A novel particle swarm Optimization algorithm based Fine Adjustment for solution of VRP

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Abstract

To solve the problem of easily trapped into local optimization and instable calculation results, a new fine-adjustment mechanism-based particle swarm optimized algorithm applicable to solution seeking of VRP model is presented in this paper. This algorithm introduces the fine-adjustment mechanism so as to get adapt to the judgment base of function directional derivative value. By adjusting the optimal value and group value, the local searching ability of algorithm in the optimal area is improved. The experiment results indicate that the algorithm presented here displays higher convergence speed, precision and stability than PSO, and is a effective solution to VRP.

Key words: particle swarm, fine adjustment, VRP, perturbation, convergence

1. Introduction

Vehicle routing problem [1] was firstly put forward by Dantzing and Ramser in 1959.VRP can be described as [2]: for a series of loading and unloading places, to plan proper vehicle route will facilitate smooth pass of vehicle and certain optimal goals can be reached by satisfying specific constraints. Generally the constraint include: goods demand, quantity, delivery time, delivery vehicles, time constraint etc., while the optimal goals may consist of shortest distance, lowest cost, the least time and so on. VRP is a very important content [3] in logistics distribution researches. A good VRP algorithm and strategy can bring huge benefit to logistics distribution.

Particle swarm optimization algorithm (PSO) [4] is an intelligent optimization algorithm put forward by

American social psychologist Kennedy is and electrical engineer Eberhart in 1995. It is sourced from group behavior theory and enlightened by the fact that bird group and fish group will develop toward a correct and proper direction as anticipated through the special information delivery among individuals. It is a self-adaption random optimization algorithm based on population searching. Due to its simplicity, convenience in actualization and high calculation speed, it has been widely applied in route planning, multi-target tracking, data classification, flow planning and decision support etc.. However, PSO also has some defects. If the initial state forces the population to converge toward the local optimal area, then the population may fail to jump out of the local area and thus be trapped, making application of PSO in VRP solution become difficult.

In this paper, a fine adjustment is introduced to PSO, acquiring FT-PSO which will conduce to enhancement of local searching of population in the optimal area at the end of searching. This method, taking the optimal value of particles as the center, forms a fine-adjustment area based on the mode defined by the algorithm. In the fine adjustment area, a fine adjustment particle swarm will form to calculate the fitness function of each fine adjustment particle. The particle with the largest function value is selected to replace the optimal value of updated particles, thus the precision of the algorithm is enhanced.

2. Establishment of VRP model

The problem of vehicle route is decried as follows: the distribution center is required to deliver goods to l clients $(1,2,\cdots,l)$, and the cargo volume of the i th client is $g_i(i=1,2,\cdots l)$, and the load weight of each vehicle is q, at the same time $g_i < q$. Here we need to find the shortest route which well satisfies the transport requirements.

In the mathematical model established

here, c_{ij} denotes the transportation cost from point i to point j, including the distance, time and expenses etc.. The distribution center is number as 0, clients numbered $i(i=1,2,\cdots,l)$, vehicle numbered k, at most m vehicles can be used. After finishing the transportation, the vehicle will return back to the distribution center. The variables are defining as follows:

$$x_{ijk}=\begin{cases}1 & \text{vehicle } k \text{ drive to } j \text{ from } i \\ 0 & \text{otherwise}\end{cases} \quad (1)$$

$$y_{ik}=\begin{cases}1 & \text{Cargo transportation task of } i \text{ to be finished by } k \\ 0 & \text{otherwise}\end{cases} \quad (2)$$

The vehicle route problem is dealing with the integer planning with constraints, that is, the sum of the clients' task on each vehicle route can not exceed the capacity of this vehicle.
The total minimal transportation cost after finishing the service is:

$$\min Z=\sum_{i=0}^{l}\sum_{j=0}^{l}\sum_{k=1}^{m} c_{ij}x_{ijk}+$$
$$R\sum_{k=1}^{m}\max(\sum_{i=1}^{l} g_i y_{ik}-q,0) \quad (3)$$

where R takes a very large positive number as the punishment coefficient.
Supposing all the vehicles used satisfying the load requirements, then we have the following expression:

$$\sum_{i=1}^{l} g_i y_{ik}\le q,\quad k=1,2,\cdots,m \quad (4)$$

Giving that only one vehicle is assigned to serve each customer only once, that is,

$$\sum_{k=1}^{m} y_{ik}=\begin{cases}1 & i=1,2,\cdots,l \\ m & i=0\end{cases} \quad (5)$$

