Priyanka M. Patel, V. H. Pradhan / International Journal of Engineering Research and Applications (IJERA) ISSN: 2248-9622 www.ijera.com
Vol. 3, Issue 1, January -February 2013, pp.169-180

Travelling Wave Solutions of BBM and Modified BBM Equations by Modified F-Expansion Method

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Abstract

A new modified F-expansion method is introduced to obtain the travelling wave solutions like soliton and periodic, of Benjamin-Bona-Mahony (BBM) equation and modified BBM equation. The method is convenient, effective and can be applied to other nonlinear partial differential equations in the mathematical physics.

Keywords : Modified F-expansion method, Benjamin-Bona-Mahony (BBM) equation, modified BBM equation, travelling wave solutions.

1. Introduction

It is well known that nonlinear partial differential equations (NLPDEs) are widely used to describe many important phenomena and dynamic processes in physics, mechanics, chemistry and biology, etc. There are many methods to solve NLPDEs such as tanh-sech method [2], homotopy analysis method [14],sine-cosine method [8], $\left(G'/G\right)$ expansion method [4,9],differential transform method [7], and so on. In this paper, we apply the modified F-expansion method [3,9,10] to the generalized BBM equation

$$u_t + u_x + \alpha u^N u_x + u_{xxx} = 0 \text{ with } N \geq 1 \qquad (1)$$

with constant parameter α. Wazwaz [2] has obtained the analytical solution of eq.(1) with tanh-sech method and sine-cosine method. Abbasbandy [14] has applied homotopy analysis method (HAM) to obtain the analytical solution of generalized BBM equation for $N=1$ and $N=2$. In the present paper we have applied modified F-expansion method to solve eq.(1) for $N=1$ and $N=2$. A set of new solutions are obtained other than the solutions obtained by previous authors.

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2. Travelling wave solutions of BBM equation

For $N=1$, eq.(1) reduces to the BBM equation or the regularized long-wave equation (RLW).

$$u_t + u_x + \alpha u u_x + u_{xxx} = 0 \qquad (2)$$

Consider the travelling wave solutions of eq. (2), under the transformation

$$u(x,t) = u(\xi) \text{ where } \xi = k(x+\omega t) \qquad (3)$$

where $k \neq 0$ and ω are constants that do determined later. By substituting eq. (3) into eq. (2) we obtain

$$k\omega u' + k u' + k\alpha u u' + k^2 u''' = 0$$
$$\omega u' + u' + \alpha u u' + k u''' = 0 \qquad (4)$$

Integrating eq. (4) with respect to 'ξ' and considering the zero constant for integration we have

$$(1+\omega)u + \alpha \frac{u^2}{2} + k u'' = 0 \qquad (5)$$

where prime denotes differentiation with respect to ξ. By balancing the order of u^2 and u'' in eq. (5), we have $2n = n+2$ then $n=2$

So we can see its exact solutions in the form

$$u(\xi) = a_0 + a_{-2} F^{-2}(\xi) + a_{-1} F^{-1}(\xi) + a_1 F(\xi) + a_2 F^2(\xi) \qquad (6)$$

where $a_0, a_{-1}, a_{-2}, a_1, a_2$ are constants to be determined later. And $F(\xi)$ satisfies the following Riccati equation

$$F'(\xi) = A + BF(\xi) + CF^2(\xi) \qquad (7)$$

where A, B, C are constants. Substituting eq. (6) into eq. (5) and using eq. (7), the left-hand side of eq. (5) can be converted into a finite series in $F^p(\xi)$, (p = -4, -3 ,-2, -1, 0, 1, 2, 3, 4). Equating each coefficient of $F^p(\xi)$ to zero yields the following set of algebraic equations.

$$\left.\begin{aligned}
&F^{-4}(\xi): \frac{a_{-2}}{2}(12A^2k + a_{-2}\alpha) \\
&F^{-3}(\xi): 2A^2ka_{-1} + 10ABka_{-2} + a_{-1}a_{-2}\alpha \\
&F^{-2}(\xi): Ak(3Ba_{-1} + 8Ca_{-2}) + \frac{a_{-1}^2\alpha}{2} + a_{-2}(1 + 4B^2k + \omega + a_0\alpha) \\
&F^{-1}(\xi): a_{-1}(1 + B^2k + 2ACk + \omega + a_0\alpha) + a_{-2}(6BCk + a_1\alpha) \\
&F^{0}(\xi): a_0 + BCka_{-1} + 2C^2ka_{-2} + ABka_1 + 2A^2ka_2 + a_0\omega + \frac{a_0^2\alpha}{2} + a_{-1}a_1\alpha + a_{-2}a_2\alpha \\
&F^{1}(\xi): a_1(1 + B^2k + 2ACk + \omega + a_0\alpha) + a_2(6ABk + a_{-1}\alpha) \\
&F^{2}(\xi): 3BCka_1 + 4B^2ka_2 + \frac{a_1^2\alpha}{2} + a_1(1 + 8ACk + \omega + a_0\alpha) \\
&F^{3}(\xi): 2C^2ka_1 + 10BCka_2 + a_1a_2\alpha \\
&F^{4}(\xi): \frac{1}{2}a_2(12C^2k + a_2\alpha)
\end{aligned}\right\} \quad (8)$$

Solving the above algebraic equations by symbolic computations , we have the following solutions:

Case 1: $A = 0$, we have

$$a_0 = a_0,\ a_{-1} = 0,\ a_{-2} = 0,\ a_1 = \frac{-12BCk}{\alpha},\ a_2 = \frac{-12C^2k}{\alpha},\ \omega = -1 - B^2k - a_0\alpha \quad (9)$$

