Performance of OFDM QPSK, 16-QAM System using Pilot-Based Channel Estimation in presence of Doppler Frequency Shift over Rayleigh Fading Channel

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Abstract—

With the rapid growth of digital communication in recent year the need for high speed data transmission is increased, OFDM is a promising solution for achieving high data rates in mobile environment, due to its resistance to ISI and ICI, which are the common problems found in high speed data communication. In this paper, the performance of different pilot based channel estimation schemes for OFDM with QPSK and 16 QAM over Rayleigh fading channel are investigated. In the block type pilot arrangement, the performance of channel estimation is analyzed with three different algorithms: LS, LMMSE and SVD algorithm. In comb type pilot arrangement, the paper introduces three method of interpolation: linear interpolation, second order interpolation and cubic spline interpolation for channel estimation. Here in this paper, it is our goal to show the way through which the Bit Error Rate (BER) result varies due to Doppler frequency shift. The analysis has been carried out with simulation studies under MATLAB environment.

Keywords- BER, Doppler Frequency, OFDM, Pilot Carrier.

I. INTRODUCTION

For high-volume and high-speed wireless mobile communication systems, Orthogonal Frequency Division Multiplexing (OFDM) is a promising modulation scheme, and will play an increasingly important role in the future development of wireless mobile communication network due to its high data rate transmission capability with high bandwidth efficiency and its robustness to multi-path delay and no Inter Symbol Interference (ISI). It has been used in wireless LAN standards such as American IEEE802.11a and the European equivalent HIPERLAN/2 and in multimedia wireless services such as Japanese Multimedia Mobile Access Communications.

When the mobile station is doing communication in motion, the frequency of the received signal will change. In multipath conditions, each multipath wave has a frequency shift, called Doppler spread. The shift of the mobile received signal frequency caused by the movement is called Doppler frequency shift, and it is proportional to the speed of mobile users.

Pilot-based channel estimation estimates the channel information by obtaining the impulse response from all sub-carriers by pilot. Compared with blind channel estimation, which uses statistical information of the received signals, pilot-based channel estimation is a practical and an effective method.

The pilot based channel estimation can be performed by either inserting pilot tones into all of the subcarriers of OFDM symbols with a specific period or inserting pilot tones into each OFDM symbol. The first one, block type pilot channel estimation, has been developed under the assumption of slow fading channel. Even with decision feedback equalizer, this assumes that the channel transfer function is not changing very rapidly. The estimation of the channel for this block-type pilot arrangement can be based on Least Square (LS) or Minimum Mean-Square-Error (MMSE). The MMSE estimate has been shown to give 10–15 dB gain in signal-to-noise ratio (SNR) for the same mean square error of channel estimation over LS estimate [2]. In [3], a low-rank approximation is applied to linear MMSE by using the frequency correlation of the channel to eliminate the major drawback of MMSE, which is complexity. The later, the comb-type pilot channel estimation has been introduced to satisfy the need for equalizing when the channel changes even in one OFDM block. The comb-type pilot channel estimation consists of algorithms to estimate the channel at pilot frequencies and to interpolate the channel. MMSE has been shown to perform much better than LS. In [4], the complexity of MMSE is reduced by deriving an optimal low-rank estimator with singular-value decomposition (SVD).

The interpolation of the channel for comb-type based channel estimation can depend on linear interpolation, second order interpolation and spline cubic interpolation. In [4], second-order interpolation has been shown to perform better than the linear interpolation. In [5], cubic spline interpolation has been proven to give lower BER compared to second order interpolation.

In this paper, our aim is to compare the performance of all of the above schemes by applying 16QAM (16 Quadrature Amplitude Modulation), QPSK (Quadrature Phase Shift Keying), as modulation schemes with multipath Rayleigh fading and Doppler frequency shift channels with Additive

White Gaussian Noise (AWGN) as channel models. In Section II, the description of the OFDM transceiver based on pilot channel estimation, channel modal and QPSK, 16QAM is given. In Section III, the estimation of the channel based on block-type pilot arrangement is discussed. In Section IV, the estimation of the channel at pilot frequencies is presented. In Section VI, the simulation environment and results are described. Section VI concludes the paper.

II. SYSTEM MODEL

In this section we cover the basic of OFDM system block diagram and working. And see the modulation method and channel that use in our simulation.

A. *OFDM Block Diagram*

The OFDM system based on pilot channel estimation is given in Fig. 1.

