# Evaluation of Spherical Form Error Using Maximum Distance Point Strategy (MDPS)

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## ABSTRACT

**Calibration of probe prior to any measurement by Coordinate Measuring Machine (CMM) is very important activity. A reference sphere is used for the calibration. Hence, measurement of spherical feature using CMM is one of the important operations in precision engineering. The operation requires use of efficient computational algorithm as it has to determine diameter and center of spherical feature. This paper proposes a strategy named as Maximum Distance Point Strategy (MDPS) for the spherical feature for minimizing sphericity from CMM measured points. The results of MDPS are compared to very well known algorithm i.e. Least Square Method (LSM). It has been found that the results are comparable if measured points are uniformly distributed. If points are not uniformly distributed, the MDPS results are better than LSM results.**

**Keywords** -Sphericity, Best-fit Sphere, Coordinate Measuring Machine, Least-Square Method, Maximum Distance Point Strategy**.**

## 1. Introduction

A measurement of geometric feature of manufactured part by Coordinate Measuring Machine (CMM) involves collection of points. These points are fitted into appropriate geometric features like plane, line, circle (hole), sphere, cylinder etc. by suitable fitting algorithm. The calibration of probeis essential before any measurement process is carried out to compensate the radius of probe. This process is carried out using reference sphere. Apart from this, sphere is an important feature of manufactured components. When these components are manufactured, closeness to required dimension is expressed in terms of sphericity.

The ANSI Dimensioning and Tolerance Standard Y14.5 [1] defines that the form tolerances on a component must be evaluated with reference to an ideal geometric feature. CMM software evaluates sphericity of spherical features by establishing a sphere as reference geometric feature from the measured points. A common way of measuring how well a function fits, is the least-squares criterion. The error is calculated by adding up the squares of the errors at each of the observation points. This is a very natural measure. The squaring is done to stop cancellations among errors with different signs. Obviously, it is most desirable to find the choice of parameters that minimizes error. It turns out that this choice can be computed efficiently. Hence, this method is called Least Squares Method (LSM) [2]. The least-square fitting method requiresoptimization algorithm to minimize non-linear objective function. Gauss-Newton Algorithm (GNA) is very prevalent algorithm to minimize such a non-linear objective function.

P. Bourdet, C. Lartigue and F. Leveauxhave appliedLSM in order to find the location of center and radius of sphere to calibrate a probe [3]. They have analyzed variouserrors involved during the steps leading to the identification of the sphereand location of its center. The errors include those associated with surfaceaccessibility for sampling points, sampling strategies, optimization algorithm,and probe diameter versus reference sphere diameter.It is inferred that a calibration strategy is proposed as a function of sampled points, optimization algorithm and the geometric surface involved.

T. Kanada[4] has suggested the calculation of the value of spherical form errors (sphericity). The iterative least-squares method and the minimum zone method are applied on simulated data. To calculate the minimum zone sphericity, the downhill simplex method is applied. It is concluded that the sphericity calculated by iterative least-square method and minimum-zone method are comparable. 3D measurement of a whole spherical surface as per the definition of sphericity can be simplified in terms of 2D measurement of cross-sectional profiles.This simplification may give rise to the evaluation errors in the sphericity, because the sphericity should be evaluated three-dimensionally. Considering this, T. Kanada[5] has determined the minimum number of cross-sectional profiles that needed to be measured in order to estimate the sphericity from roundness values accurately. A recommended number of cross-sectional profile measurements is proposed by means of a statistical process. The accuracy of the sphericity values estimated using this procedure is not investigated in the work.

C. M. Shakarji [6] suggested use of Levenberg-Marquardt Algorithm (LMA) to minimize the square of error distances for various features including sphere. LMA is a trust-region strategy which provides a numerical solution to the problem of minimizing non-linear function. LMA is more robust than GNA. However, even for well-behaved functions and reasonable starting parameters, LMA tends to be a bit slower than GNA. LMA can also be viewed as improved GNA with trust region approach [7, 8]. Also, convergence of the solution is highly dependent on choice of Levenberg-Marquardt parameter and its selection is challenging.

Theoretical derivation of the minimum zone criteria of sphericity error based on the principle of minimum potential energy is proposed by K. C. Fan and J. C. Lee [9]. They have converted the problem of finding the minimum zone sphericity error into the problem of finding the minimum elastic potential energy of the corresponding mechanical system.