Case 2: $B = 0$, we have

$$\left.\begin{aligned}
&a_0 = a_0, a_{-1} = 0, a_{-2} = 0, a_1 = 0, a_2 = \frac{-12C^2k}{\alpha}, \omega = -1 - 8ACk - a_0\alpha \\
&a_0 = a_0, a_{-1} = 0, a_{-2} = \frac{-12A^2k}{\alpha}, a_1 = 0, a_2 = \frac{-12C^2k}{\alpha}, \omega = -1 - 8ACk - a_0\alpha
\end{aligned}\right\} \quad (10)$$

So we can list the solutions of u as follows:

When $A = 0, B = 1, C = -1$, from appendix and case 1, we have

$$u_1(\xi) = a_0 + \frac{3k}{\alpha}\sec h^2\left(\frac{\xi}{2}\right) \quad (11)$$

where $\xi = k(x - (1 + k + a_0\alpha)t)$

When $A = 0, B = -1, C = 1$, from appendix and case 1, we have

$$u_2(\xi) = a_0 - \frac{3k}{\alpha}\csc h^2\left(\frac{\xi}{2}\right) \quad (12)$$

where $\xi = k(x - (1 + k + a_0\alpha)t)$

When $A = \frac{1}{2}, B = 0, C = -\frac{1}{2}$, from appendix and case 2, we have

$$u_3(\xi) = a_0 - \frac{3k}{\alpha}\left[\coth(\xi) \pm \csc h(\xi)\right]^2 \quad (13)$$

$$u_4(\xi) = a_0 - \frac{3k}{\alpha}\left[\coth(\xi) \pm \csc h(\xi)\right]^{-2} - \frac{3k}{\alpha}\left[\coth(\xi) \pm \csc h(\xi)\right]^2 \quad (14)$$

$$u_5(\xi) = a_0 - \frac{3k}{\alpha}\left[\tanh(\xi) \pm i\sec h(\xi)\right]^2 \quad (15)$$

$$u_6(\xi) = a_0 - \frac{3k}{\alpha}\left[\tanh(\xi) \pm i\operatorname{sec}h(\xi)\right]^{-2} - \frac{3k}{\alpha}\left[\tanh(\xi) \pm i\operatorname{sec}h(\xi)\right]^{2} \quad (16)$$

where $\xi = k\left(x - (1 - 2k + a_0\alpha)t\right)$

When $A = 1, B = 0, C = -1$, from appendix and case 2, we have

$$u_7(\xi) = a_0 - \frac{12k}{\alpha}\tanh^2(\xi) \quad (17)$$

$$u_8(\xi) = a_0 - \frac{12k}{\alpha}\tanh^{-2}(\xi) - \frac{12k}{\alpha}\tanh^2(\xi) \quad (18)$$

$$u_9(\xi) = a_0 - \frac{12k}{\alpha}\coth^2(\xi) \quad (19)$$

$$u_{10}(\xi) = a_0 - \frac{12k}{\alpha}\coth^2(\xi) - \frac{12k}{\alpha}\coth^{-2}(\xi) \quad (20)$$

where $\xi = k(x - (1 - 8k + a_0\alpha)t)$

When $A = \frac{1}{2}, B = 0, C = \frac{1}{2}$, from appendix and case 2, we have

$$u_{11}(\xi) = a_0 - \frac{3k}{\alpha}\left[\sec(\xi) + \tan(\xi)\right]^2 \quad (21)$$

$$u_{12}(\xi) = a_0 - \frac{3k}{\alpha}\left[\sec(\xi) + \tan(\xi)\right]^{-2} - \frac{3k}{\alpha}\left[\sec(\xi) + \tan(\xi)\right]^2 \quad (22)$$

$$u_{13}(\xi) = a_0 - \frac{3k}{\alpha}\left[\csc(\xi) - \cot(\xi)\right]^2 \quad (23)$$

$$u_{14}(\xi) = a_0 - \frac{3k}{\alpha}\left[\csc(\xi) - \cot(\xi)\right]^{-2} - \frac{3k}{\alpha}\left[\csc(\xi) - \cot(\xi)\right]^2 \quad (24)$$

where $\xi = k\left(x - (1 + 2k + a_0\alpha)t\right)$

When $A = -\frac{1}{2}, B = 0, C = -\frac{1}{2}$, from appendix and case 2, we have

$$u_{15}(\xi) = a_0 - \frac{3k}{\alpha}\left[\sec(\xi) - \tan(\xi)\right]^2 \quad (25)$$

$$u_{16}(\xi) = a_0 - \frac{3k}{\alpha}\left[\sec(\xi) - \tan(\xi)\right]^{-2} - \frac{3k}{\alpha}\left[\sec(\xi) - \tan(\xi)\right]^2 \quad (26)$$

$$u_{17}(\xi) = a_0 - \frac{3k}{\alpha}\left[\csc(\xi) + \cot(\xi)\right]^2 \quad (27)$$

$$u_{18}(\xi) = a_0 - \frac{3k}{\alpha}\left[\csc(\xi) + \cot(\xi)\right]^{-2} - \frac{3k}{\alpha}\left[\csc(\xi) + \cot(\xi)\right]^2 \quad (28)$$