The binary information is first grouped and mapped according to the modulation in "signal mapper." After inserting pilots either to all sub-carriers with a specific period or uniformly between the information data sequence, IDFT block is used to transform the data sequence of length $N\{X(k)\}$ into time domain signal $\{x(n)\}$ with the following equation:

$$\{x(n)\} = IDFT\{x(k)\}, n = 1,2,...,N-1$$
$$= \sum_{k=0}^{N-1} X(k) e^{j(2\pi kn/N)} \qquad (1)$$

Where N is the DFT length. We used inverse fast Fourier transform/fast Fourier transform (IFFT/ FFT) in place of IDFT/DFT to reduce computation complexity. Following IDFT block, guard time, which is chosen to be larger than the expected delay spread, is inserted to prevent ISI. This guard time includes the cyclically extended part of OFDM symbol in order to eliminate inter-carrier interference (ICI). The resultant OFDM symbol is given as follows:

$$x_f(n) = \begin{cases} x(N+n), & n = -N_g, -N_g+1, ..., -1 \\ x(n), & n = 0,1,......, N-1 \end{cases} \qquad (2)$$

Where N_g is the length of the guard interval. The transmitted signal $x_f(n)$ will pass through the frequency selective time varying fading channel with additive noise. The received signal is given by:

$$y_f(n) = x_f(n) \otimes h(n) + w(n) \qquad (3)$$

where $w(n)$ is AWGN and $h(n)$ is the channel impulse response. The channel response $h(n)$ can be represented by [5]:

$$h(n) = \sum_{i=0}^{r-1} h_i e^{j(2\pi/N) f_{Di} T_n} \quad \delta \quad (ʎ-\tau_i),$$
$$0 \le n \le N-1 \quad (4)$$

where r is the total number of propagation paths, h_i is the complex impulse response of the i^{th} path, f_{Di} is the i^{th} path Doppler frequency shift, ʎ is delay spread index, T is the sample period and τ_i is the i^{th} path delay normalized by the sampling time. At the receiver, after passing to discrete domain through A/D and low pass filter, guard time is removed:

$$y_f(n) \quad \text{for } -N_g \le n \le N-1$$
$$y(n) = y_f(n+N_g) \qquad n = 0,1,2.....,N-1 \qquad (5)$$

Then $y(n)$ is sent to DFT block for the following operation:

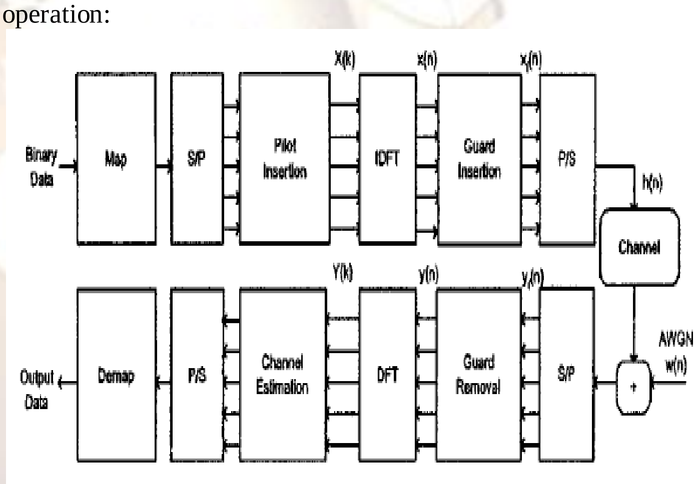

Fig.1. OFDM system for simulation [1].

$$Y(k) = DFT\{y(n)\} \qquad k = 0,1,2.....,N-1.$$
$$= \frac{1}{N} \sum_{n=0}^{N-1} y(n) e^{j(2\pi kn/N)}. \qquad (6)$$

Assuming there is no ISI shows the relation of the resulting $Y(k)$ to $H(k) = DFT\{h(n)\}, I(k)$ that is ICI because of Doppler frequency and $w(k) = DFT\{w(n)\}$, with the following equation [4]:

$$Y(k) = X(k)H(k) + I(k) + W(k), k = 0,1....,N-1 \qquad (7)$$

where

$$H(k)=\sum_{i=0}^{r-1} h_i e^{j\pi f_{Di}T\frac{\sin(\pi f_{Di}T)}{\pi f_{Di}T}} e^{-j(2\pi\tau_i/N)k},$$

$$I(k)=\sum_{i=0}^{r-1}\sum_{\substack{K=0,\\K\neq k}}^{N-1}\frac{h_i X(K)}{N}\frac{1-e^{j2\pi(f_{Di}T-k+K)}}{1-e^{j(2\pi\pi/N)(f_{Di}T-k+K)}}e^{-j(2\pi\tau_i/N)K}.$$

Following DFT block, the pilot signals are extracted and the estimated channel $H_e(k)$ for the data sub-channels is obtained in channel estimation block. Then the transmitted data is estimated by:

$$X_e=\frac{Y(k)}{H_e(k)}, k=0,1,....,N-1 \qquad (8)$$

Then the binary information data is obtained back in "signal demapper" block.

