. Bhattacharyya, A new approach to Digital PID Controller Design, *IEEE Transactions on Automatic Control*, 48(4), Apr. 2003, 687-692.

[11] Ming-Tzu Ho and Yi-Wei Tu, PID controller for flexible link manipulator, *Proceedings of the 44th IEEE Conference on Decision and Control, and the European Control Conference*, 2005, 6841-6846.

[12] T. Mansour, A. Konno and M. Uchiyama, Modified PID Control of a Single- Link flexible Robot, *Advanced Robotics*, 22(4), 2008, 433-449.