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# Viscous Dissipation And Soret Effects On An Unsteadymhd Convection Flow Past A Semi-Infinite Vertical Permeable Moving Porous Platewith Thermal Radiation

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## Abstract

**The objective of this paper is to analyse the radiation and soret effects on an unsteady two-dimensional laminar mixed convective boundary layer flow of a viscous, electrically conducting chemically reacting fluid, along a semi-infinite vertical permeable moving plate embedded in a uniform porous medium. In the presence of transverse magnetic field, by taking into account the effects of viscous dissipation. The dimensionlessgoverning equations are solved numerically by a finite element method. Computations are performed for a wide range of the governing flow parameters, viz., the thermal Grashof number, solutal Grashof number, Magnetic field parameter, Permeability parameter, Prandtal number, Heat sourceparameter, Radiation parameter,Soret number, Schmidt number, Chemical reaction parameter and Eckert number. The effects of these flow parameters on the velocity, temperature and concentration are shown graphically. Finally, the effects of various parameters on the skin-friction coefficient, Nusselt number and Sherwood number are shown in Tables.**

**Key words:** MHD, Viscous dissipation, Soret, Thermal radiation, Finite element method.

## INTRODUCTION

The research area of laminar flow is continuously growing, and it is the subject of intensive studies in recent years because of its application in engineering, particularly in aeronautical engineering. One of the most important applications of laminar flow is the calculation of friction drag of bodies in a flow, for example, the drag of a plate at zero incidences, the friction drag of ship, an air foil. It is also important for heat transfer between a body and the fluid around it. The problem of mixed convection under the influence of magnetic field has attracted numerous researchers in view of its applications in geophysics and astrophysics. Soundalgekar et al. [1] investigated the problem of free convection effects on Stokes problem for a vertical plate with transverse applied magnetic field whereas Elbasheshy [2] studied MHD heat and mass transfer problem along a vertical plate under the combined buoyancy effects of thermal and spices diffusion. Magnetohydrodynamics (MHD) natural convection heat transfer flow in porous and non-porous media is of considerable interest in the technical field due to its frequent occurrence in industrial technology and geothermal applications, high-temperature plasmas applicable to nuclear fusion energy conversion, liquid metal fluids, and (MHD) power generation systems. Sparrow and Cess [3] studied the effect of magnetic field on the natural convection heat transfer. Takhar and Ram [4] studied the magneto hydrodynamic free convection flow of water through a porous medium. Damesh [5] studied the magnetohydrodynamics – mixed convection heat transfer problem from a vertical surface embedded in a porous media. El-Kabeir et al. [6] studied the unsteady MHD combined convection over a moving vertical sheet in a fluid saturated porous medium. Hayat et al. [7] examined the effects of Hall current and heat transfer on the rotating flow of a second grade fluid subject to a transverse applied magnetic field past a porous plate. Ali and Mahmood [8] studied the unsteady boundary layer flow of a viscous fluid through porous medium with uniform suction/injection at the wall.