where $\xi = k\left(x - (1 + 2k + a_0\alpha)t\right)$

When $A = 1(-1), B = 0, C = 1(-1)$, from appendix and case 2, we have

$$u_{19}(\xi) = a_0 - \frac{12k}{\alpha}\tan^2(\xi) \quad (29)$$

$$u_{20}(\xi) = a_0 - \frac{12k}{\alpha}\tan^{-2}(\xi) - \frac{12k}{\alpha}\tan^2(\xi) \quad (30)$$

$$u_{21}(\xi) = a_0 - \frac{12k}{\alpha}\cot^2(\xi) \quad (31)$$

$$u_{22}(\xi) = a_0 - \frac{12k}{\alpha}\cot^{-2}(\xi) - \frac{12k}{\alpha}\cot^{2}(\xi) \quad (32)$$

where $\xi = k\left(x - (1 + 8k + a_0\alpha)t\right)$

2.1 Discussion

Travelling wave solutions from eq.(11) to eq.(20) represents the soliton solutions and from eq. (21) to eq.(32) represents the periodic solutions.

We discuss one travelling wave solution given by eq. (11). The travelling wave solution (11) is a solitary like solution which does not change its shape and propagates at constant speed. It is a stable solution. The first observation of this kind of wave was made in 1834 by John Scott Russell [5]. If we draw the solution with a constant speed, we will get the following figure (A.1) which shows the right moving wave at different values of t.

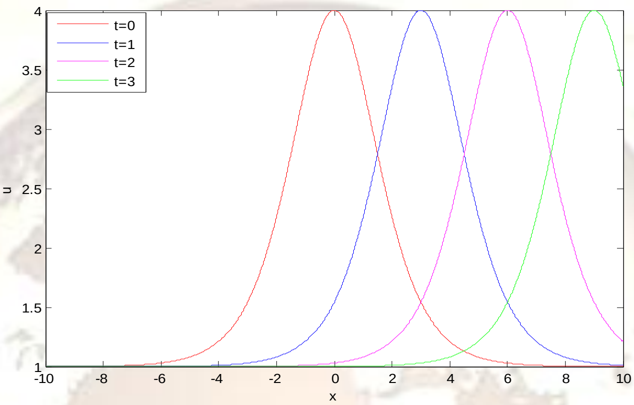
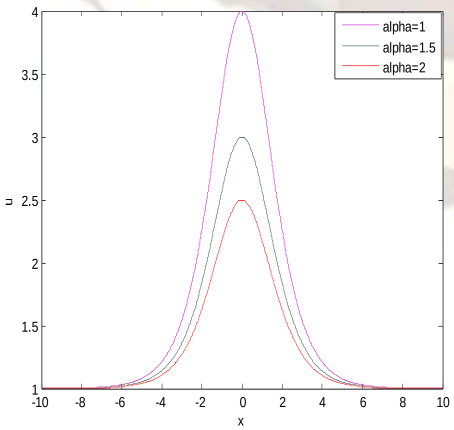


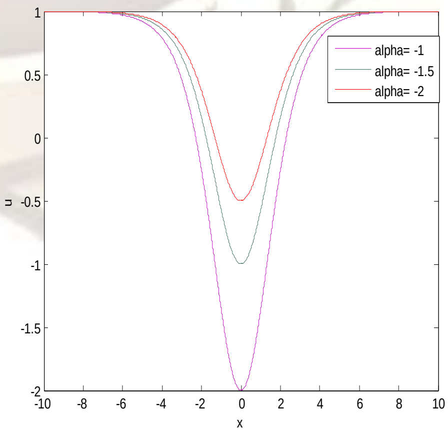
Fig. A.1. Plot of travelling wave solution eq. (11) with $k = \alpha = a_0 = 1$ and $t = 0,1,2,3$

If we draw the travelling wave solution (11) with different positive values of α ($\alpha = 1, 1.5, 2$), we will get the following figure (A.2-I) which shows the value of positive amplitude in decreasing. In other words the value of positive amplitude tends to zero. From this physically we can say that the fluid becomes more spread out. If we take negative values of α ($\alpha = -1, -1.5, -2$), we will get the following figure (A.2-II) which shows the value of negative amplitude. Here negative amplitude means the wave is going to look upside down. From figures (A.2-I) and (A.2-II) we can say that positive and negative amplitudes are the same

.



(I)



(II)

Fig. A.2. Different solitary waves profiles given by eq. (11) at $t = 0$, $a_0 = k = 1$ for (I) $\alpha = 1, 1.5, 2$ and (II) $\alpha = -1, -1.5, -2$

If we draw the travelling wave solution (11) with different values of k ($k=1,2,3$), we will get the following figure (A.3) which describes the value of amplitude in increasing. The amplitude is proportional to the velocity, which means that larger pulse travel faster than smaller ones.

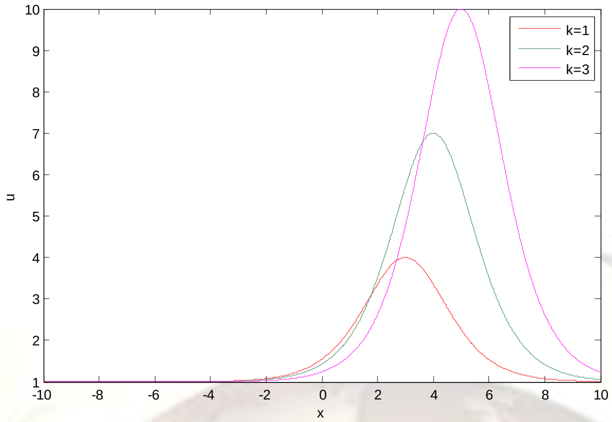


Fig. A.3. Different solitary waves profiles given by eq. (11) at $t=1, a_0=\alpha=1$ for $k=1,2,3$

The other solitary and periodic solutions are graphically shown in fig.(A.4) to fig.(A.10).

