Liu jun jian/ International Journal of Engineering Research and Applications (IJERA) ISSN: 2248-9622 www.ijera.com
Vol. 2, Issue6, November- December 2012, pp.1364-1366

Two properties of Prequasi-invexity

Liu jun jian
(Teaching affairs office，Chongqing Jiaotong University，Chongqing 400074，P. R. China

Abstract

A classe of prequasi-invex functions, which introduced by Yang.et al. [6], is further studied in this paper. First, a new sufficient condition of prequasi-invex functions is given under weaker certain conditions. Then, the property of the composite to pre(quasi)-invex functions is obtained. Our results extend some known results in the literature.

Key Word : Prequasi-invexity, semistrictly prequasi-invex functions, composite functions.
Clasification: AMS(2000)90C26,26B25

I. Introduction

Convexity and generalized convexity play a central role in mathematical economics and optimization theories. Especially, the research on convexity or generalized convexity becomes one of the most important aspects in mathematical programming. A significant generalization of convex functions termed preinvex functions was introduced by [1]. Yang and Li obtained some new properties of preinvex functions, strictly preinvex functions and semistrictly preinvex functions in [2][4].Yang also further discussed the relationships among convexity, semistrictly convexity and strictly convexity in [3]. Recently, Yang.et established some characterizations of prequasi-invex functions and semistrictly prequasi-invex functions in [6].

Motivated by the the work mentioned above, in the paper, we mainly further discuss the prequasi-invexity, which introduced by Yang.et al. [6]. First, we give a new sufficient condition of prequasi-invex functions under weaker certain conditions. Then, we obtain the property about the composite of pre(quasi)-invex functions. Our results extend some known results in the literature.

Firstly, we give the following definitions .

Definition 1.1[1] A set $K \subseteq R^n$ is said to be invex if there exist a vector function $\eta : R^n \times R^n \to R^n$, such that

$$\forall x, y \in K, \forall \lambda \in [0,1] \Rightarrow y + \lambda\eta(x, y) \in K.$$

Definition 1.2[6]. Let $K \subseteq R^n$ be an invex set with respect to $\eta : R^n \times R^n \to R^n$. Let $f : K \to R$. We say that f is prequasi-invex on K, if $\forall x, y \in K, \lambda \in [0,1]$,

$$f(y + \lambda\eta(x, y)) \le \max\{f(x), f(y)\}.$$

Definition 1.3[6]. Let $K \subseteq R^n$ be an invex set with respect to $\eta : R^n \times R^n \to R^n$. Let $f : K \to R$ .We say that f is semistrictly prequasi-invex on K,

if $\forall \lambda \in (0,1)$ $\forall x, y \in K, f(x) \ne f(y)$,

$$f(y + \lambda\eta(x, y)) < \max\{f(x), f(y)\}.$$

Condition C[5]. Let $\eta : R^n \times R^n \to R^n$. We say that the function η satisfies Condition C, if for any $x, y \in K$, for any $\lambda \in [0,1]$,

$$\eta(y, y + \lambda\eta(x, y)) = -\lambda\eta(x, y),$$

$$\eta(x, y + \lambda\eta(x, y)) = (1-\lambda)\eta(x, y).$$

Condition D[6]. Let the set Γ be invex with respect to $\eta : R^n \times R^n \to R^n$ and let $f : \Gamma \to R$. Then,

$$f(y + \eta(x, y)) \le f(x), \forall x, y \in \Gamma.$$

Example：Let $\boldsymbol{\eta(x, y)} = \begin{cases} \boldsymbol{x - y, (x \ge 0, y \ge 0)} \\ \boldsymbol{x - y, (x < 0, y < 0)} \\ \boldsymbol{-1 - y, (x > 0, y \le 0)} \\ \boldsymbol{1 - y, (x \le 0, y > 0)} \end{cases}$

From the definition of **Condition C,** we can verify η satisfies the Condition C. Another example that η satisfies the Condition C may refer Example 2.4 in [5].

II. Main Results

In this paper, we always assume that:

(i) $K \subseteq R^n$ is an invex set with respect to $\eta : R^n \times R^n \to R^n$,

(ii) $\eta : R^n \times R^n \to R^n$ is a vector function; $f : K \to R$ is a real-valued function on K.

Now, we give a new sufficient condition of prequasi-invex functions.

Theorem 2.1 $\eta : R^n \times R^n \to R^n$ satisfies ConditionC. Let $f : K \to R$ be a semistrictly prequasi-invex function and satisfy Condition D for the same η, and if for each pair of $x, y \in K$, there exists an $\alpha \in (0,1)$, such that

$$f(y + \alpha\eta(x, y)) < \max\{f(x), f(y)\} \tag{2.1}$$

Then f is preqausi-invex on K with respect to η.