In the model the vehicle reaching and leaving each client are the same, which is expressed as follows:

$$\sum_{i=0}^{l} x_{ijk}=y_{kj},\quad j=0,1,2,\cdots l;\ \forall k \quad (6)$$

$$\sum_{j=0}^{l} x_{ijk}=y_{ki},\quad i=0,1,2,\cdots l;\ \forall k \quad (7)$$

Given the variable value constraint as 0 or 1, and the expression is:

$$x_{ijk}=0 \text{ or } 1,\quad i,j=0,1,\cdots,l;\quad k=1,2,\cdots,m \quad (8)$$

$$y_{ik}=0 \text{ or } 1,\quad i=0,1,\cdots,l;\quad k=1,2,\cdots,m \quad (9)$$

3. Particle swarm algorithm

Particle swarm algorithm [5] essentially is enlightened by the modeling and simulation research results of many bird populations, wherein the modeling and simulation algorithm mainly utilizes the mode of biologist Hepper. It is an optimal algorithm based on group intelligent theory that will generate group intelligent to guide the optimal searching through cooperation and competition of particles, and has been widely applied [6] now. PSO algorithm can be expressed as: initializing a particle swarm with quantity of m, wherein the number of iteration n, and the position of the i th particle $X_i=(x_{i1},x_{i2},\cdots x_{in})$, the speed $V_i=(v_{i1},v_{i2},\cdots v_{in})$. During each iteration, particles are keeping upgrade its speed and position through individual extremum $P_i=(P_{i1},P_{i2},\cdots P_{in})$ and global extremum $G=(g_1,g_2,\cdots g_n)$, accordingly reaching optimization solution seeking [7]. The upgrade equation is

$$v_{k-1}=c_0v_k+c_1(pbest_k-x_k)+c_2(gbest_k-x_k) \quad (10)$$

$$x_{k+1}=x_k+v_{k-1} \quad (11)$$

where v_k denotes the particle speed, x_k is the position of the current particles, $pbest_k$ is the position where particles find the optimal solution, $gbest_k$ is the position of the optimal solution of the population, c_0, c_1 and c_2 denote the population recognition coefficient, c_0 usually is a random number in interval (0,1), c_1, c_2 is a random number [8] in (0,2).

4 Particle swarm algorithm improvement

4.1 Mechanism of fine adjustment

According to the flow of PSO, after calculation of all parties, $pbest$ and a $gbest$ will be acquired, wherein the quantity and value of $pbest$ depends on the total number of population. In fine adjustment, $gbest$ is adjusted as the object of fine adjustment with the goal of searching the optimal solution in the neighboring area of $gbest$.

Whereas the excellent performance of PSO in fine adjustment principle, we should make full use of the existing advantages in the adjustment. Fine adjustment is not necessarily implemented in each generation unless meeting specific requirements. Directional derivative is intended to be used here as the determinant benchmark. Determination methods are introduced as follows:

Given Y the function of X, $X=[x_1,x_2,\cdots,x_n]$, the directional derivative of Y at g direction is defined as :

$$D_g Y=\frac{\Delta Y}{\|X\|} \tag{12}$$

In a specific generation j, the target function fitness Y_g corresponding to the overall optimal value of the population shows the position of $gbest$ in the design space of j generation that can be expressed as:

$$gbest^j = x_1^j e_{x1} + x_2^j e_{x_2} \tag{13}$$

Supposing the population going through p iterations, $gbest$ arrives at the new position, then the displacement vector of these two positions are:

$$g=(x_1^{j+p}-x_1^j)u+(x_2^{j+p}-x_2)v \tag{14}$$

The difference of the fitness of these two objective functions is

$$\Delta Y = Y_g^j - Y_g^{j+p} \tag{15}$$

Using Equation A, the directional derivative of function Y_g in $j+p$th generation is:

$$D_g Y=\frac{\Delta Y}{\|g\|} \tag{16}$$

After calculation, if $D_g Y$ acquired is larger, it means the population is performing global searching and in a variable situation. Or otherwise the distribution of population particles are gathering in the area where the current population locates for regional searching, while the movement of population is also slow relatively. In this case, the algorithm determines whether to initiate fine adjustment mechanism by using $D_g Y$.