The effects of radiation on MHD flow and heat transfer problem have become more important industrially. At high operating temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. Bestman [9] examined the natural convection boundary layer with suction and mass transfer in a porous medium. His results confirmed the hypothesis that suction stabilizes the boundary layer and affords the most efficient method in boundary layer control yet know. AbdusSattar and Hamid Kalim [10] investigated the unsteady free convection interaction with thermal radiation in a boundary layer flow past a vertical porous plate. Makinde [11] examined the transient free convection interaction with thermal radiation of an absorbing – emitting fluid Muthucumaraswamy and Ganesan [12] studied effect of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate. Deka et al. [13] studied the effect of the first order homogeneous chemical reaction on the process of an unsteady flow past an infinite vertical plate with a constant heat and mass transfer. Soundalgekar and Patti [14] studied the problem of the flow past an impulsively started isothermal infinite vertical plate with mass transfer effects. The effect of foreign mass on the free-convection flow past a semi-infinite vertical plate was studied by Gebhart [15]. Chamkha [16] assumed that the plate is embedded in a uniform porous medium and moves with a constant velocity in the flow direction in the presence of a transverse magnetic field. Raptis [17] investigate the steady flow of a viscous fluid through a vary porous medium bounded by a porous plate subjected to a constant suction velocity by the presence of thermal radiation. Effects of the chemical reaction and radiation absorption on free convection flow through porous medium with variable suction in the presence of uniform magnetic field were studied by SudheerBabu and Satyanarayana [18]. Recently Dulal Pal et al [19] studied Perturbation analysis of unsteady magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction.Cogley[20]studied the Differential approximation for irradiative transfer in a mongrel-gas near equilibrium.Satyanarayana [21] studied theviscous dissipation and thermal radiation effects on an unsteady MHD convection flow past a semi-infinite vertical permeablemoving porous plate.Shivaiah[22] discussed Chemical reaction effects on an unsteady MHD free convective flow past and infinite vertical porous plate with constant suction and heat source.

The object of the present paper is to study the thermal radiation effect on an unsteady magnetohydrodynamic convective flow past a semi-infinite vertical permeable moving porous plate in the presence of soret. The problem is governed by the system of coupled non-linear partial differential equations whose exact solutions are difficult to obtain, if possible. So, Galerkin finite element method has been adopted for its solution, which is more economical from computational point of view.

## MATHEMATICAL ANALYSIS

An unsteady two-dimensional hydromagnetic laminar mixed convective boundary layer flow of a viscous, incompressible, electrically conducting and chemically reacting fluid in an optically thin environment, past a semi-infinite vertical permeable moving plate embedded in a uniform porous medium, in the presence of thermal radiation is considered. The $x'$ - axis is taken in the upward direction along the plate and $y'$ - axis normal to it. A uniform magnetic field is applied in the direction perpendicular to the plate. The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field is negligible. Also, it is assumed that the there is no applied voltage, so that the electric field is absent. The concentration of the diffusing species in the binary mixture is assumed to be very small in comparison with the other chemical species which are present, and hence the Soret and Dufour effects are negligible. Further due to the semi-infinite plane surface assumption, the flow variables are functions of normal distance $y'$ and $t'$ only. Now, under the usual Boussinesq's approximation, the governing boundary layer equations are:

Continuity equation:

$$\frac{\partial v'}{\partial y'}=0 \tag{1}$$

Momentum equation:

$$\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}=g\beta(T'-T'_\infty)+g\beta^*(C'-C'_\infty)-\frac{1}{\rho}\frac{\partial p'}{\partial x'}+\nu\frac{\partial^2 u'}{\partial y'^2}-\frac{\nu}{K'}u'-\frac{\sigma B_0^2}{\rho}u' \tag{2}$$

Energy Equation:

$$\frac{\partial T'}{\partial t'}+v'\frac{\partial T'}{\partial y'}=\frac{1}{\rho C_P}\left[k\frac{\partial^2 T'}{\partial y'^2}-\frac{\partial q'}{\partial y'}\right]-\frac{Q_o}{\rho C_P}(T'-T'_\infty)+\frac{\nu}{c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{3}$$

Concentration Equation:

$$\frac{\partial C'}{\partial t'}+v'\frac{\partial C'}{\partial y'}=D\frac{\partial^2 C'}{\partial y'^2}-K'_\Gamma(C'-C'_\infty)+\frac{D_m k_T}{T_m}\frac{\partial^2 T'}{\partial y'^2} \tag{4}$$

The boundary conditions for the velocity, temperature and concentration fields are:

$$u'=u'_p\ , \qquad T'=T'_\infty+\varepsilon(T'_W-T'_\infty)e^{n't'}\ ,C'=C'_\infty+\varepsilon(C'_W-C'_\infty)e^{n't'} \qquad at \qquad y'=0$$

$$u' = U'_\infty = U_O(1+\varepsilon A e^{n't'}), \qquad T' \to T'_\infty, \qquad C' \to C'_\infty, \qquad as \quad y' \to \infty \tag{5}$$

Where $u'_p$ is the plate velocity, $T'_W$ and $C'_W$ Dimensional temperature and concentration respectively, $U'_\infty$ the free stream velocity, $U_O$ and $n'$ the constants.

