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Iterative Back-Projection Algorithm Based Signal Processing Approach To Enhance An Image Resolution

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ABSTRACT

In all image processing applications, there is huge demand for high-resolution images. Image resolution depends on the resolution of the image acquisition device like digital camera. One way to increase resolution is to go for very high-resolution cameras but the major difficulty associated with using high-resolution cameras is, it increases the cost. An alternative to expensive high resolution cameras, super-resolution (SR) is used to obtain a high-resolution (HR) image from a sequence of multiple low-resolution (LR) observations of the same scene. SR image reconstruction is one of the most spotlighted research areas, because it can overcome the inherent resolution limitation of the imaging system and improve the performance of most digital image processing applications. In this paper, we demonstrate the Iterative-Back Projection (IBP) algorithm to obtain high-resolution images. The analysis is first carried out in the 1-D domain to demonstrate the dealiasing capability of the super-resolution and extended to synthetic images.

Keywords – Image reconstruction, IBP, LR, HR, SR.

I. INTRODUCTION

The SR image reconstruction is proved to be useful in many practical cases where images with high resolution are often required, in areas like medical imaging, satellite imaging, and video applications. Image resolution is defined as the smallest measurable detail in a visual representation. In digital image processing, the term resolution can be divided into three different classes - spatial resolution, brightness resolution and temporal resolution [1]. The spatial resolution of an image is determined by the imaging sensors.

In charge-coupled device (CCD) camera, an image resolution is determined by the size of its photo-detector. One way to increase resolution is by reducing the size of pixels. With decreasing the pixel size, the amount of light available for each pixel is also decreases and the image quality is degraded due to the enhancement of short noise. Thus it is difficult for a single CCD sensor to capture a high resolution image because of the limitaion on the size of current CCD sensing elements. It has been estimated that the minimum size of a photo-detectors should be approximately $50\mu m^2$. This limit has already been attained by current CCD technology. Therefore, new techniques are needed to provide a high resolution image beyond the physical device performance which is possible using signal processing approach.

An algorithmic way of enhancing the resolution is super-resolution. SR is process in which a high-resolution image is constructed from a set of LR images. Fundamentally, the task involves dealiasing, deblurring and denoising [1]-[3]. SR can be considered as a second generation problem of image reconstruction. Super-resolution algorithms can be broadly divided into two categories: motion based [4] and motion-free [5]-[6]. In comparison to motion-free methods, motion-based approach has attracted significantly more attention from the imaging community. In motion-based super-resolution, sequences of multiple, sub-pixel shifted low-resolution observations are used to generate a high-resolution image.

The aim of super-resolution (SR), for the motion-free case, is to remove the effects of blurring and aliasing, by making use of the information in the given set of defocused observations. The LR images are captured as sequence with different sensors which represent different"looks" at the same scene. The high resolution is constructed from these sub-sampled, noisy and blurred as well as shifted with sub-pixel LR images. If the LR images are shifted by integer units, then there is no new information that can used to reconstruct an HR image. If the LR images are shifted by sub-pixel values, then using this information an HR image can be reconstructed. Multiple scenes can be obtained from one camera with several captures or from multiple cameras located in different positions. These scene motions can occur due to the movement of local object or imaging systems. If these scene motions are known and if we combine these LR images, SR image reconstruction is possible.

This paper is organized as follows. Section II, discusses the choice of the observation model. The IBP formulation of the problem is given in section III. Implementation issues [7] are given in section IV. Experimental results are given in section V while section VI concludes the paper.