4.2 Operation of fine adjustment

The core concept of fine adjustment is to take $gbest$ as the center. More specifically, it divides the area and range by the defined method and forms a fine-adjustment area based on the mode defined by the algorithm. In the fine adjustment area, a fine adjustment particle swarm will form to calculate the fitness function of each fine adjustment particle. The particle with the optimal adjustment is selected to compare with Y_g and replace the optimal value of updated particles and becomes a new $gbest$ if its fitness excels Y_g .

5. Algorithm improvement and VRP solving

5.1 Model improvement

Particle swarm algorithm is a solution algorithm [9] for continuous space, whereas VRP involves integer planning, thus the model should be improved to construct particles in the algorithm so as to solve the constraint issue. Letting X_v of particle denotes the number of vehicle serving each task point, X_r the delivery order of this node. Particle fitness function is the minimal cost value satisfying the constraints.

The key of this paper lies in finding a proper expression with which the particles correspond to the model. For this, the particle swarm optimized VRP issue is constructed as a space with $(l+m-1)$ dimensions corresponding to l tasks. Numbering of clients is represented by natural number, i denotes the i th clients and 0 denotes the delivery center. For the VRP with l clients and m vehicles, $m-1$ of 0 are inserted into the client series which will be then divided into m sections, each of which denotes the route of vehicles. Each particle corresponds to a $l+m-1$-dimensional vector."

5.2 Algorithm procedure

The VRP solution by using the algorithm presented here consists of the following procedures:

Step 1: given the proper proportion of random particle number q , execute the determined values p of fine adjustment operation and D_{cr}^f of directional derivative as well as the particle number n of population.

Step 2: When $c = p+1$, if the directional derivative $D_g Y$ of f_g is less than or equal to D_{cr}^f, then implement step (3) and (4) for fine adjustment, or otherwise if $D_g Y$ is more than D_{cr}^f, implement step (5).

Step 3: Perform fine adjustment on $gbest_i$, use equation (17) to generate fine-adjustment particles that are randomly generated and evenly distributed in the fine-adjustment area centered by $gbest_i$.

$$x_{td} = gbest_d + l \times [rand() - 0.5] \qquad (17)$$

Step 4: For the fine-adjustment instance generated, its fitness function values Y_t are calculated and arranged in order to select the optimal find-adjustment instance and the corresponding optimal function fitness. If there is an optimal function fitness in the original population, then it can be replaced by the optimal fin-adjustment particles and fitness function values.

Step 5: Calculate the fitness function value Y_i in accordance with the known target function; make a comparison between the fitness Y_i of each particle at the current position and the optimal solution $Pbest_i$, if $P_i < Pbest_i$, then new fitness function value can replace the previous optimal solution, and the new particles replace the previous ones, e.g., $P_i = Pbest_i$, $x_i = xbest_i$, or otherwise implement step (7).

Step 6: Compare the optimal fitness $Pbest_i$ of each particle with the optimal fitness $gbest_i$ of all particles, if $Pbest_i < gbest_i$, then replace the optimal fitness of all particles with the one of each particles, at the same time reserve the current state of particles, e.g., $gbest_i = Pbest_i$, $xbest_i = xbest_i$.

Step 7: use equation (10) and (11) to update the speed and position of particles.

Step 8: determine the end condition, and exit if the condition is satisfied, or otherwise transfer to Step 2 for operation until the iterations are finished or the preset precision is satisfied.