G. L. Samuel and M. S. Shunmugam [10] have developed methods based on the computational geometry to establish the assessment spheres. The suggested methods start with construction of 3-D hull. The 3-D convex hull is established using computational geometric concept. For establishing a 3-D inner hull, a new heuristic method is suggested. A new concept of 3-D equidistant (ED) line is introduced in the method. Based on this concept, the authors have constructed 3-D farthest and nearestequidistance diagrams for establishing the assessment spheres. Algorithms proposed are implemented and validated with the simulated data.

Techniques for evaluating circularity and sphericity error from CMM data are presented by G. L Samuel and M.S. Shunmugam [11]. It is summarized that the form error can be evaluated directly from CMM data by employing sphere as assessment features and using normal deviations. The CMM data can also be transformed by applying appropriate methods that not only suppress the size but also introduce distortion. In the work, the form error is evaluated from the transformed data by employing limacon/limacoid as assessment features and using linear deviations. Also, the methods for handling CMM and transformed data are presents.

The authors of this paper have developed a novel strategy to evaluate circularity which is named as Maximum Distance Point Strategy (MDPS) [12]. In the present work, the same strategy is extended to evaluate the spherical form error. Circularity is 2-dimensional feature and a sphere is 3-dimensional feature. Hence, the procedure for selection of the points at maximum distance is different for sphere than that of circle.Point selection procedure should be capable of eliminating coplanar points. The proposed strategy is compared with least-square fitting method usingGauss-Newton algorithm and other published methods/algorithm in literature for the sphericity form error and sum of square of deviation.

This is a customized approach to find the best fit sphere for evaluating sphericity rather than addressing a general unconstrained nonlinear problem. It is based on the postulate that “A unique sphere passes through any four non-coplanar points in space, where any three points are not in a line.” Hence, selection procedure of four points (quadruplet) is suggested in the next section.

## 2. Point Selection Procedure

The selection procedure for quadruplet (A, B, C, D) is as follow.

1. Let $P_i(x_i, y_i, z_i)$, $i = 1,2,\dots,n$ and $n > 3$, be the set of $n$ points.
2. Select any point from $P_i$ and name it as A, which is first point in quadruplet.
3. Calculate distance from point A to each point $P_i$ using equation 2.1.

$$AP_i = \sqrt{(x_a - x_i)^2 + (y_a - y_i)^2 + (z_a - z_i)^2} \qquad (2.1)$$

where, $AP_i$ is distance from point A to point $P_i$, $i = 1,2,\dots,n$

$x_a, y_a, z_a$ are coordinates of point A.

$x_i, y_i, z_i$ are coordinates of point $P_i$.

4. Second point in quadruplet (point B) is selected which has maximum value of $AP_i$.
5. Select third point in quadruplet (point C) from $P_i$, $i = 1,2,\dots,n$ such that its normal distance from line AB to the selected point is maximum. To determine normal distance the equation 2.3 is used.

$$d(P_i)_{AB} = ((x_a - x_i)^2 + (y_a - y_i)^2 + (z_a - z_i)^2)((x_b - x_i)^2 + (y_b - y_i)^2 + (z_b - z_i)^2) - \big((x_a - x_b)(x_a - x_i) + (y_a - y_b)(y - y_i) + (z_a - z_b)(z_a - z_i)\big)^2 \qquad (2.2)$$

where, $d(P_i)_{AB}$ is normal distance parameter from point $P_i$ to line AB, $i = 1,2,\dots,n$

$x_b, y_b, z_b$ are coordinates of point B.

Note: The point with $d(P_i)_{AB} = 0$ should be discarded [11].

6. The normal distance parameter from point $P_i$ to the plane passing through selected points A, B and C is determined by the equation 2.3.

$$d(P_i)_{ABC} = abs\left(\begin{vmatrix} x_i - x_a & y_i - y_a & x_i - z_a \\ x_b - x_a & y_b - y_a & y_c - y_a \\ x_c - x_a & x_b - z_a & z_c - y_a \end{vmatrix}\right) \qquad (2.3)$$

where, $d(P_i)_{ABC}$ is normal distance parameter from point $P_i$ to plane passing through points A, B and C, $i = 1,2,\ldots,n$
$x_c, y_c, z_c$ are coordinates of point C.

7. Point D, is selected with maximum value of $d(P_i)_{ABC}$ from the available points.

The selection procedure is followed for each point $P_i$. Hence, there are n quadruplet and n candidate spheres.