B. *M-ary PSK and 16QAM Modulation in OFDM model*

The general analytic expression for M-ary PSK waveform is:

$$s_i(t)=A\cos(w_c t+\varphi_i(t)); \quad i=0,1,2,...,M-1 \qquad (14)$$

Where $A=\sqrt{\frac{2E_s}{T_s}}$, $\varphi=\frac{2\pi m}{M}$, $m=0,1,2,...,M-1$

The parameter E_s is symbol energy, T_s is symbol time duration, and $0\le t\le T$. For BPSK modulation, *M*=2, and for QPSK modulation *M*=4, and the modulation data signal shifts the phase of the waveform, $s_i(t)$. The BPSK bandwidth efficiency is 1 bit/Hz, while QPSK bandwidth efficiency is 2 bits/Hz.

16-QAM is a one type of M-ary QAM, where *M* = 16. In 16-QAM modulation scheme we can send ($k = log_2 M = log_2 16 = 4$) 4 bit information per symbol. The general analytic expression for M-ary QAM waveform is:

$$s_i(t)=\sqrt{E_{\min}}a_i\cos(2\pi f_c t)+\sqrt{E_{\min}}b_i\sin(2\pi f_c t) \qquad (15)$$

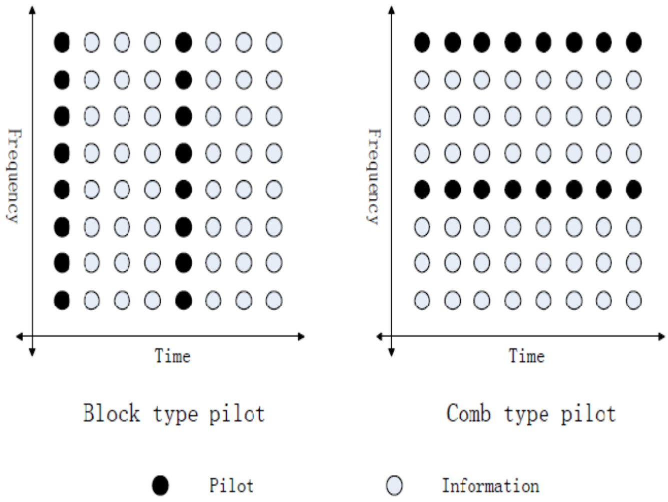

Fig.2. Block type Pilot and Comb type pilot arrangement[2].

$\sqrt{E_{\min}}$ means the energy of symbol with minimum amplitude. $a_i, b_i\ (i=1,2,....M-1)$ are a pair of independent integer numbers that could be determined by constellation.

C. *Pilot arrangement*

The pilot channel estimation methods are based on the pilot channel and pilot symbol. However, due to two-dimensional time-frequency structure of OFDM system, pilot symbol assisted modulation (PSAM) is more flexible [6]. The fading channel of the OFDM system can be viewed as a 2D lattice in a time-frequency plane, because signal is transmitted in the fixed position. And the 2D sampling should satisfy the Nyquist sampling theorem in order to eliminate the distortion. So the minimum limit of pilot symbols inserted is decided by Nyquist theorem. From Nyquist theorem, the interval of time domain N_t and frequency domain N_f should satisfy [7]

$f_m.T.N_t \le 1/2$, and $\tau_{max}.w_f.N_f \le 1/2$

Where w_f is bandwidth of sub-carrier, *T* is period of signal, $\tau_{\max}$ is the maximum multipath time delay and f_m is the maximum Doppler shift.

The two basic channel estimations in OFDM systems, block-type pilot and comb-type pilot, are illustrated in Fig.2. In the block-type pilot channel estimation, we inserting pilot tones into all subcarriers of OFDM symbols with a specific period in time and in comb-type pilot channel estimation, we inserting pilot tones into certain subcarriers of each OFDM symbol, where the interpolation is needed to estimate the conditions of date subcarriers.