From Equation (1) it is clear that the suction velocity at the plate is either a constant or function of time only. Hence the suction velocity normal to the plate is assumed in the form

$$v' = -V_O(1+\varepsilon A e^{n't'}) \tag{6}$$

Where $A$ is the suction parameter and $\varepsilon A << 1$. Here $V_O$ is mean suction velocity, which are a non-zero positive constant and the minus sign indicates that the suction is towards the plate.
Outside the boundary layer, Equation (2) gives

$$-\frac{1}{\rho}\frac{\partial p'}{\partial x'} = \frac{dU'_\infty}{dt'} + \frac{\nu}{K'}U'_\infty + \frac{\sigma}{\rho}B_o^2 U'_\infty \tag{7}$$

Since the medium is optically thin with relatively low density, the radiative heat flux given by Equation (3), in thespirit of Cogley et al. [20], becomes

$$\frac{\partial q'}{\partial y'} = 4(T' - T'_\infty)I' \tag{8}$$

$$I' = \frac{R\rho c_p V_o^2}{4\nu}$$

In order to write the governing equations and the boundary conditions in dimensionless form, the following non dimensional quantities are introduced.

$$u = \frac{u'}{U_o}, v = \frac{v'}{V_o}, y = \frac{V_o y'}{\nu}, t = \frac{t' V_o^2}{\nu}, P_\Gamma = \frac{\rho c_p \nu}{k}, S_c = \frac{\nu}{D}, T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, R = \frac{4\nu I'}{\rho C_p V_o^2}$$

$$C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, K = \frac{K' V_o^2}{\nu^2}, n = \frac{\nu n'}{V_o^2}, G_\Gamma = \frac{g\beta\nu(T'_w - T'_\infty)}{U_o V_o^2}, G_m = \frac{\nu g\beta(C'_w - C'_\infty)}{U_o V_o^2}, Q = \frac{\nu Q_o}{\rho C_p V_o^2} \tag{9}$$

$$M = \frac{\sigma B_o^2 \nu}{\rho V_o^2}, K_\Gamma = \frac{K'_\Gamma \nu}{V_o^2}, S_\Gamma = \frac{D_m k_T (T'_w - T'_\infty)}{\nu T_m (C'_w - C'_\infty)}, E_C = \frac{U_o^{\ 2}}{C_p(T'_w - T'_\infty)}, U_\infty = \frac{U'_\infty}{U_o}, U_p = \frac{u'_p}{U_o}$$

In view of Equations (5), (6), (7), (8) and (9), Equations (1), (2), (3) and (4) reduce to the following dimensionless form:

$$\frac{\partial u}{\partial t} - (1+\varepsilon A e^{nt})\frac{\partial u}{\partial y} = \frac{dU_\infty}{dt} + G_\Gamma\theta + G_m C + \frac{\partial^2 u}{\partial y^2} + N(U_\infty - u) \tag{10}$$

$$\frac{\partial T}{\partial t} - (1+\varepsilon A e^{nt})\frac{\partial T}{\partial y} = \frac{1}{P_\Gamma}\left[\frac{\partial^2 T}{\partial y^2} - RT\right] + E_C\left(\frac{\partial u}{\partial y}\right)^2 - QT \tag{11}$$

$$\frac{\partial C}{\partial t} - (1+\varepsilon A e^{nt})\frac{\partial C}{\partial y} = \frac{1}{S_C}\frac{\partial^2 C}{\partial y^2} + S_\Gamma\frac{\partial^2\theta}{\partial y^2} - K_\Gamma C \tag{12}$$

where $N = M + \left(\frac{1}{K}\right)$ and $G_\Gamma$, $G_m$, $P_\Gamma$, $R$, $E_C$, $S_C$, $K$, $S_\Gamma$ and $Q$ are the thermal Grashof number, solutalGrashof Number, Prandtl Number, Radiation parameter, Eckert number, Schmidt number, chemical reaction parameter,Soret number and Heat sourceparameter respectively.