II. OBSERVATION MODEL

The In the process of recording a digital image, there is a natural loss of spatial resolution caused by the optical distortions, motion blur due to limited shutter speed, noise that occurs within the sensor or during transmission, and insufficient sensor density. Thus, the recorded image usually suffers from blur, noise, and aliasing effects. Therefore, the goal of SR techniques is to restore an HR image from noisy, blurred and aliased LR images. The first step in analyzing the problem is to formulate a suitable observation model that relates the original HR image to the LR observations. The relation between a lexicographically ordered LR observation and the original HR image can be expressed as

$$\underline{y_r} = DH_rW_r\underline{x} + \underline{n_r}, \quad 1 \le r \le m \qquad (1)$$

Where,

$\underline{x}$ = Original HR image of size $(N_1 \times N_2)$ recorded in a vector of size $(N_1N_2 \times 1)$,

$\underline{y_r}$ = r^{th} LR observation of size $(M_1 \times M_2)$ recorded in a vector of size $(M_1M_2 \times 1)$

D = Down-sampling matrix $(M_2M_1 \times N_1N_2)$,

H_r= Camera defocus blur matrix for the r^{th} frame of dimension $(N_1N_2 \times N_1N_2)$,

W_r= Geometric warping matrix for the r^{th} frame of dimension $(N_1N_2 \times N_1N_2)$,

$\underline{n_r}$ = Noise in the r^{th} frame which is assumed to be Gaussian,

m = Number of LR observations.

The motion that occurs during the image acquisition is represented by the warp matrix W_r. It may contain translation and rotation. Since this information is generally unknown, therefore it is necessary to estimate the scene motion for each frame with reference to one particular frame. Blurring may be caused by an optical system and relative motion between the imaginary system and original scene, and the point spread function (PSF) of the LR sensor. It can be modulated as linear space invariant (LSI) or linear space variant (LSV), and represented by the matrix H_r.

In the SR image reconstruction, the finiteness of dimension in LR sensors is an important factor of blur and the characteristics of the blur are assumed to be known. The sub-sampling matrix D generates aliased LR images from the warped and blurred HR image. The observation model that we have discussed here is for an image and it also can be suitably changed for 1D case.

III. ITERATIVE BACK-PROJECTION (IBP) FORMULATION

In this algorithm, the HR image is estimated by back projecting the difference between simulated LR images and the observed LR images. Starting with an initial estimate for the HR image, the back-projection process is repeated iteratively for each incoming LR image.

Eq. (1) can be expressed in matrix-vector form as

$$\begin{pmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_m \end{pmatrix} = \begin{pmatrix} DH_1W_1 \\ DH_2W_2 \\ \cdot \\ \cdot \\ \cdot \\ DH_mW_m \end{pmatrix} \underline{x} + \begin{pmatrix} n_1 \\ n_2 \\ \cdot \\ \cdot \\ \cdot \\ n_m \end{pmatrix} \leftrightarrow \underline{x_L} = A\underline{x_H} + \underline{e} \qquad (2)$$

For practical reasons, it is assumed the noise is uncorrelated and has uniform variance. In this case, the maximum likelihood solution is found by minimizing the function $E(\underline{x_H})$, which can be expressed as

$$E(\underline{x_H}) = \frac{1}{2} \left\| \underline{x_L} - A(\underline{x_H}) \right\|^2 \qquad (3)$$

Taking the derivation of E with respect to $\underline{x_H}$ and setting the gradient to zero:

$$\nabla E = 0 \rightarrow A^T(A\underline{x_H} - \underline{x_L}) = 0$$

$$\leftrightarrow \sum_{r=1}^{m} W_r^T H_r^T D^T \left(DH_rW_r\underline{x_H} - \underline{x_L^r}\right) = 0 \qquad (4)$$

Where the matrix A^TA operates on the vectors $\underline{x_H}$ and matrix A^T operates on the $\underline{x_L}$.For the simplest implementation of the eq. (4) IBP algorithm is used. For the r^{th} LR image, the basic update equation can be written as:

$$\underline{x_H}(m+1) = \underline{x_H}(m) + \sum_{r=1}^{m} W_r^T H_r^T D^T (\underline{x_L^r} - DH_rW_r\underline{x_H}(m)) \qquad (5)$$