D. Channel Model

In simulation, we take three OFDM channel model: AWGN channel, Rayleigh fading channel and Doppler spread channel model. In a multipath environment, it is reasonably intuitive to visualize that an impulse transmitted from the transmitter will reach the receiver as a train of pulses. When there are large numbers of paths, applying Central Limit Theorem, each path can be modeled as circularly complex Gaussian random variable with time as the variable. This model is called Rayleigh fading channel model in Fig.3. A circularly symmetric complex Gaussian random variable is of the form,

$$Z = X + jY \quad (9)$$

where real and imaginary parts are zero mean independent and identically distributed Gaussian random variables. For a circularly symmetric complex Gaussian random variable Z, Z which has a probability density,

$$E[z] = E\left[e^{i\theta} Z\right] = e^{i\theta} E[z] \quad (10)$$

The statistics of a circularly symmetric complex Gaussian random variable is completely specified by the variance, $\sigma^2 = E\left[z^2\right]$ Now, the magnitude

$$p(z) = \frac{z}{\sigma^3} e^{\frac{-z^2}{2\sigma^2}} \qquad z \ge 0 \quad (11)$$

is called a Rayleigh random variable. This model called Rayleigh fading channel model, is reasonable for an environment where there are large number reflectors. The channel impulse response (CIR) of Rayleigh multipath channel could be expressed as,

$$h(t,\tau) = \sum_{k=0}^{L-1} u_k e^{j\varphi_k} \delta(t - \tau_k) \quad (12)$$

where L is the number of multipath, τ_k is delay of the k^{th} path, $u_k e^{j\varphi_k}$ is gain coefficient, respectively. For Doppler spread channel CIR is given by,

$$h(t) = ue^{j\varphi} e^{j2\pi f_D t} \quad (13)$$

III. CHANNEL ESTIMATION BASED ON BLOCK –TYPE PILOT ARRANGEMENT

In block-type pilot based channel estimation, OFDM channel estimation symbols are transmitted periodically, in which all sub-carriers are used as pilots. If the channel is constant during the block, there will be no channel estimation error since the pilots are sent at all carriers. The estimation can be performed by using either LS or MMSE [2], [3]. If ISI is eliminated by the guard interval, we write (7) in matrix notation:

$$Y = XFh + W \quad (16)$$

where

$$X = diag\{X(0), X(1), ...X(N-1)\}$$

$$Y = \left[Y(0)Y(1).......Y(N-1)\right]^T$$

$$W = \left[W(0)W(1)......W(N-1)\right]^T$$

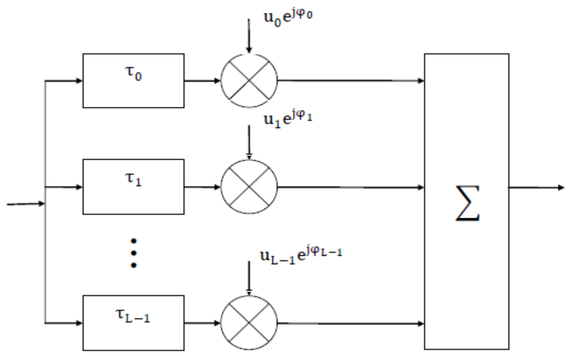

Fig. 3 Rayleigh channel model.

$$H = \left[H(0)H(1).......H(N-1)\right]^T = DFT_N\{h\}$$

$$F = \begin{pmatrix} W_N^{00} & \cdots & W_N^{0(N-1)} \\ \vdots & \ddots & \vdots \\ W_N^{(N-1)0} & \cdots & W_N^{(N-1)(N-1)} \end{pmatrix}$$

and $W_N^{nk} = \frac{1}{N} e^{-j2\pi(n/N)k}$.

If the time domain channel vector h is Gaussian and uncorrelated with the channel noise W , the frequency domain MMSE estimate of h is given by [3]:

$$H_{MMSE} = FR_{hY} R_{YY}^{-1} Y \quad (17)$$

where

$$R_{hY} = E\{hY\} = R_{hh} F^H X^H$$

$$R_{YY} = E\{YY\} = XFR_{hh} F^H X^H + \sigma^2 I_N \quad (18)$$

are the cross covariance matrix between h and Y and the auto covariance matrix of Y. R_{hh} is the auto-covariance matrix of h and σ^2 represent the noise variance $E\left\{W(k)^2\right\}$. The LS estimate is represented by:

$$H_{LS} = X^{-1} Y, \quad (20)$$

which minimizes $(Y - XFh)^H (Y - XFh)$.