The corresponding dimensionless boundary conditions are:

$$u = U_p, \quad T = 1 + \varepsilon e^{nt}, \quad C = 1 + \varepsilon e^{nt} \qquad at \quad y = 0$$

$$u \to U_\infty, \quad T \to 0, \quad C \to 0 \qquad as \quad y \to \infty \tag{13}$$

The mathematical formulation of the problem is now completed. Eqs. (10)-(12) are coupled non-linear systems

Of partial differential equations, and are to be solved by using the initial and boundary conditions given in Eq(13). However, exact solutions are difficult if possible. Hence these equations are solved by Galerkin finite element method.

## METHOD OF SOLUTION.

Applying the Galerkin finite element method for Eq. (10) Over the element $(e)$ $(y_j \le y \le y_k)$ is :

$$\int_{y_J}^{y_K} N^{(e)^T}\left(\frac{\partial^2 u^{(e)}}{\partial y^2} + B\frac{\partial u^{(e)}}{\partial y} - \frac{\partial u^{(e)}}{\partial t} - Nu^{(e)} + R_1\right)dy = 0 \tag{14}$$

Where

$$B = 1 + \varepsilon A e^{nt} \quad R_1 = n\varepsilon e^{nt} + G_\Gamma \theta + G_m C + NU_\infty \quad N = M + \frac{1}{K}$$

Let the linear piecewise approximation solution

$$u^{(e)} = N_j(y)u_j(t) + N_k(y)u_k(t) = N_j u_j + N_k u_k$$

Where

$$N_j = \frac{y_k - y}{y_k - y_j} \qquad N_k = \frac{y - y_j}{y_k - y_j} \qquad N^{(e)^T} = \begin{bmatrix} N_j & N_k \end{bmatrix}^T = \begin{bmatrix} N_j \\ N_k \end{bmatrix}$$

The Galerkin expansion for the differential equation (14) becomes

$$\left. N^{(e)^T}\frac{\partial u^{(e)}}{\partial y}\right\}_{y_j}^{y_K} - \int_{y_j}^{y_K}\left\{\frac{\partial N^{(e)^T}}{\partial y}\frac{\partial u^{(e)}}{\partial y} - N^{(e)^T}\left(B\frac{\partial u^{(e)}}{\partial y} + \frac{\partial u^{(e)}}{\partial t} + Nu^{(e)} - R_1\right)\right\}dy = 0 \tag{15}$$

Neglecting the first term in equation (15)we gets

$$\int_{y_j}^{y_K}\left\{\frac{\partial N^{(e)^T}}{\partial y}\frac{\partial u^{(e)}}{\partial y} - N^{(e)^T}\left(B\frac{\partial u^{(e)}}{\partial y} - \frac{\partial u^{(e)}}{\partial t} - Nu^{(e)} + R_1\right)\right\}dy = 0$$

$$\frac{1}{l^{(e)}}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix} - \frac{B}{2}\begin{bmatrix}-1 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix} + \frac{l_{(e)}}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}\dot{u}_j\\ \dot{u}_k\end{bmatrix} + \frac{Nl^{(e)}}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix} = R_1\frac{l_{(e)}}{2}\begin{bmatrix}1\\ 1\end{bmatrix}$$

Where $l^{(e)} = y_k - y_j = h$ and dot denotes the differentiation with respect to $t$.