Amongst all candidate spheres, spheres which are far from the solution are eliminated heuristically as discussed in section 3.3. The average of center coordinates of the selected spheres and the average radii of these spheres represent the center and radius of the best fit sphere for a given set of points.

## 3. Formulations

For $P_i(x_i, y_i, z_i)$ $i = 1,2,\ldots,n$ and $n \geq 4$

### 3.1 Least Square Method

A sphere with the center $(x_0, y_0, z_0)$ and radius $r_0$ is found such that it minimizes the sum of squared deviations. The sphere equation in an implicit form can be written as

$$f(x,y,z) = (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 - r_0^2 = 0$$

The deviation of distance for a point $P_i$, $i = 1,2,\ldots,n$ may be explicitly written as

$$e_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} - r_0; \; where\; i = 1,2,\ldots,n \quad (3.1)$$

The sum of squared deviations is then described as

$$e_s = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} \left[\sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} - r_0\right]^2 \quad (3.2)$$

### 3.2 Sphericity error

Denote the maximum value among the deviations $e_i$, $i = 1,2,\ldots,n$ as $e_{max}$ and the minimum value as $e_{min}$. Then, the sphericity error h can be computed as

$$h = e_{max} - e_{min} \quad (3.3)$$

According to the minimum zone criterion given by ANSI Standard Y14.5 [1], the center $(x_0, y_0, z_0)$ and radius $r_0$ of an ideal circle should be determined such that the sphericity h is the minimum.

### 3.3 Maximum Distance Point Strategy

Fix a point, say, $P_k$ and select three other points as explained in section 2. Let the coordinates of the points be $(x_a, y_a, z_a)$, $(x_b, y_b, z_b)$, $(x_c, y_c$, zcand xd, yd, zd. Solve the following system of linear algebraic equations

$$2(x_b - x_a)x + 2(y_b - y_a)y + 2(z_b - z_a)z = (x_b^2 - x_a^2) + (y_b^2 - y_a^2) + (z_b^2 - z_a^2)$$

$$2(x_c - x_a)x + 2(y_c - y_a)y + 2(z_c - z_a)z = (x_c^2 - x_a^2) + (y_c^2 - y_a^2) + (z_c^2 - z_a^2)$$

$$2(x_d - x_a)x + 2(y_d - y_a)y + 2(z_d - z_a)z = (x_d^2 - x_a^2) + (y_d^2 - y_a^2) + (z_d^2 - z_a^2)$$

for center of the sphere $(x_0, y_0)$ and calculate $r_0$ using $r_0 = \sqrt{(x_a - x_0)^2 + (y_a - y_0)^2 + (z_i - z_0)^2}$. This is the sphere passing through $(x_a, y_a, z_a)$, $(x_b, y_b, z_b)$, $(x_c, y_c, z_c)$ and $(x_d, y_d, z_d)$. Repeating the procedure by fixing each point $P_i$, $i = 1, 2, \ldots n$, n–centers and n–radii are found. To select the best fit sphere following heuristic method is used.

1. Let
$$e_k = \sum_{i=1}^{n} \left[\sqrt{(x_i - a_k)^2 + (y_i - b_k)^2 + (z_i - c_k)^2} - r_k\right]^2$$
$(a_k, b_k, c_k)$ is center and $r_k$ is radius of $k^{th}$ sphere where, $k = 1,2,\ldots n$.
2. The mean and standard deviation of $e_k$, $k = 1,2,\ldots n$ are found.
3. The spheres with $e_k$ less than or equal to mean of $e_k$, $k = 1,2,\ldots n$ are selected.
4. Calculate the mean of the coordinates of centers and radii of these selected spheres. This gives center and radius of the best fit sphere.
5. Calculate sphericity using equation (3.3) for the sphere found in step 4.
6. Steps 1 to 5 are followed for (n – m) number of spheres; where m is number of spheres which are not selected in step 3.
7. If sphericity calculated in step 5 is less than sphericity calculated in previous iteration, go to step 1. Otherwise go to step 8.
8. Stop iterations. The sphere found is the claimed best fit sphere.

## 4. Results and Discussion

MATLAB programs for evaluating sphericity by MDPS and LSM were executed on computer with Intel atom processor, 800 MHz clock speed and 1 GB RAM. The programs were run for CMM measured data set. A reference sphere is measured using SCAN facility available on CMM. The SCAN facility ensures that points in the dataset are uniformly spaced. These measured points are tabulated in Table 1.