Since LS estimate is susceptible to noise and ICI, MMSE is proposed while compromising complexity. Since MMSE includes the matrix inversion at each iteration, the simplified linear MMSE estimator is suggested in [8]. In this simplified version, the inverse is only need to be calculated once. In [4], the complexity is further reduced with a low-rank approximation by using singular value decomposition.

Assuming the same signal constellation on all tones and equal probability on all constellation points we get,

$$E\left\{X^H X\right\} = E\left\{\frac{1}{|X_k|^2}\right\} I$$

and average Signal to Noise Ratio (SNR) is $\overline{SNR} = E\left\{|X_k|^2\right\} / \sigma_N^2$, the term $\sigma_N^2 \left(X^H X\right)^{-1}$ is the approximated by $\dfrac{\beta}{SNR} I$, where

$$\beta = \frac{E\left\{|X_k|^2\right\}}{E\left\{\left|\frac{1}{X_k}\right|^2\right\}} . \quad (21)$$

β is a constant which depends only on the signal constellation for QPSK and 16-QAM. Then the modified MMSE estimator in terms of Linear-MMSE is given by:

$$H_{LMMSE} = R_{HH}\left(R_{HH} + \frac{\beta}{SNR} I\right)^{-1} H_{LS} . \quad (22)$$

Since LS estimate is susceptible to noise and ICI, MMSE is proposed while compromising complexity. Since MMSE includes the matrix inversion at each iteration, the simplified linear MMSE estimator is suggested in [8]. In this simplified version, the inverse is only need to be calculated once. In [4], the complexity is further reduced with a low-rank approximation by using SVD.

The optimal rank reduction of the estimator in (22), using the SVD, is obtained by exclusion of base vectors corresponding to the smallest singular values [9]. We denote the SVD of the channel correlation matrix:

$$R_{hh} = U \Lambda U^H \quad (23)$$

where U is a matrix with orthonormal columns $u_0, u_1, ...u_{N-1}$ and Λ is a diagonal matrix, containing the singular values $\lambda_0 \geq \lambda_1 \geq ...... \geq \lambda_{N-1} \geq 0$, on its diagonal. This allows the estimator in(22) to be written:

$$H_{SVD} = U □ U^H H_{LS} ,$$

where □ is a diagonal matrix containing the values

$$\delta_k = \lambda_k / \left(\lambda_k + \frac{\beta}{SNR}\right), \quad k = 0,1,....., N-1.$$

on its diagonal. The best rank – p approximation of the estimator in (22) then becomes

$$H_{SVD} = U \begin{pmatrix} □_p & 0 \\ 0 & 0 \end{pmatrix} U^H H_{LS} . \quad (24)$$

where $□_p$ is the upper left $p * p$ corner of □ .

A block diagram of the rank-*p* estimator in (24) is shown in

Fig. 3, where the LS-estimator is calculated from *y* by multiplying by X^{-1}.

IV. CHANNEL ESTIMATION BASED ON COMB-TYPE PILOT ARRANGEMENT

In Comb-type pilot based channel estimation, an efficient interpolation technique is necessary in order to estimate

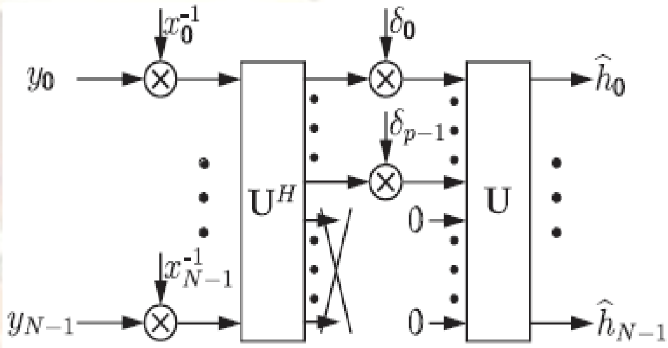

Fig.4 Block diagram of the rank-*p* channel estimator [3].

channel at data sub-carriers by using the channel information at pilot sub-carriers. Let N_p pilot signals are uniformly inserted into $X(k)$ according to the following equation:

$$X(k) = X(mL + l) \quad (25)$$
$$= \begin{cases} x_p(m), & l = 0 \\ \inf .data & l = 1, ...L-1 \end{cases}$$

where L = number of carriers / N_p and $x_p(m)$ is the m^{th} pilot carrier value. we define $\left\{H_p(k), k = 0,1,....N_p\right\}$ as the frequency response of the channel at pilot sub-carriers. The estimate

of the channel at pilot sub-carriers based on LS estimation is given by:

$$H_e = \frac{Y_p}{X_p} \qquad k = 0,1,...., N_p - 1 \quad (26)$$

where $Y_e(k)$ and $X_e(k)$ are output and input at the k^{th} pilot sub carrier respectively.