We write the element equations for the elements $y_{i-1} \le y \le y_i$ and $y_i \le y \le y_{i+1}$ assemble three element equations, we obtain

$$\frac{1}{l^{(e)^2}}\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{bmatrix} - \frac{B}{2l^{(e)}}\begin{bmatrix}-1 & 1 & 0\\ -1 & 0 & 1\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{bmatrix} + \frac{1}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}\dot{u}_{i-1}\\ \dot{u}_i\\ \dot{u}_{i+1}\end{bmatrix} + \frac{N}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{bmatrix} = \frac{R_1}{2}\begin{bmatrix}1\\ 2\\ 1\end{bmatrix} \tag{16}$$

Now put row corresponding to the node $i$ to zero, from equation (16) the difference schemes is

$$\frac{1}{l^{(e)^2}}[-u_{i-1} + 2u_i - u_{i+1}] - \frac{B}{2l^{(e)[\,]}}[-u_{i-1} + u_{i+1}] + \frac{1}{6}[\dot{u}_{i-1} + 4\dot{u}_i + \dot{u}_{i+1}] + \frac{N}{6}[u_{I-1} + 4u_i + u_{i+1}] = R_1$$

Applying the Trapezoidal rule, following system of equations in Crank-Nicholson method are obtained:

$$A_1 u_{i-1}^{j+1} + A_2 u_i^{j+1} + A_3 u_{i+1}^{j+1} = A_4 u_{i-1}^{j} + A_5 u_i^{j} + A_6 u_{i+1}^{j} + R^* \tag{17}$$

Where

$$A_1 = 2 - 6r + 3Brh + Nk \qquad A_4 = 2 + 6r - 3Brh - Nk$$

$$A_2 = 8 + 12r + 4Nk \qquad A_5 = 8 - 12r - 4Nk$$

$$A_3 = 2 - 6r - 3Brh + Nk \qquad A_6 = 2 + 6r + 3Brh - Nk$$

$$R^* = 12(G_\Gamma)k\theta_i^j + 12(G_m)kC_i^j + 12kNU_\infty + 12kn\varepsilon e^{nt}$$

Applying similar procedure to equation (11) and (12) then we gets

$$B_1 T_{i-1}^{j+1} + B_2 T_i^{j+1} + B_3 T_{i+1}^{j+1} = B_4 T_{i-1}^{j} + B_5 T_i^{j} + B_6 T_{i+1}^{j} + R^{**} \tag{18}$$

$$C_1 C_{i-1}^{j+1} + C_2 C_i^{j+1} + C_3 C_{i+1}^{j+1} = C_4 C_{i-1}^{j} + C_5 C_i^{j} + C_6 C_{i+1}^{j} + R^{***} \tag{19}$$

Where

$B_1 = 4P_\Gamma - 12r + 6Drh + 2Fk$ $\quad$ $B_2 = 16P_\Gamma + 24r + 8Fk$

$B_3 = 4P_\Gamma - 12r - 6Drh + 2Fk$ $\quad$ $B_4 = 4P_\Gamma + 12r - 6Drh - 2Fk$

$B_5 = 16P_\Gamma - 24r - 8Fk$ $\quad$ $B_6 = 4P_\Gamma + 12r + 6Drh - 2Fk$

$R^{**} = 24P_\Gamma r E_C (u[i+1] - u[i])^2$

$C_1 = 2S_C - 6r + 3BS_C rh + S_C K_\Gamma k$ $\quad$ $C_2 = 8S_C + 12r + 4S_C K_\Gamma k$

$C_3 = 2S_C - 6r - 3BS_C rh + S_C K_\Gamma k$ $\quad$ $C_4 = 2S_C + 6r - 3BS_C rh - S_C K_\Gamma k$

$C_5 = 8S_C - 12r - 4S_C K_\Gamma k$ $\quad$ $C_6 = 2S_C + 6r + 3BS_C rh - S_C K_\Gamma k$