Where $\underline{x_H}(m)$ is a initial estimate and m represents the number of iterations. The matrices W_r, H_r and D model the image formulation process and their implementation is simply the image warping, blurring and sub-sampling respectively. The transpose matrices are also implemented as follows. $D^T$is implemented by up- sampling the image without interpolation. For a convolution blur H_r^T is implemented by convolution with the flipped blur kernel. For space-variant blur H_r^T is implemented by forward projection of the values, using the weights of the original blur kernel. W_r^T is implemented by backward warping of the motion. A small residual error suggests that the scene estimate is an accurate one; whereas a significant residual error indicates that the estimate is poor therefore the error information can be used to improve the scene estimate. By a process called back-projection the error residual is used to form an updated estimate of the scene which is a better approximation of the original. The approximation is better in the sense that the result of the simulated imaging process using the updated estimate has reduced residual error as compared with the earlier estimate. This process is iterative and reduces the simulation residual error to a minimum. The iterative process thus comprises two steps: simulation of the observed images, and back-projection of the error to correct the estimate of the original scene.

IV. IMPLEMENTATION ISSUES

4.1 Warping

The warping operation is typically performed using bilinear interpolation. Each pixel value in the warped image is calculated from its four neighboring pixels using the interpolation coefficients. Assuming a 2×2 pixel image

$$a = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \quad (6)$$

which must be warped by sub-pixel shifts δ_x and δ_y in the x and y directions respectively. The values of the pixels corresponding to the warped image is

$$b = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} \quad (7)$$

Where,

$$b_1 = (1-\delta_x)(1-\delta_y)a_1 + \delta_y(1-\delta_x)a_2 + \delta_x(1-\delta_y)a_3 + \delta_x\delta_y a_4,$$
$$b_2 = (1-\delta_x)(1-\delta_y)a_2 + \delta_x(1-\delta_y)a_4,$$
$$b_3 = (1-\delta_x)(1-\delta_y)a_1 + \delta_y(1-\delta_x)a_4,$$
$$b_4 = (1-\delta_x)(1-\delta_y)a_4$$

The warping operation can be expressed as

$$b = Wa$$

Where,

$$W = \begin{bmatrix} W_A & W_B & W_C & W_D \\ 0 & W_A & 0 & W_C \\ 0 & 0 & W_A & W_B \\ 0 & 0 & 0 & W_A \end{bmatrix} \quad (8)$$

Here,

$$W_A = (1-\delta_x)(1-\delta_y), \quad W_B = \delta_y(1-\delta_x), \quad W_C = \delta_x(1-\delta_y), \text{ and } W_D = \delta_x\delta_y$$

The transpose operation can be expressed as

$$a = W^T b$$

Where,

$$W^T = \begin{bmatrix} W_A & 0 & 0 & W_D \\ W_B & W_A & 0 & W_C \\ W_C & 0 & W_A & W_B \\ W_D & W_C & W_B & W_A \end{bmatrix} \quad (9)$$

The matrix W^T spreads the intensity of a pixel proportional to the interpolation coefficients. Note that W and W^T are not inverse of each other.

4.2 Blurring

The convolution operation is performed using linear convolution. Assume a vector of size 1×4, $a = [a_1 \quad a_2 \quad a_3 \quad a_4]$ and the blur kernel $h = [h_0 \quad h_1 \quad h_2]$. Then, convolution of a by h can be written as

$$[c] = [a_1 \quad a_2 \quad a_3 \quad a_4] * [h_0 \quad h_1 \quad h_2] \quad (10)$$
$$= [a_1h_0,\ a_2h_0 + a_1h_1,\ a_3h_0 + a_2h_1 + a_1h_2,\ a_4h_0 + a_3h_1 + a_2h_2,\ a_4h_1 + a_3h_2,\ a_4h_2]$$

For length of convolved vector as the a, this can be written as

$$[c] = [a_2h_0 + a_1h_1,\ a_3h_0 + a_2h_1 + a_1h_2,\ a_4h_0 + a_3h_1 + a_2h_2,\ a_4h_1 + a_3h_2]$$