6. Simulation experiment

Experiment 1: Experimental analysis of the example in literature[10] is performed. In this example, there are 8 transportation tasks (number $1, 2, \cdots, 8$), and the cargo volume of each task g (unit: ton), loading (or unloading) time T_i (unit: hour) and the time range $[ET_i, LT_i]$ of each task are given in table 1. These tasks are to be finished by three vehicles with capacity of 8 ton respectively from parking 0. The Distance between the parking and each task point and the intervals between these tasks points are given in Table 2. Supposing the driving time of vehicle is negatively proportional to distance, and the average driving speed of each vehicle is 50kh/h, than the driving time from i to j is $t_{ij} = d_{ij}/50$. Taking the distance between each point as the expense $c_{ij} = d_{ij} (i, j = 0, 1, \cdots, 8)$, and the unit punishment of exceeding time window is $c_1 = c_2 = 50$, wherein $n = 80$; $w = 1$; $c_1 = 1.5$; $c_2 = 1.5$, given the maximal iteration number 200, each of the three algorithm runs 1000 times and the operation results are given in Table 3.

Table 1 Cargo volume, loading and unloading time and time window at each distribution point

Task number	1	2	3	4	5	6	7	8
Freight volume (g_i)	2	1.5	4.5	3	1.5	4	2.5	3
T_i	1	2	1	3	2	2.5	3	0.8
ET_i, LT_i	1, 4	4, 6	1, 2	4, 7	3, 5.5	2, 5	5, 8	1.5, 4

Table 2 Distance between the parking and each task point and the intervals between these task points

	0	1	2	3	4	5	6	7	8
0	0	40	60	75	90	200	100	160	80
1	40	0	65	40	100	50	75	110	100
2	60	65	0	75	100	100	75	75	75
3	75	40	75	0	100	50	90	90	150
4	90	100	100	100	0	100	75	75	100
5	20	50	10	50	10	0	70	90	75

	0		0		0				
6	100	75	75	90	75	70	0	70	100
7	160	110	75	90	75	90	70	0	100
8	80	100	75	150	100	75	100	100	0

Table 3 Comparison of different results gained by different algorithm

Algorithm	Search success rate /(%)	Average search time /s
Basic PSO	72.3	0.34
Genetic algorithm	54.7	0.28
Predatory search algorithm	93.8	0.55
FA-PSO	96.1	0.39

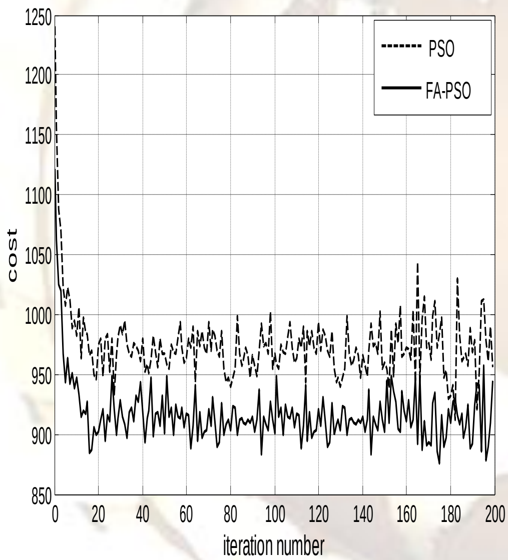

Figure 1: Optimal solution curve of two algorithms

Experimental results show that the successful rate of the algorithm presented here is higher than that of other algorithms. While enhancing the precisions, it does not show any significant increase in average searching time. Taking time and precision into consideration, the algorithm here is of more practical value.

Experiment 2: inspect the situation with multiple dimensions. Model and parameter are referred to literature [8]. Clients are distributed in the interval $[0,100]^2$ and divided into high, middle and low ends based on their needs, e.g., $U(1,9) \sim low$, $U(5,15) \sim middle$, $U(10,20) \sim high$. Given the anticipation of the cargo on the vehicle as $\overline{f} = \sum_{i=1}^{n} \frac{E[D_i]}{gQ}$, where g denotes the quantities of vehicles. $E(D_i) = \frac{8+10+15}{3} = 11$. Letting $\overline{f}' = \max\{0, \overline{f} - 1\}$ denotes the anticipation of route failures. $Q = 11n/(1+\overline{f}')$, where n represents the number of client nodes to be served. In the experiment, $\overline{f}' = 0.7$ and $\overline{f}' = 0.9$. Taking $n = 20,30,40,50,60$ respectively, the improvement rate is compared with greedy algorithm, The simulation result is shown in the following table:

Table 2: Comparison of VRP solution performance by different algorithm

parameters	Improvement rate /%				Time/s			
$(n, Q, \overline{f}')$	PSO	ACO	FA-PSO	CPSO	PSO	ACO	FA-PSO	CPSO
(20,129,0.7)	4.21	4.46	4.81		2.04	1.83	1.82	
(20,116,0.9)	4.47	4.72	5.20		1.63	1.51	1.54	
(30,194,0.7)	4.13	4.37	4.76		6.52	6.04	6.08	
(30,174,0.9)	4.61	4.83	5.28		5.96	5.42	5.41	
(40,259,0.7)	5.07	5.38	5.85		16.27	15.11	14.88	
(40,232,0.9)	4.68	4.98	5.36		12.76	11.80	11.52	
(50,324,0.7)	5.50	5.80	6.43		68.25	62.39	62.33	
(50,289,0.9)	5.18	5.52	6.01		62.20	55.94	56.05	
(60,388,0.7)	4.76	5.02	5.29		114.64	103.87	104.92	
(60,347,0.9)	4.68	4.94	5.14		105.53	95.27	95.98	
Average	4.72	5.03	5.40		\	\	\	

Seeing from the simulation result, to

solve VRP by FA-PSO achieves better effect than ACO and PSO with an average improvement rate of 5.40% by contrast with the primitive solution (greedy algorithm solution). Besides, regarding the operation time, the improved algorithm requires the least operation time, showing that the convergence of the algorithm here has been improved in solving VRP.

7. Conclusion

Through analysis of the shortcomings of PSO, a fine adjustment-based algorithm FA-PSO, which is suitable for VRP solving, is proposed here. This algorithm enhances the effectiveness of initial samples and at the same time solve PSO's problem of being easily trapped into local optimization, thereby improving the global searching ability. The experimental result of VRP model indicates that the algorithm presented here features better speed and precision than the previous algorithm and thus is of good application value in solving VRP.

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References

[1] Claudia Archetti;Dominique Feillet;Michel Gendreau,etl. Complexity of the VRP and SDVRP. Transportation Research Part C: Emerging Technologies. 2011,19(5): 741-750.

[2] Eksioglu B, Vural A V, Reisman A. The Vehicle Routing Problem:A Taxonomic Review. Computers & Industrial Engineering, 2009, 57(4): 1472-1483.

[3] Zagrafos K G, Androutsopulos K N. A Heuristic Algorithm for Solving Hazardous Materials Distribution Problems. European Journal of Operational Research, 2004, 152(2): 507-519.

[4] Ying Li, Bendu Bai, Yanning Zhang. Improved particle swarm optimization algorithm for fuzzy multi-class SVM. Journal of Systems Engineering and Electronics.2010,21(3): 509-513.

[5] Z. Fang, G. F. Tong, X. H. Xu. Particle swarm optimized particle filter. Control and Decision, 2007. 273-277.

[6] ZHANG W , LIU Y T. Adaptive particle swarm optimization for reactive power and voltage control in power systems. Lecture Note in Computer Science , 2005 , 449～452.

[7] Bergh F van den, Engelbrecht A P. A Study of Particle Swarm Optimization Particle Trajectories. Information Sciences. 2006, 176(8): 937-971.

[8] Halil Karahan. Determining rainfall-intensity-duration-frequency relationship using Particle Swarm Optimization. KSCE Journal of Civil Engineering.2012,16(4):667-675.

[9] Xudong Guo,Cheng Wang, Rongguo Yan. An electromagnetic localization method for medical micro-devices based on adaptive particle swarm optimization with neighborhood search. Measurement.2011,44(5):852-858.

[10] Jun LI, Yaohuang GUO. Optimization of logistics delivery vehicle scheduling theory and methods.2001