$R^{***} = 12rS_C S_\Gamma (\theta[i-1] - 2\theta[i] + \theta[i+1])$

Here $r = \dfrac{k}{h^2}$, $D = BP_\Gamma$, $F = R + P_\Gamma Q$, and $h, k$ are the mesh sizes along $y$-direction and time $t$ -direction respectively. Index $i$ refers to the space and $j$ refers to the time. In Equations (17)-(19), taking $i$ = 1(1) $n$ and using initial and boundary conditions (14), the following system of equations are obtained:

$$A_i X_i = B_i \qquad i = 1(1)3 \tag{20}$$

Where $A_i$ 's are matrices of order $n$ and $X_i, B_i$ 's column matrices having $n$ – components. The solutions of above system of equations are obtained by using Thomas algorithm for velocity, temperature and concentration. Also, numerical solutions for these equations are obtained by
C-program. In order to prove the convergence and stability of finite element method, the same
C-program was run with slightly changed values of $h$ and $k$ and no significant change was observed in the values of $u, T$ and $C$. Hence, the finite element method is stable and convergent.

## SKRIN FRICTION

The skin-friction, Nusselt number and Sherwood number are important physical parameters for this type of boundary layer flow. The skin friction, rate of heat and mass transfer are

Skin friction coefficient ( $C_f$ ) is given by

$$C_f = \frac{\tau_w'}{\rho U_o V_o} = \left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{21}$$

The rate of heat transfer coefficient, which in the non-dimensional form in terms of the Nusselt number is givenby

$$Nu = -x \frac{\left(\frac{\partial T'}{\partial y'}\right)_{y'=0}}{T_w' - T_\infty'} \Rightarrow Nu \, \mathrm{Re}_x^{-1} = -\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{22}$$

The rate of mass transfer coefficient, which in the non-dimensional form in terms of the Sherwood number, isgiven by

$$Sh = -x \frac{\left(\frac{\partial C'}{\partial y'}\right)_{y'=0}}{C_w' - C_\infty'} \Rightarrow Sh \, \mathrm{Re}_x^{-1} = -\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

$$Sh = \left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{23}$$

Where $\mathrm{Re}_x = \dfrac{V_o x}{\nu}$ is the local Reynolds number.

## RESULTS AND DISCUSSION

The formulation of the problem that accounts for the effects of the chemical reaction on an unsteady MHD convection flow past a semi-infinite vertical porous plate with viscous dissipation and soret effects is performed in the preceding sections. The governing equations of the flow field are solved analytically by using a finite element method. The expressions for the velocity, temperature, concentration, skin-friction, Nusselt number, and Sherwood number are obtained. To get a physical insight of the problem, the above physical quantities are computed numerically for different values of the governing parameters, viz., the thermal Grashof number $G_r$, the solutal Grashof number $G_m$, the magnetic parameter $M$, the thermal radiation the permeability parameter $K$, the Prandtl number $P_r$, the Radiation parameter $R$, the Eckert number $E_C$, the Schmidt number $S_C$, the chemical reaction parameter $K_r$, the Soret number $S_r$ and Heat source parameter $Q$ Here we fixed $\varepsilon = 0.1, n = 0.1, t = 1.0, A = 0.5, U_p = 0.5$. In order to ascertain the accuracy of the numerical results; the present results are compared with the existed results of SatyaNarayana et al [21].The results of these comparisons are found to be in very good agreement [See table 1].

The influence of the thermal Grashof number on the velocity is presented in Fig 1. The thermal Grashof number signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Also, as $G_r$ increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity.