- The linear interpolation method is shown to perform better than the piecewise-constant interpolation in [9]. The channel estimation at the data-carrier k, $mL < k < (m+1)L$, using linear interpolation is given by:

$$H_e(k) = H_e(mL+l),\ 0 \le l \le L$$
$$= \left(H_p(m+1) - H_p(m)\right)\frac{l}{L} + H_p(m) \quad (27)$$

- The second-order interpolation is shown to fit better than linear interpolation [4]. The channel estimated by second-order interpolation is given by:

$$H_e(k) = H_e(mL+l)$$
$$= c_1 H_p(m-1) + c_0 H_p(m) + c_{-1} H_p(m+1)$$
(28)

$$\text{where} \begin{cases} c_1 = \dfrac{\alpha(\alpha-1)}{2}, \\ c_0 = -(\alpha-1)(\alpha+1), \qquad \alpha = \dfrac{l}{N}. \\ c_{-1} = \dfrac{\alpha(\alpha+1)}{2}. \end{cases}$$

- The cubic spline interpolation is shown to fit better than second-order interpolation [4]. The channel estimated by cubic spline interpola

$$H_e(k) = \alpha_1 H_e(m+1) + \alpha_0 H_e(m+1) + L\alpha_1 H_p'(m+1) - L\alpha_0 H_p'(m)$$
(29)

where $H_p'(m)$ is the first order derivative of $H_p(m)$,

$$\text{and} \begin{cases} \alpha_1 = \dfrac{3(L-l)^2}{L^2} - \dfrac{2(L-l)^3}{L^3} \\ \alpha_0 = \dfrac{3l^2}{L^2} - \dfrac{2l^3}{L^3} \end{cases}.$$

V. SIMULATION

A. System parameters for simulation

OFDM system parameters used in the simulation are indicated in Table I: We assume to have perfect synchronization since the aim is to observe channel estimation performance. Moreover, we have chosen the guard interval to be greater than the maximum delay spread in order to avoid inter-symbol interference. Simulations are carried out for different sig SNR ratios and for different Doppler spreads.

B. Simulation Results

To analyze and compare the performances of different channel estimation schemes for QPSK and 16QAM modulation in OFDM over Rayleigh fading channel, we perform simulation using MATLAB.

Fig. 5 shows the BER performance of Block -type pilot based channel estimation in OFDM system under 16QAM and QPSK modulation, we could find LMMSE with SVD rank p=16,20 have better performance over LS estimator. 16 QAM shows worse performance. However, the bandwidth efficiency, 16QAM is more feasible in practice.

TABLE I:
SIMULATION PARAMETRES

Parameters	Specification
Channel Bandwidth	1MHz
Number of Sub-Carriers	128
IFFT/FFT size	128 bin points
Pilot Ratio	1/8
Guard Type	Cyclic Extension
Cyclic Prefix Length	16 Samples
Number of Multipath	5
Multipath Delays	0, 2e-6, 4e-6, 8e-6, 12e-6
Sub-Carrie Frequency Spacing	7.8125KHz
SNR	40 dB

Fig.6 shows that, the cubic spline interpolation estimator has better performance than linear and second order interpolation, when we used 16 QAM modulation. From Fig. 7 we can say that after performing MATLAB simulation with Block-type pilot based channel estimation for QPSK modulation in presence of Doppler spread, LS and LMMSE has better performance over LMMSE (SVD) rank p=5.

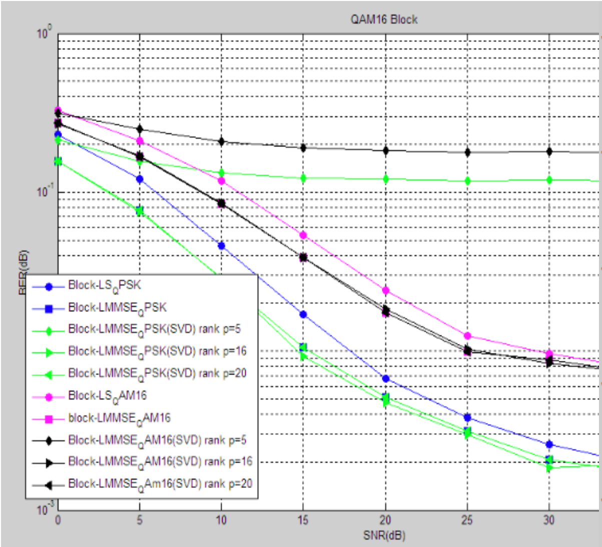

Fig. 5. Comparison of QPSK and 16QAM under Block type pilot(Doppler frequency = 40Hz)