Solitary solutions

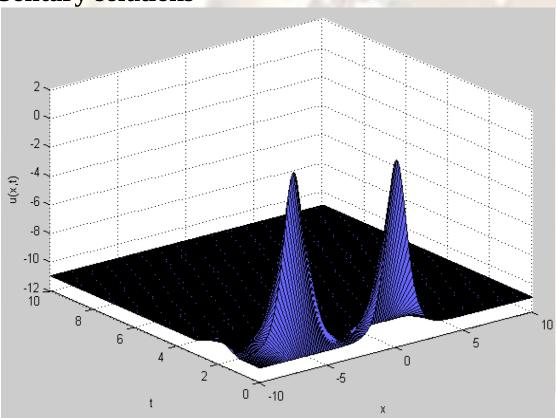


Fig.A.4

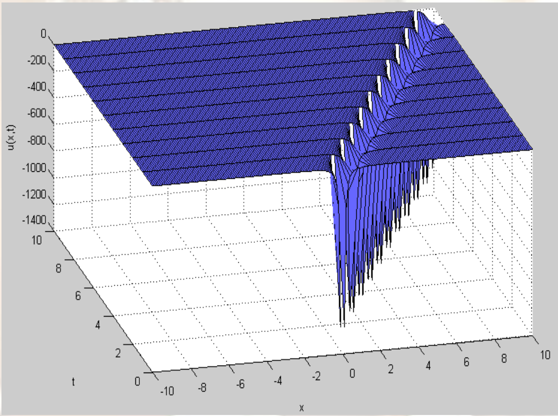


Fig.A.5

Fig.A.4. Travelling wave solution given by eq. (17) with values $a_0=\alpha=k=1$ and Fig.A.5. Travelling wave solution given by eq.(14) with values $a_0=2$ and $\alpha=k=1$

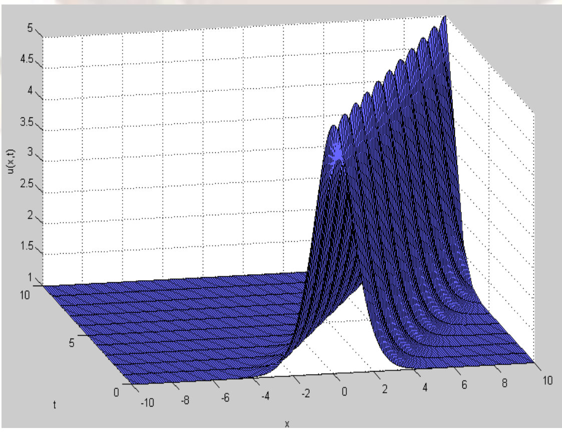
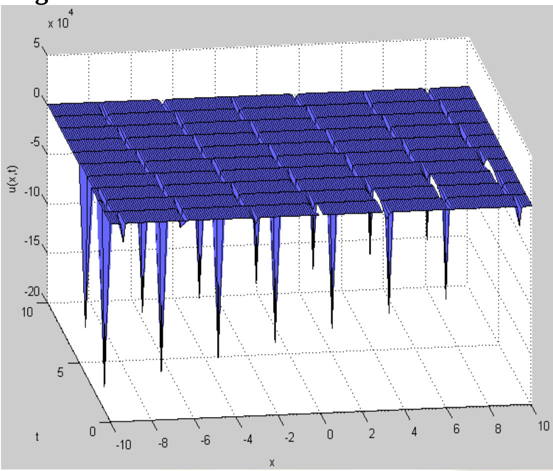


Fig.A.6

Fig.A.6 Travelling wave solution given by absolute of eq. (15) with values $a_0=2$ and $\alpha=k=1$

Trigonometric solutions :



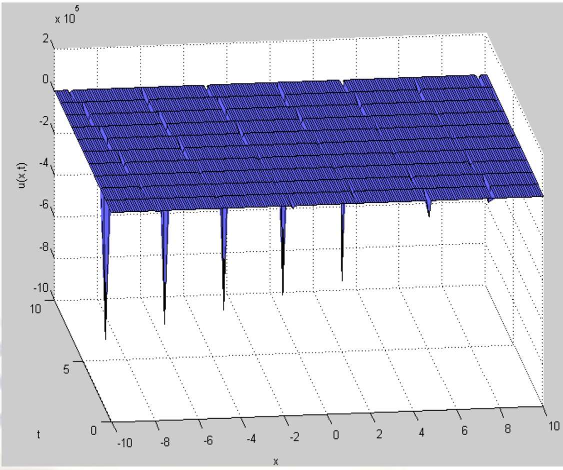
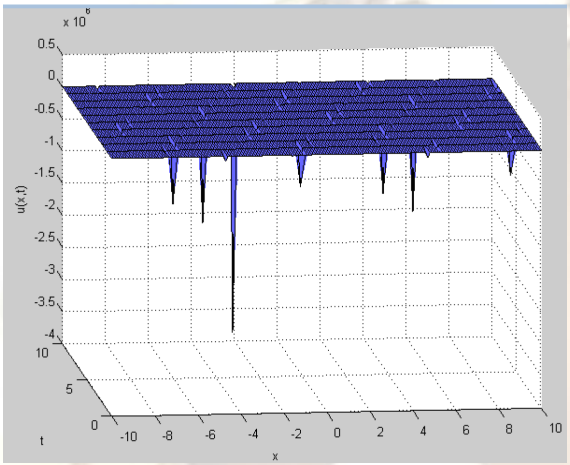