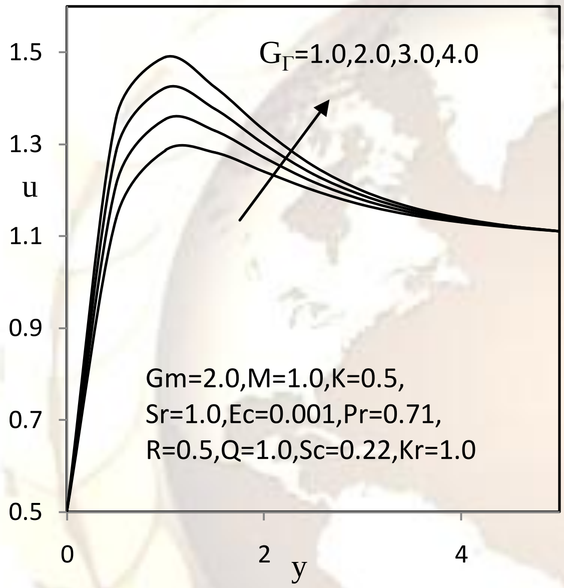



Fig. 1: .Effects of $G_r$ on Velocity

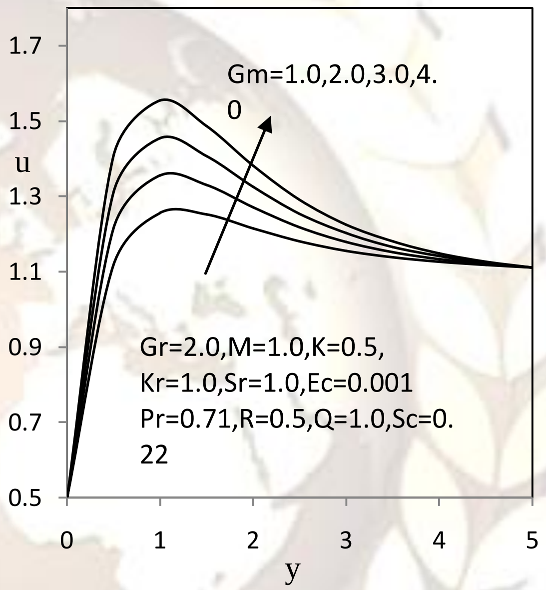



Fig. 2: Effects of $G_m$ on Velocity

Fig 2 presents typical Velocity profiles in the boundary layer for various values of the solutal Grashof number $G_m$, while all other parameters are kept at some fixed values. The solutal Grashof number $G_m$ defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decays smoothly to approach the free stream value.It is noticed that velocity increases with increasing in solute Grashof number.

For various values of the Magnetic parameter $M$, the Velocity profiles are plotted in Fig 3. It can be seen that as $M$ increases, the velocity decreases. This result qualitatively agrees with the expectations, since the magnetic field exerts a retarding force on the flow.