The blurring operation can be expressed as $c = Ha$,
Where,

$$H = \begin{bmatrix} h_1 & h_0 & 0 & 0 \\ h_2 & h_1 & h_0 & 0 \\ 0 & h_2 & h_1 & h_0 \\ 0 & 0 & h_2 & h_1 \end{bmatrix}$$

If instead of h, convolve a with a flipped version of h, then

$$[c] = [a_2h_2 + a_1h_1,\ a_3h_2 + a_2h_1 + a_1h_0,\ a_4h_2 + a_3h_1 + a_2h_0,\ a_4h_1 + a_3h_0]$$

In matrix-vector form $c = H_{flip}\, a$
Where,

$$H_{flip} = \begin{bmatrix} h_1 & h_2 & 0 & 0 \\ h_0 & h_1 & h_2 & 0 \\ 0 & h_0 & h_1 & h_2 \\ 0 & 0 & h_0 & h_1 \end{bmatrix}$$

Note that H_{flip} is the same as H^T, i.e., the operation H^Ta is the convolution of the vector with flipped version of h.

4.3 Down-sampling

Consider a small image of size 4×4 pixels value

$$a = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \quad (11)$$

After down-sampling by a factor of 2,

$$b = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} \quad (12)$$

Where,

$$b_1 = \frac{a_1 + a_2 + a_5 + a_6}{4},$$
$$b_2 = \frac{a_3 + a_4 + a_7 + a_8}{4},$$
$$b_3 = \frac{(a_9 + a_{10} + a_{13} + a_{14})}{4},$$
$$b_4 = \frac{a_{11} + a_{12} + a_{15} + a_{16}}{4}$$

The transpose operation yields the expanded image as

$$b^T = \frac{1}{4}\begin{bmatrix} b_1 & b_1 & b_2 & b_2 \\ b_1 & b_1 & b_2 & b_2 \\ b_3 & b_3 & b_4 & b_4 \\ b_3 & b_3 & b_4 & b_4 \end{bmatrix} \quad (13)$$

The matrix b^T spreads the intensity value in a lower dimension to a higher dimension. Note that b and b^T are not inverse of each other.

V. EXPERIMENTAL RESULTS

In the experiments, IBP algorithm was tested on both 1D and 2D signals respectively. In experiment, Mean Square Error (MSE) is calculated, which can be expressed as

$$MSE = \frac{1}{N_1 N_2} \left\| X - \hat{X} \right\|^2 \qquad (14)$$

Where,

X : Original image,

$\hat{X}$: Super-resolved image and

$N_1 N_2$: Number of rows and columns in the image

In the first experiment, a sine wave (Fig. 1(a)) with $f_m = 400Hz$ is considered. It is sampled by $f_s = 1000Hz$. Four LR observations are generated from a good quality sine wave of size 1 × 2500 by warping to the sub pixel shift parameters (0.1), (0.25), (0.5) and (0.6). Each of the LR observations is blurred by Gaussian blur of standard deviation (0.000005) and down-sampled by a factor of 2. Gaussian noise of variance (0.000005) is then added. In this experiment, we observed that aliasing is taken place in LR signal as shown in Fig. 1(b). Using interpolation, it is not possible to get back original signal as shown in Fig. 1(c). Therefore, interpolation methods are not considered as SR. Using IBP technique with low blur (0.000005) and low noise (0.000005), it is possible to get signal same as original signal. The reconstructed results of the super-resolution algorithm are shown in Fig. 1(d).

Next, to verify the algorithm, increase/decrease the value of both standard deviation and variance for blur, noise respectively. Results of sine wave with low blur and low noise (ideal case) is shown in Fig. 2(a) and Fig. 2(b). With increase in noise (0.33), noisy samples have appeared and the decreasing MSE has increased as shown in Fig. 2(c) and Fig. 2(d). With increasing the blur (0.5), blurred samples at frequency 100, 900 are not completely suppressed and MSE is increased as shown in Fig. 2(e) and Fig. 2(f).