Fig.A.7 Fig.A.8

Fig.A.7. Travelling wave solution given by eq. (23) and Fig.A.8 travelling wave solution given by eq.(25) with values $a_0 = \alpha = k = 1$



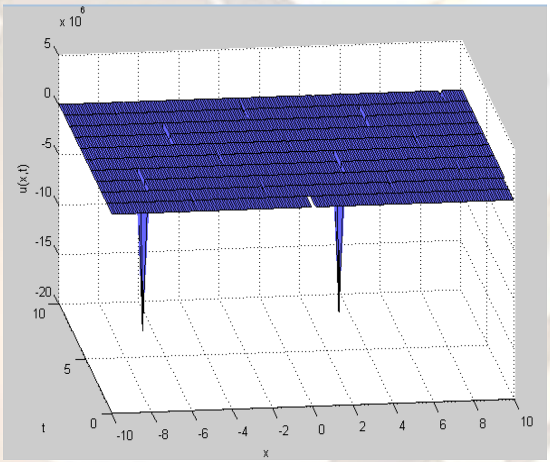


Fig.A.9 Fig.A.10

Fig.A.9. Travelling wave solution given by eq. (29) and Fig.A.10. Travelling wave solution given by eq.(31) with values $a_0 = \alpha = k = 1$

3. Travelling wave solutions of modified BBM equation

For $N = 2$, eq.(1) reduces to the modified BBM equation.

$$u_t + u_x + \alpha u^2 u_x + u_{xxx} = 0 \tag{33}$$

Now, we construct the travelling wave solutions of eq. (33), under the transformation

$$v(x,t) = v(\xi) \text{ where } \xi = k(x + \omega t) \tag{34}$$

where $k \neq 0$ and ω are constants. Substituting eq. (34) into the eq. (33), we get

$$k\omega v' + k v' + \alpha k v^2 v' + k^3 v''' = 0 \tag{35}$$

Integrating eq. (35) with respect to 'ξ' and considering the zero constant for integration we have

$$(1+\omega)v + \frac{\alpha}{3}v^3 + k^2 v'' = 0 \tag{36}$$

where prime denotes differentiation with respect to ξ. By balancing the order of v^3 and v'' in eq. (36), we have $3m = m + 2$ then $m = 1$. So we can write

$$v(\xi) = a_0 + a_{-1}F^{-1}(\xi) + a_1 F(\xi) \tag{37}$$

where a_0, a_{-1}, a_1 are constants to be determined later. And $F(\xi)$ satisfies Riccati equation given by eq. (7). Substituting eq. (37) into eq. (36) and using eq. (7), the left-hand side of eq. (36) can be converted into a finite series in $F^{p}(\xi)$, ($p=$ -3 ,-2,-1, 0, 1, 2, 3). Equating each coefficient of $F^{p}(\xi)$ to zero yields the following set of algebraic equations.

$$\left.\begin{aligned}
&F^{-3}(\xi): 2A^2k^2a_{-1}+\frac{a_{-1}{}^3\alpha}{3}\\
&F^{-2}(\xi): 3ABk^2a_{-1}+a_0a_{-1}{}^2\alpha\\
&F^{-1}(\xi): a_{-1}+B^2k^2a_{-1}+2ACk^2a_{-1}+a_{-1}\omega+a_0{}^2a_{-1}\alpha+a_{-1}{}^2a_1\alpha\\
&F^{0}(\xi): a_0+BCk^2a_{-1}+ABk^2a_1+a_0\omega+\frac{a_0{}^3\alpha}{3}+2a_0a_{-1}a_1\alpha\\
&F^{1}(\xi): a_1+B^2k^2a_1+2ACk^2a_1+a_1\omega+a_0{}^2a_1\alpha+a_{-1}a_1{}^2\alpha\\
&F^{2}(\xi): 3BCk^2a_1+a_0a_1{}^2\alpha\\
&F^{3}(\xi): 2C^2k^2a_1+\frac{a_1{}^3\alpha}{3}
\end{aligned}\right\} \tag{38}$$

Solving the algebraic equations above by symbolic computations, we have the following solutions:

Case 3: $A=0$, we have

$$a_0=\pm i\,Bk\sqrt{\frac{3}{2\alpha}},\ a_{-1}=0,\ a_1=\pm i\,Ck\sqrt{\frac{6}{\alpha}},\ \omega=-1+\frac{B^2k^2}{2} \tag{39}$$

Case 4: $B=0$, we have

$$\left.\begin{aligned}
&a_0=0, a_{-1}=0,\ a_1=\pm i\,Ck\sqrt{\frac{6}{\alpha}},\ \omega=-1-2ACk^2\\
&a_0=0, a_{-1}=\pm i\,Ak\sqrt{\frac{6}{\alpha}},\ a_1=\pm i\,Ck\sqrt{\frac{6}{\alpha}}, \omega=-1+4ACk^2
\end{aligned}\right\} \tag{40}$$

So we can list the solutions of v as follows:

When $A=0, B=1, C=-1$, from appendix and case 3, we have

$$v_1(\xi)=\mp i\sqrt{\frac{3}{2\alpha}}\,k\tanh\left(\frac{\xi}{2}\right) \tag{41}$$

where $\xi=k\left(x+\left(-1+\frac{k^2}{2}\right)t\right)$

When $A=0, B=-1, C=1$, from appendix and case 3, we have

$$v_2(\xi)=\mp i\,k\sqrt{\frac{3}{2\alpha}}\coth\left(\frac{\xi}{2}\right) \tag{42}$$

where $\xi=k\left(x+\left(-1+\frac{k^2}{2}\right)t\right)$

When $A=\frac{1}{2}, B=0, C=-\frac{1}{2}$, from appendix and case 4, we have

$$v_3(\xi)=\mp\frac{ik}{2}\sqrt{\frac{6}{\alpha}}\left[\coth(\xi)\pm\operatorname{csc}h(\xi)\right] \tag{43}$$

$$v_4(\xi)=\mp\frac{ik}{2}\sqrt{\frac{6}{\alpha}}\left[\tanh(\xi)\pm i\operatorname{sec}h(\xi)\right] \tag{44}$$

where $\xi = k\left(x-\left(1-\frac{k^2}{2}\right)t\right)$

$$v_5(\xi) = \pm\frac{ik}{2}\sqrt{\frac{6}{\alpha}}\left[\coth(\xi)\pm\csc h(\xi)\right]^{-1} \mp \frac{ik}{2}\sqrt{\frac{6}{\alpha}}\left[\coth(\xi)\pm\csc h(\xi)\right] \quad (45)$$

$$v_6(\xi) = \pm\frac{ik}{2}\sqrt{\frac{6}{\alpha}}\left[\tanh(\xi)\pm i\sec h(\xi)\right]^{-1} \mp \frac{ik}{2}\sqrt{\frac{6}{\alpha}}\left[\tanh(\xi)\pm i\sec h(\xi)\right] \quad 46)$$

where $\xi = k\left(x-\left(1+k^2\right)t\right)$

When $A=1, B=0, C=-1$, from appendix and case 4, we have

$$v_7(\xi) = \mp ik\sqrt{\frac{6}{\alpha}}\tanh(\xi) \quad (47)$$

$$v_8(\xi) = \mp ik\sqrt{\frac{6}{\alpha}}\coth(\xi) \quad (48)$$

where $\xi = k\left(x-\left(1-2k^2\right)t\right)$

$$v_9(\xi) = \pm ik\sqrt{\frac{6}{\alpha}}\tanh^{-1}(\xi) \mp ik\sqrt{\frac{6}{\alpha}}\tanh(\xi) \quad (49)$$

$$v_{10}(\xi) = \pm ik\sqrt{\frac{6}{\alpha}}\coth^{-1}(\xi) \mp ik\sqrt{\frac{6}{\alpha}}\coth(\xi) \quad (50)$$

where $\xi = k\left(x-\left(1+4k^2\right)t\right)$

When $A=\frac{1}{2}, B=0, C=\frac{1}{2}$, from appendix and case 4, we have

$$v_{11}(\xi) = \pm\frac{ik}{2}\sqrt{\frac{6}{\alpha}}\left[\sec(\xi)+\tan(\xi)\right] \quad (51)$$

$$v_{12}(\xi) = \pm\frac{ik}{2}\sqrt{\frac{6}{\alpha}}\left[\csc(\xi)-\cot(\xi)\right] \quad (52)$$

where $\xi = k\left(x-\left(1+\frac{1}{2}k^2\right)t\right)$

$$v_{13}(\xi) = \pm\frac{ik}{2}\sqrt{\frac{6}{\alpha}}\left[\sec(\xi)+\tan(\xi)\right]^{-1} \pm \frac{ik}{2}\sqrt{\frac{6}{\alpha}}\left[\sec(\xi)+\tan(\xi)\right] \quad (53)$$

$$v_{14}(\xi) = \pm\frac{ik}{2}\sqrt{\frac{6}{\alpha}}\left[\csc(\xi)-\cot(\xi)\right]^{-1} \pm \frac{ik}{2}\sqrt{\frac{6}{\alpha}}\left[\csc(\xi)-\cot(\xi)\right] \quad (54)$$

where $\xi = k\left(x+\left(-1+k^2\right)t\right)$

When $A=-\frac{1}{2}, B=0, C=-\frac{1}{2}$, from appendix and case 4, we have

$$v_{15}(\xi) = \mp\frac{ik}{2}\sqrt{\frac{6}{\alpha}}\left[\sec(\xi)-\tan(\xi)\right] \quad (55)$$

$$v_{16}(\xi) = \mp\frac{ik}{2}\sqrt{\frac{6}{\alpha}}\left[\csc(\xi)+\cot(\xi)\right] \quad (56)$$

where $\xi = k\left(x-\left(1+\frac{k^2}{2}\right)t\right)$

$$v_{17}(\xi) = \mp\frac{ik}{2}\sqrt{\frac{6}{\alpha}}\left[\sec(\xi)-\tan(\xi)\right]^{-1} \mp\frac{ik}{2}\sqrt{\frac{6}{\alpha}}\left[\sec(\xi)-\tan(\xi)\right] \quad (57)$$

$$v_{18}(\xi) = \mp\frac{ik}{2}\sqrt{\frac{6}{\alpha}}\left[\csc(\xi)+\cot(\xi)\right]^{-1} \mp\frac{ik}{2}\sqrt{\frac{6}{\alpha}}\left[\csc(\xi)+\cot(\xi)\right] \quad (58)$$