Table 2 shows results of sphericity (h) evaluation for the dataset presented in Table 1. The results are expressed up to six decimal places. It can be observed that sphericity error obtained by MDPS is not less than that of LSM and CMM result. But, it can be observed that the order of sphericity error is $10^{-2}$μm. Table 2 also shows the comparison of sum of squared deviation ($e_s$). It can be observed that sum of squared deviation of MDPS is less than that of CMM result and more than LSM. The order of sum of squared deviation is $10^{-8}$ mm.

Since, the points are uniformly distributed, the results obtained from each method is very precise and near to each other. Three more datasets of points are manually measured for the same reference sphere. These datasets contain 44, 37 and 23 points respectively. Table 3 shows sphericity obtained by MDPS and LSM. It can be seen that sphericity achieved by MDPS is less than that of LSM. The table 3 also indicates that the sum of squared deviation of MDPS is not less than that of LSM, which is not objective of the method.

## 5. Conclusions

The present paper shows that the Maximum Distance Point Strategy (MDPS) suggested for circle [11] is extended to the sphere. It is concluded that the points are very well uniformly distributed the MDPS gives comparable results with Least Square Method (LSM). The MDPS gives better results compared to LSM when points are not uniformly distributed. This method can be used as starting solution for Simplex Search. The developed methodology has great potential for implementation in CMM software for evaluation of spherical features.

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Table 1: CMM measured dataset