In the next experiment, “Lena” image (Fig. 3(a)) is considered of dimension 132 × 132 pixels from which four low-resolution observations are generated by warping, blurring and down-sampling. Sub-pixel shifts (0, 0), (0, 1), (1, 0), and (1, 1) are used. One such LR observation and bilinear interpolation output are shown are Fig. 3(b) and Fig. 3(c). The reconstructed results of the super-resolution algorithm are shown in Fig. 3(d) which is close to the original image.

Results of ’Lena’ image with low blur and low noise (ideal case) is shown in Fig. 4(a). An image with high noise (0.15), low blur (0.000005) and low noise (0.000005), high blur (0.5) are shown in Fig. 4(b) and Fig. 4(c). The mean squared error (MSE) per pixel between the original and the reconstructed image is also given for comparison. With increase in noise and blur, the quality of an image is decreased as compared to the original image.

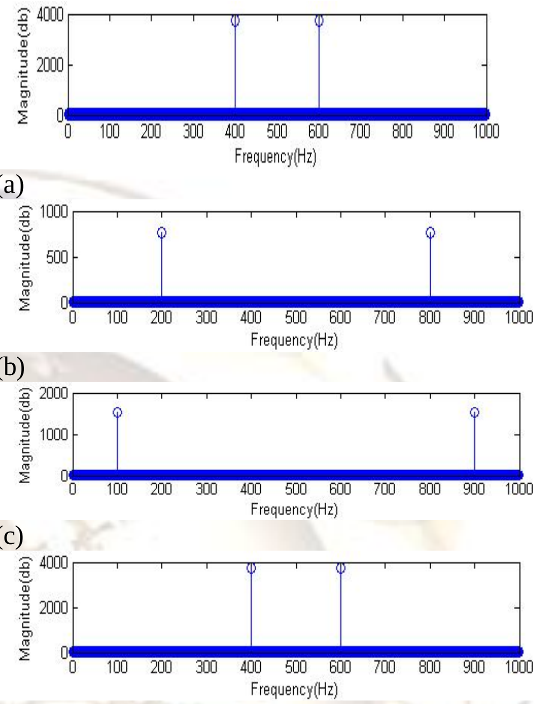
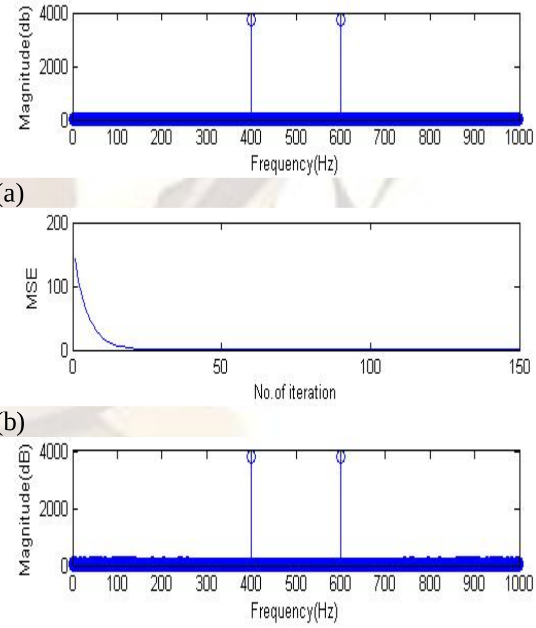
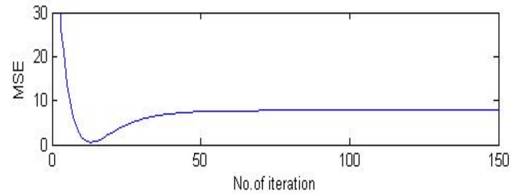
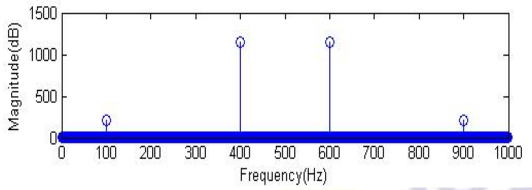


Fig. 1. Results for a sine wave spectrum of (a) Original Signal. (b) An LR observation. (c) Interpolation output. and (d) Super-resolved signal..