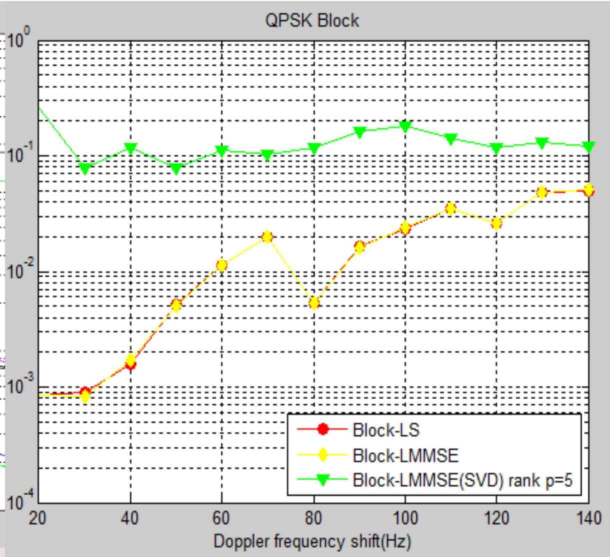

Fig. 7 Comparison of Block type under QPSK modulation(SNR=40db)

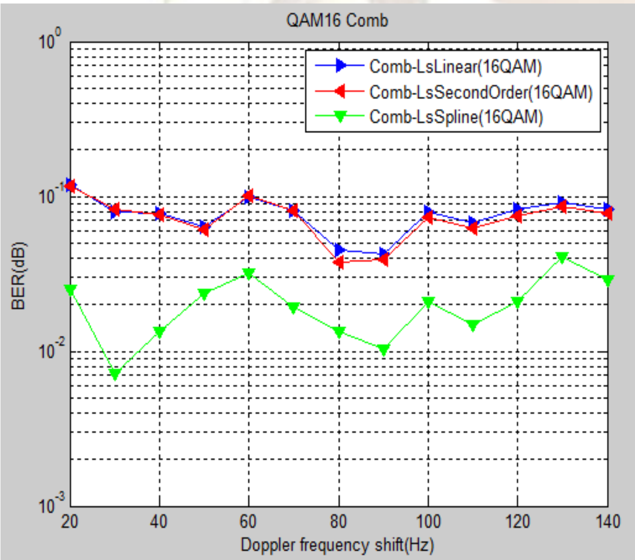

Fig. 6 Comparison of comb type under 16QAM modulation(SNR=40db)

As seen in Fig. 8 the performances of BER for QPSK, we can say that cubic spline interpolation has better results over linear and second order interpolation, but cubic spline interpolation give fluctuating BER.

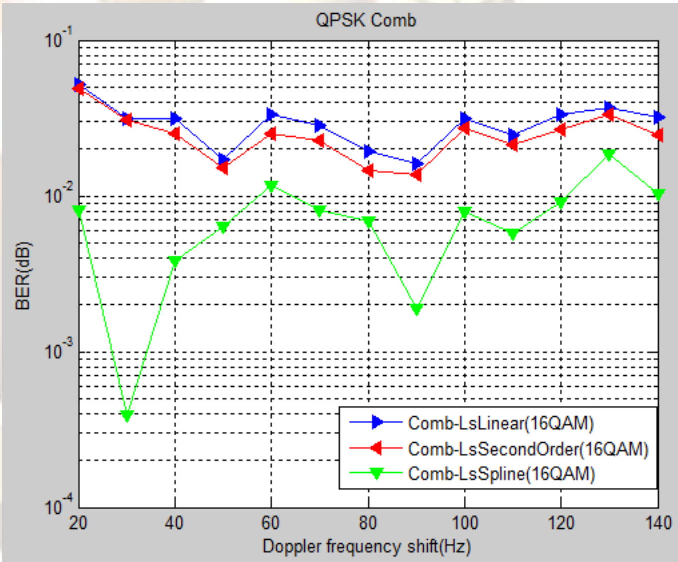

Fig. 8 Comparison of comb type under QPSK modulation(SNR=40db)

From Fig. 9 we see that, when we used interpolation technique in place of block –type estimator we get opposite results from Fig. 7, means for Comb –type pilot estimator LS and LMMSE has worse performances.