Proof: Firstly, From Condition D, if $\lambda = 0,1$, we have

$$f(y+\lambda\eta(x,y)) \le \max\{f(x), f(y)\}$$

$, \forall x, y \in K$

Then, we assume that there exist $x, y \in K$, $\lambda \in (0,1)$ such that

$$f(y+\lambda\eta(x,y)) > \max\{f(x), f(y)\}. \quad (2.2)$$

Let $z = y+\lambda\eta(x,y)$, then

$$f(z) > \max\{f(x), f(y)\} \quad (2.3)$$

If $f(x) \ne f(y)$, it follows from the semistrictly prequasi-invex function of f that

$$f(z) < \max\{f(x), f(y)\}$$

which contradicts (2.3) . Therefore we have $f(x) = f(y)$ and , also by (3)

$$f(z) > f(x) = f(y) \quad (2.4)$$

Note that the pair $x, y \in K$ .From inequality (1) ,there exists an $\alpha \in (0,1)$, such that

$$f(y+\alpha\eta(x,y)) < f(x) = f(y) \quad (2.5)$$

Denote $\bar{x} = y + \alpha\eta(x,y)$. If $\lambda < \alpha$, define $u = (\alpha - \lambda)/\alpha$, then $u \in (0,1)$ .From condition C we have

$$\bar{x} + u\eta(y,\bar{x}) = y + \alpha\eta(x,y) + u\eta(y, y+\alpha\eta(x,y)) =$$

$$y + (\alpha - u\alpha)\eta(x,y) = y + \lambda\eta(x,y) = z$$

By (5) and since f is semistrictly prequasi-invex function on K,

$$f(z) = f(\bar{x} + u\eta(y,\bar{x})) < \max\{f(\bar{x}), f(y)\} = f(y)$$

This conctradicts (4). If $\lambda > \alpha$, define $v = (\lambda - \alpha)/(1-\alpha)$, so $v \in (0,1)$ .From condition C we obtain

$$\bar{x} + v\eta(x,\bar{x}) = y + \alpha\eta(x,y) + v\eta(x, y+\alpha\eta(x,y)) =$$

$$y + [\alpha + v(1-\alpha)]\eta(x,y) = y + \lambda\eta(x,y) = z$$

Since f is semistrictly prequasi-invex function on K and (5) holds, we have

$$f(z) = f(\bar{x} + v\eta(x,\bar{x})) < \max\{f(\bar{x}), f(x)\} = f(x)$$

which contradicts (4). This completes the proof.

Remark2.1. In Theorem 2.1, a uniform $\alpha \in (0,1)$ is not needed, so the corresponding result of [6] has been improved to a great extent.

Corrolary 2.2 $\eta : R^n \times R^n \to R^n$ satisfies ConditionC. Let $f : K \to R$ be a semistrictly preinvex function and satisfy ConditionD for the same η, and if for each pair of $x, y \in K$, there exists an $\alpha \in (0,1)$, such that

$$f(y+\alpha\eta(x,y)) < \alpha f(x) + (1-\alpha) f(y)$$

Then f is preinvex on K with respect to η.

Now we will discuss some new properties of pre(quasi)-invexity.

Theorem 2.3 Let $f : K \to R$ be a prequasi-invex function for the same η, and let $g : I \to R$ be a convex and not decreasing function, where $range(f) \subset I.$ Then the composite function $g(f)$ is a prequasi-invex function on $K.$

Proof. Since f is prequasi-invex function, for any $x, y \in K,\ \alpha \in [0,1]$, we have $f(y+\lambda\eta(x,y)) \le \max\{f(x), f(y)\}$, from the convex and not decreasing properties of g, we obtain

$$g[f(y+\alpha\eta(x,y))] \le g[\max\{f(x), f(y)\}] = \max\{g(f(x)), g(f(y))\}$$

Hence

$$g[f(y+\alpha\eta(x,y))] \le \max\{g(f(x)), g(f(y))\}$$

Thus $g(f)$ is a prequasi-invex function on $K.$ This completes the proof.

From Theorem 2.3 we can get the Corllary 2.4 as follows.

Corllary 2.4. Let $f : K \to R$ be a preinvex function for the same η, and let $g : I \to R$ be a convex and not decreasing function, where $range(f) \subset I.$ Then the composite function $g(f)$ is a preinvex function on $K.$

Theorem 2.5. Let $f : K \to R$ be a semistrictly prequasi-invex function for the same η, and let $g : I \to R$ be a convex and strictly increasing function, where $range(f) \subset I.$ Then the composite function $g(f)$ is a semistrictly prequasi-invex function on $K.$

Proof. The proof is similar to the proof of Theorem2.2.

Remark2.2. Theorem 2.3 and 2.5 generalize theorem

3.2 in [4] from the semistrictly preinvex case to the (semistrictly) prequasi-invex case.

Corllary 2.6[4]. Let $f: K \to R$ be a semistrictly preinvex function for the same η, and let $g: I \to R$ be a convex and strictly increasing function, where $range(f) \subset I$. Then the composite function $g(f)$ is a semistrictly preinvex function on K.

References

[1] Weir T,Mond B. Preinvex Functions in Multiple Objective Optimization[J].J Math Anal Appl 1988,136:29-38.
[2] Yang X M,Li D. On Properties of Preinvex Functions[J].J Math Anal Appl 2001,256:229-241.
[3] Yang X M. Semistrictly convex Functions[J].Opsearch, 1994, 31(1):15-27.
[4] Yang X M, Li D. Semistrictly Preinvex Functions[J].J Math Anal Appl 2001,258:287-308.
[5] Mohan S R, Neogy S K. On Invex sets and Preinvex Functions[J]. J Math Anal Appl,1995,189:901-908.
[6] Yang X M,Yang X Q,Teo K L. Characterizations and Applications of Prequasi-invex Functions[J], Journal of optimization theory and Applications 2001,110(3): 645-668.
[7] Yang X M,Yang X Q,Teo K L.Generalized Invexity and Invariant monotonicicy[J].J Optim Theory Appl 2003,117:607-625.