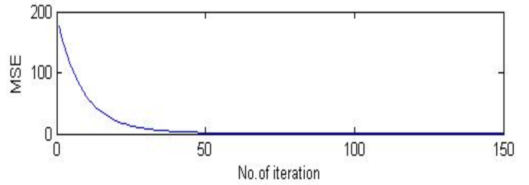




(d)



(e)



(f)

Fig. 2. I. Results for a sine wave spectrum of (a) Super-resolved signal with low blur and low noise (Ideal case). (c) Super-resolved signal with low blur and high noise. (e) Super-resolved signal with high blur and low noise.
II. Results for a MSE plot of (b) MSE with low blur and low noise (Ideal case). (d) MSE with low blur and high noise. and (f) MSE with high blur and low noise.


(a)


(b)


(c)


(d)

Fig. 3. Results for an image. (a) Original image. (b) An LR observation. (c) Initial estimate. (d) Super-resolved image.

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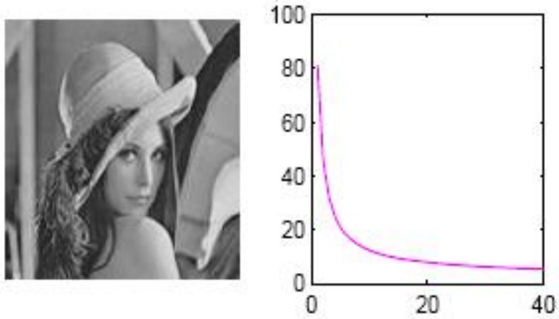
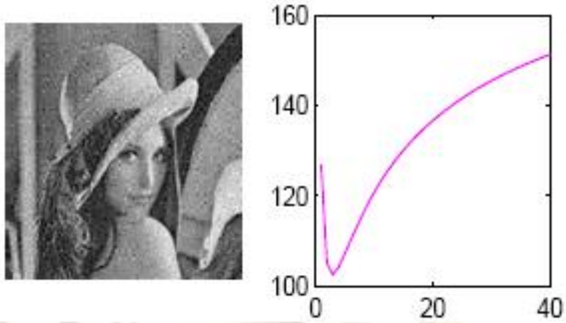

(a)


(b)

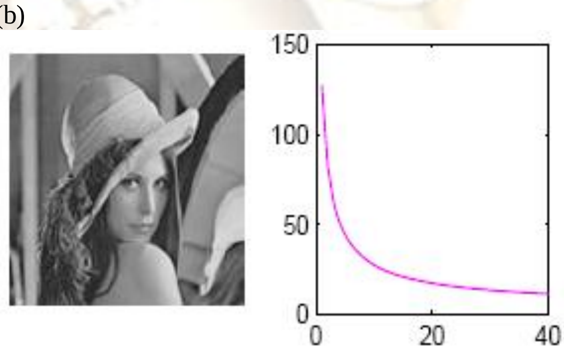

(c)

Fig. 4. Results for an image. (a) An image with low blur and low noise (Ideal case). (b) An image with low blur and high noise and (c) An image with high blur and low noise.

VI. CONCLUSION

The IBP algorithm has been demonstrated in this paper to enhance an image resolution. Algorithm uses the information available from multiple observations to obtain high quality image. An analysis is carried out in the 1D and 2D domain and it is observed that the resolution of image has increased after performing super-resolution on the LR observations. The performance of the IBP algorithm is good in case of low blur and low noise but it is highly sensitive to noise and blur. In IBP algorithm, an improvement can be obtained under noisy conditions by regularization.

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