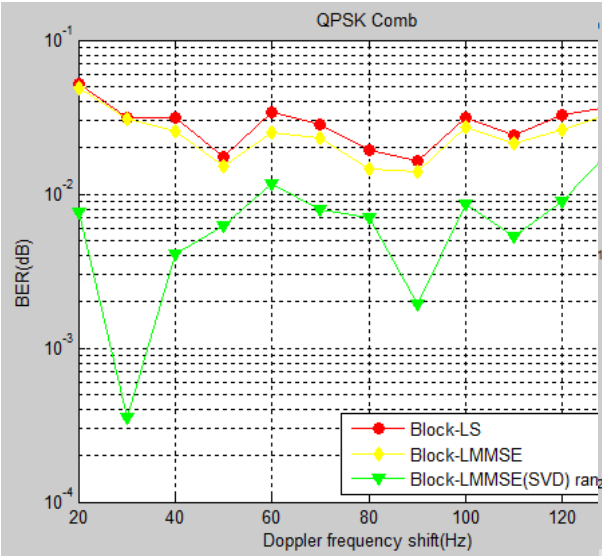

Fig. 9 Comparison of comb type under QPSK modulation(SNR=40db)

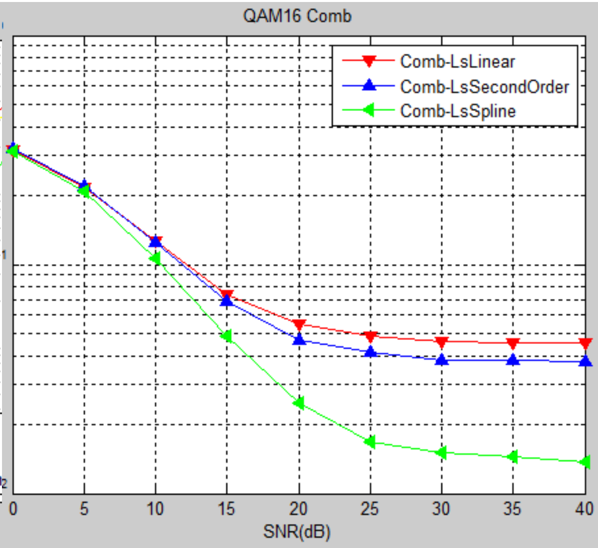

Fig. 11 Comparison of comb type under 16QAM modulation (Doppler frequency=80Hz)

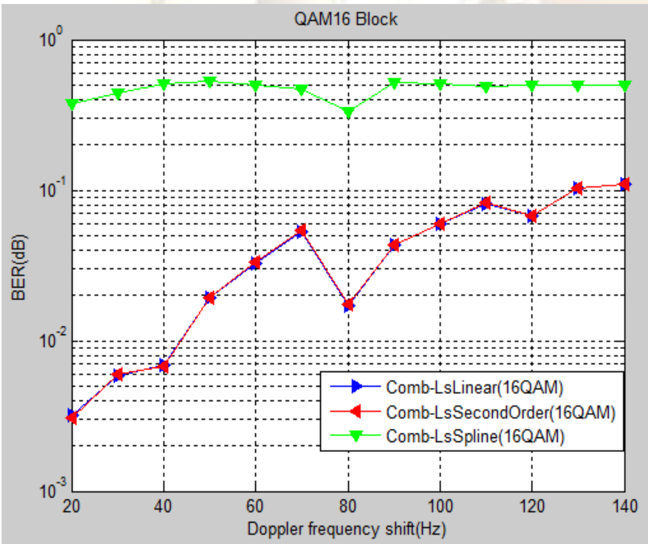

Figure 10 Comparison of Block type under 16QAM modulation(SNR=40db)

From Fig. 10 we see that, when we used Block-type pilot technique in place of Comb –type pilot estimator we get opposite results from Fig. 6, means for block –type pilot estimator cubic spline interpolation has worse performances.

Fig. 11 shows, the comparison between different interpolation method using 16 QAM modulation.

VI. CONCLUSION

Finally in this paper, we conclude that, in presence of Doppler frequency shift the performance of OFDM system degrade. For higher Doppler spread, BER increase, the reason is the existence of severe ICI caused by Doppler shift. The simulation results shows that, Comb-type pilot based channel estimation with cubic spline interpolation for QPSK and16QAM performs the best as compared to linear and second order interpolation. For Block –type pilot based channel estimation with LS and LMMSE estimator gives better performance as compared to LMMSE (SVD) rank p=5.

But when we used Block-type pilot for interpolation estimator, linear and second order interpolation perform better.

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