where $\xi = k\left(x+\left(-1+k^2\right)t\right)$

When $A=1(-1), B=0, C=1(-1)$, from appendix and case 4, we have

$$v_{19}(\xi) = \pm(\mp)ik\sqrt{\frac{6}{\alpha}}\tan(\xi) \quad (59)$$

$$v_{20}(\xi) = \pm(\mp)ik\sqrt{\frac{6}{\alpha}}\cot(\xi) \quad (60)$$

where $\xi = k\left(x-\left(1+2k^2\right)t\right)$

$$v_{21}(\xi) = \pm(\mp)ik\sqrt{\frac{6}{\alpha}}\tan^{-1}(\xi)\pm(\mp)ik\sqrt{\frac{6}{\alpha}}\tan(\xi) \quad (61)$$

$$v_{22}(\xi) = \pm(\mp)ik\sqrt{\frac{6}{\alpha}}\cot^{-1}(\xi)\pm(\mp)ik\sqrt{\frac{6}{\alpha}}\cot(\xi) \quad (62)$$

where $\xi = k\left(x+\left(-1+4k^2\right)t\right)$

The travelling wave solutions given by eq.(41) to eq.(50) represents soliton-like solutions and eq.(51) to eq.(62) represents periodic solutions. Some absolute of solitary and periodic solutions are shown in fig.(B.1) to fig.(B.6).

Solitary solutions:

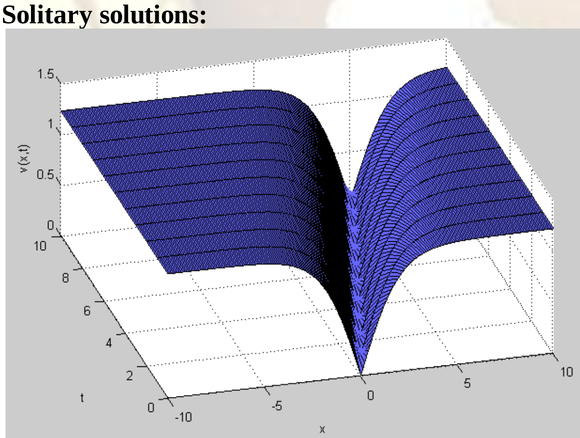


Fig.B.1

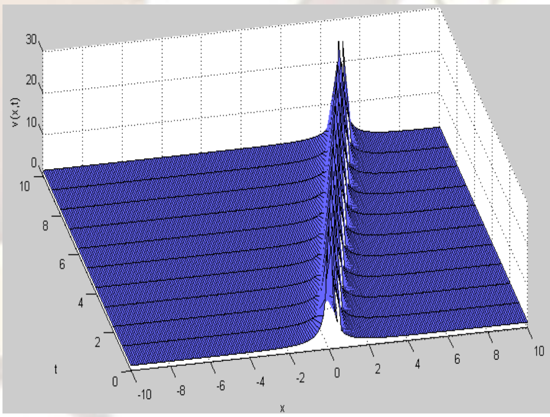


Fig.B.2

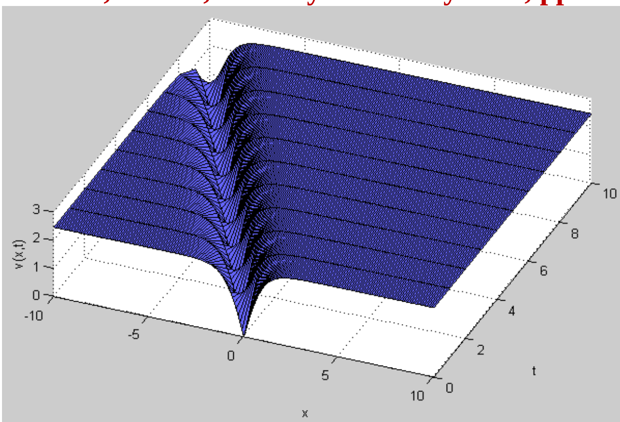


Fig.B.3

Fig.B.1 Travelling wave solution given by eq.(41), Fig.B.2 Travelling wave solution given by eq.(42) and Fig.B.3 Travelling wave solution given by eq.(47) with values $\alpha = k = 1$

Trigonometric solutions:

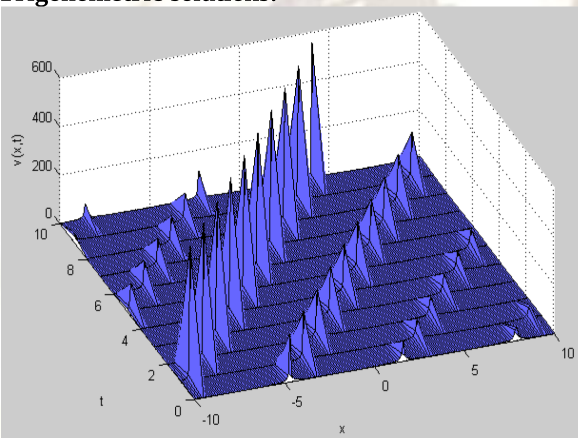


Fig.B.4

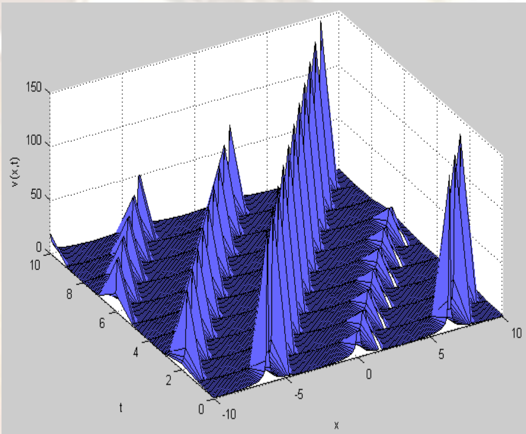


Fig.B.5

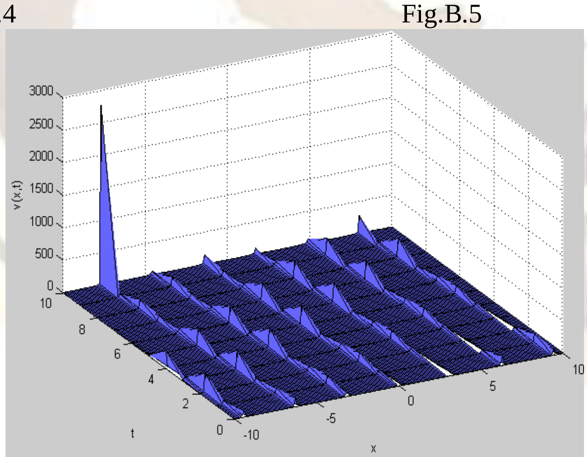


Fig.B.6

Fig.B.4 Travelling wave solution given by eq. (51), Fig.B.5 Travelling solution given by eq.(56) and Fig.B.6 Travelling wave solution given by eq.(60) with values $\alpha = k = 1$

4. Conclusion

In this paper, we have used a modified F-expansion method to construct more exact solutions of the nonlinear BBM and modified BBM equations with the aid of Mathematica. We have shown that this method is direct, concise, effective and can be applied to other NLPDEs in the mathematical physics. All the obtained solutions satisfies BBM and modified BBM equations.

Appendix A

Relations between values of (A, B, C) and corresponding $F(\xi)$ in Riccati equation $F'(\xi) = A + BF(\xi) + CF^2(\xi)$

A	B	C	$F(\xi)$
0	1	-1	$\frac{1}{2}+\frac{1}{2}\tanh\left(\frac{\xi}{2}\right)$
0	-1	1	$\frac{1}{2}-\frac{1}{2}\coth\left(\frac{\xi}{2}\right)$
$\frac{1}{2}$	0	$-\frac{1}{2}$	$\coth(\xi) \pm \csc h(\xi), \tanh(\xi) \pm i \sec h(\xi)$
1	0	-1	$\tanh(\xi), \coth(\xi)$
$\frac{1}{2}$	0	$\frac{1}{2}$	$\sec(\xi)+\tan(\xi), \csc(\xi)-\cot(\xi)$
$-\frac{1}{2}$	0	$-\frac{1}{2}$	$\sec(\xi)-\tan(\xi), \csc(\xi)+\cot(\xi)$
$1(-1)$	0	$1(-1)$	$\tan(\xi), \cot(\xi)$
0	0	$\neq 0$	$-\frac{1}{C\xi+m}$ (m is an arbitrary constant)
arbitrary constant	0	0	$A\xi$
arbitrary constant	$\neq 0$	0	$\frac{\exp(B)-A}{B}$

References

[1]. A.I.Volpert, Vitaly A. Volpert, Vladimir A. Volpert, "Travelling wave solutions of parabolic systems", American Mathematical Society, (1993).

[2]. A.Wazwaz, "New travelling wave solutions of different physical structures to generalized BBM equation", Elsevier, (2006), pp.358-362.

[3]. G.Cai, Q.Wang, "A modified F-expansion method for solving nonlinear PDEs", Journal of Information and Computing Science", (2007), pp.03-16.

[4]. J.Manafianheris, "Exact solutions of the BBM and MBBM equations by the generalized (G'/G)-expansion method", International Journal of Genetic Engineering, (2012), pp.28-32.

[5]. J-H. He, "Soliton perturbation",Selected works of J-H.He, (2009), pp. 8453-8457.

[6]. M.D.Abdur Rab, A.S.Mia, T.Akter, "Some travelling wave solutions of Kdv-Burger equation", International Journal of Mathematical Analysis, (2012), pp.1053-1060.

[7]. M.T.Alquran, "Applying differential transform method to nonlinear partial differential equations: A modified approach", Applications and Applied Mathematics:An International Journal, (2012), pp.155-163.

[8]. M.T.Alquran, "Solitons and periodic solutions to nonlinear partial differential equations by the Sine-Cosine method", Applied Mathematics & Information

Sciences: An International Journal, (2012), pp.85-88.

[9]. M.T.Darvishi, Maliheh Najafi, Mohammad Najafi, "Travelling wave solutions for the (3+1)-dimensional breaking soliton equation by (G'/G)-expansion method and modified F-expansion method", International Journal of Computational and Mathematical Sciences 6:2, (2012), pp.64-69.

[10]. M.T.Darvishi, Maliheh Najafi, Mohammad Najafi, "Travelling wave solutions for foam drainage equation by modified F-expansion method", Food and Public health, (2012), pp. 6-10.

[11]. M.Wadati, "Introduction to solitons", Pramana Journal of Physics, (2001), pp.841-847.

[12]. N.Taghizadeh, M.Mirzazadeh, "The modified extended tanh method with the Riccati equation for solving nonlinear partial differential equations", Mathematica Aeterna, vol.2, (2012), pp.145-153.

[13]. P.G.Estevez, S.Kuru, J.Negro and L.M.Nieto, "Travelling wave solutions of the generalized Benjamin-Bona-Mahony equation", Cornell University Library, (2007), pp.1-19.

[14]. S.Abbasbandy, "Homotopy analysis method for generalized Benjamin-Bona-Mahony equation", ZAMP, (2008), pp.51-62.