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | 193.6395 | 449.6373 | -489.2791 | 41. | 174.5898 | 459.3663 | -485.0077 |
| 2. | 193.381 | 452.1639 | -489.2791 | 42. | 172.6499 | 457.7269 | -485.0077 |
| 3. | 192.6185 | 454.5816 | -489.2791 | 43. | 171.1076 | 455.7077 | -485.0077 |
| 4. | 191.3827 | 456.7966 | -489.2791 | 44. | 170.0412 | 453.4188 | -485.0077 |
| 5. | 189.7255 | 458.7156 | -489.2791 | 45. | 169.4875 | 450.941 | -485.0077 |
| 6. | 187.7236 | 460.255 | -489.2791 | 46. | 169.4812 | 448.3883 | -485.0077 |
| 7. | 185.4375 | 461.3658 | -489.2791 | 47. | 170.0211 | 445.9123 | -485.0077 |
| 8. | 182.9779 | 461.9903 | -489.2791 | 48. | 171.0772 | 443.6143 | -485.0077 |
| 9. | 180.4467 | 462.1048 | -489.2791 | 49. | 172.6045 | 441.5925 | -485.0077 |
| 10. | 177.947 | 461.7068 | -489.2791 | 50. | 174.5259 | 439.9484 | -485.0077 |
| 11. | 175.5631 | 460.8051 | -489.2791 | 51. | 176.7545 | 438.7544 | -485.0077 |
| 12. | 173.4296 | 459.4521 | -489.2791 | 52. | 179.1957 | 438.0638 | -485.0077 |
| 13. | 171.6114 | 457.6967 | -489.2791 | 53. | 181.7355 | 437.9145 | -485.0077 |
| 14. | 170.1802 | 455.6046 | -489.2791 | 54. | 184.2403 | 438.3139 | -485.0077 |
| 15. | 169.2004 | 453.2646 | -489.2791 | 55. | 186.5973 | 439.2404 | -485.0077 |
| 16. | 168.7137 | 450.7755 | -489.2791 | 56. | 188.6988 | 440.6494 | -485.0077 |
| 17. | 168.7419 | 448.2211 | -489.2791 | 57. | 190.4487 | 442.4752 | -485.0077 |
| 18. | 169.2817 | 445.7492 | -489.2791 | 58. | 191.7708 | 444.6418 | -485.0077 |
| 19. | 170.3135 | 443.4282 | -489.2791 | 59. | 192.5949 | 447.0365 | -485.0077 |
| 20. | 171.7879 | 441.3704 | -489.2791 | 60. | 190.7176 | 449.6317 | -481.2513 |
| 21. | 173.6443 | 439.6542 | -489.2791 | 61. | 190.3819 | 452.1481 | -481.2513 |
| 22. | 175.8157 | 438.3435 | -489.2791 | 62. | 189.3965 | 454.4868 | -481.2513 |
| 23. | 178.2024 | 437.4996 | -489.2791 | 63. | 187.8376 | 456.4777 | -481.2513 |
| 24. | 180.7174 | 437.1542 | -489.2791 | 64. | 185.8181 | 457.9869 | -481.2513 |
| 25. | 183.2553 | 437.3253 | -489.2791 | 65. | 183.4654 | 458.9185 | -481.2513 |
| 26. | 185.6967 | 438.0035 | -489.2791 | 66. | 180.9368 | 459.2004 | -481.2513 |
| 27. | 187.9497 | 439.1597 | -489.2791 | 67. | 178.4324 | 458.8085 | -481.2513 |
| 28. | 189.9258 | 440.7492 | -489.2791 | 68. | 176.1218 | 457.7746 | -481.2513 |
| 29. | 191.5412 | 442.7068 | -489.2791 | 69. | 174.1603 | 456.1676 | -481.2513 |
| 30. | 192.7221 | 444.9373 | -489.2791 | 70. | 172.6964 | 454.1142 | -481.2513 |
| 31. | 192.8863 | 449.6328 | -485.0077 | 71. | 171.8159 | 451.7317 | -481.2513 |
| 32. | 192.6111 | 452.1626 | -485.0077 | 72. | 171.5938 | 449.1912 | -481.2513 |
| 33. | 191.8008 | 454.5652 | -485.0077 | 73. | 172.0445 | 446.7014 | -481.2513 |
| 34. | 190.498 | 456.7317 | -485.0077 | 74. | 173.1329 | 444.4158 | -481.2513 |
| 35. | 188.7602 | 458.57 | -485.0077 | 75. | 174.778 | 442.4999 | -481.2513 |
| 36. | 186.6678 | 459.9937 | -485.0077 | 76. | 176.8722 | 441.0785 | -481.2513 |
| 37. | 184.3112 | 460.9379 | -485.0077 | 77. | 179.265 | 440.2563 | -481.2513 |
| 38. | 181.8087 | 461.353 | -485.0077 | 78. | 181.7975 | 440.0905 | -481.2513 |
| 39. | 179.2695 | 461.2198 | -485.0077 | 79. | 184.2738 | 440.5927 | -481.2513 |
| 40. | 176.8231 | 460.5445 | -485.0077 | 80. | 186.5341 | 441.7271 | -481.2513 |
| 81. | 188.419 | 443.4148 | -481.2513 | 87. | 178.0569 | 452.5809 | -477.5433 |
| 82. | 189.796 | 445.5387 | -481.2513 | 88. | 176.9389 | 450.3482 | -477.5433 |
| 83. | 185.422 | 449.6341 | -477.5433 | 89. | 177.2727 | 447.8446 | -477.5433 |
| 84. | 184.6802 | 452.0413 | -477.5433 | 90. | 178.939 | 445.9814 | -477.5433 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 85. | 182.7367 | 453.6017 | -477.5433 | 91. | 181.3477 | 445.3688 | -477.5433 |
| 86. | 180.2401 | 453.809 | -477.5433 | 92. | 183.7082 | 446.2146 | -477.5433 |

Table 2: Results of sphericity evaluation for measured points tabulated in table 1.

| | $x_0$ *(mm)* | $y_0$ *(mm)* | $z_0$ *(mm)* | $r_0$ *(mm)* | *h(µm)* | *Sum of squared deviation,* $e_s$ |
|---|---|---|---|---|---|---|
| MDPS | 181.150541 | 449.935688 | -489.279092 | 12.488976 | 0.082138 | $5.7158\times10^{-8}$ |
| LSM | 181.150541 | 449.635692 | -489.279098 | 12.488985 | 0.073402 | $5.2312\times10^{-8}$ |
| CMM result | 181.150541 | 449.635693 | -489.279077 | 12.489 | 0.080474 | $5.8423\times10^{-8}$ |

Table 2: Results of sphericity evaluation for points manually.

| Number of points | MDPS | | LSM | |
|---|---|---|---|---|
| | *h(µm)* | *Sum of squared deviation,* $e_s$ | *h(µm)* | *Sum of squared deviation,* $e_s$ |
| 44 | 2.0791 | 7.9354 | 2.1182 | 6.9776 |
| 37 | 2.0810 | 7.4879 | 2.0852 | 6.0300 |
| 23 | 2.0101 | 3.4039 | 2.0854 | 3.4771 |