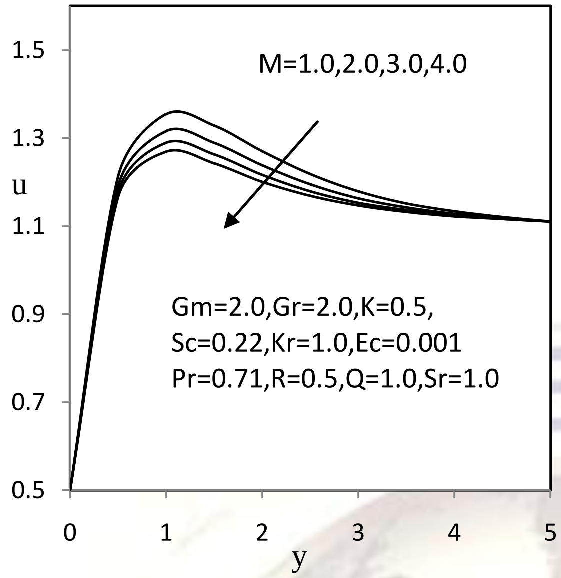



Fig. 3: Effects of $M$ on Velocity

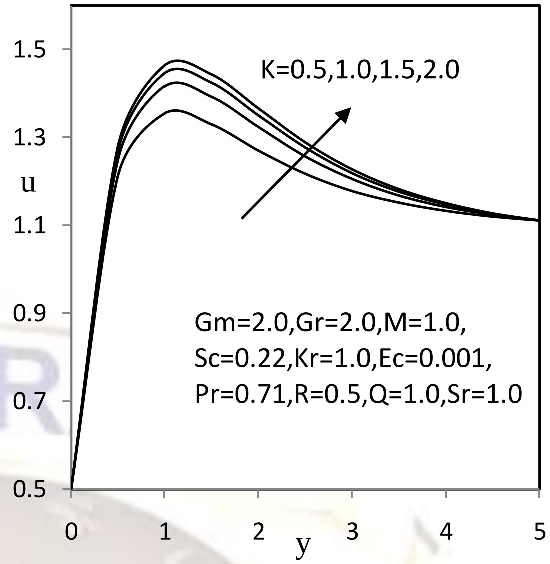
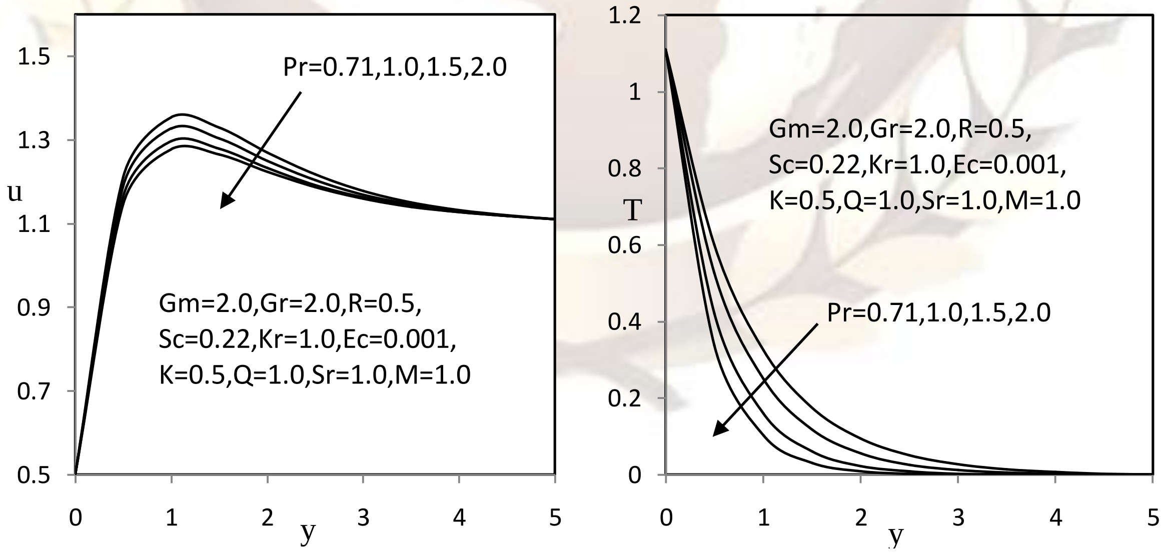



Fig. 4: Effects of $K$ on Velocity.

The effect of the Permeability parameter $K$ on the Velocity field is shown in Fig 4. An increase the resistance of the porous medium which will tend to increase the velocity. This behaviour is evident from Fig 4.

Figs 5(a) and 5(b) illustrate the Velocity and Temperature profiles for different values of the Prandt number $P_r$. The Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. The numerical resultsshow that the effect of increasing values of Prandtl number results in a decreasing velocity (Fig 5 (a)). From Fig5(b), it is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of $P_r$ are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated plate more rapidly than for higher values of $P_r$. Hence in the case of smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced.




(a) (b)

Fig.5: Effects of $P_r$ on a) Velocity and b) Temperature profile

Figs 6(a) and 6(b) illustrate the Velocity and Temperature profiles for different values of Radiation parameter $R$, the numerical results show that the effect of increasing values of radiation parameterresult in a decreasing velocity and temperature.

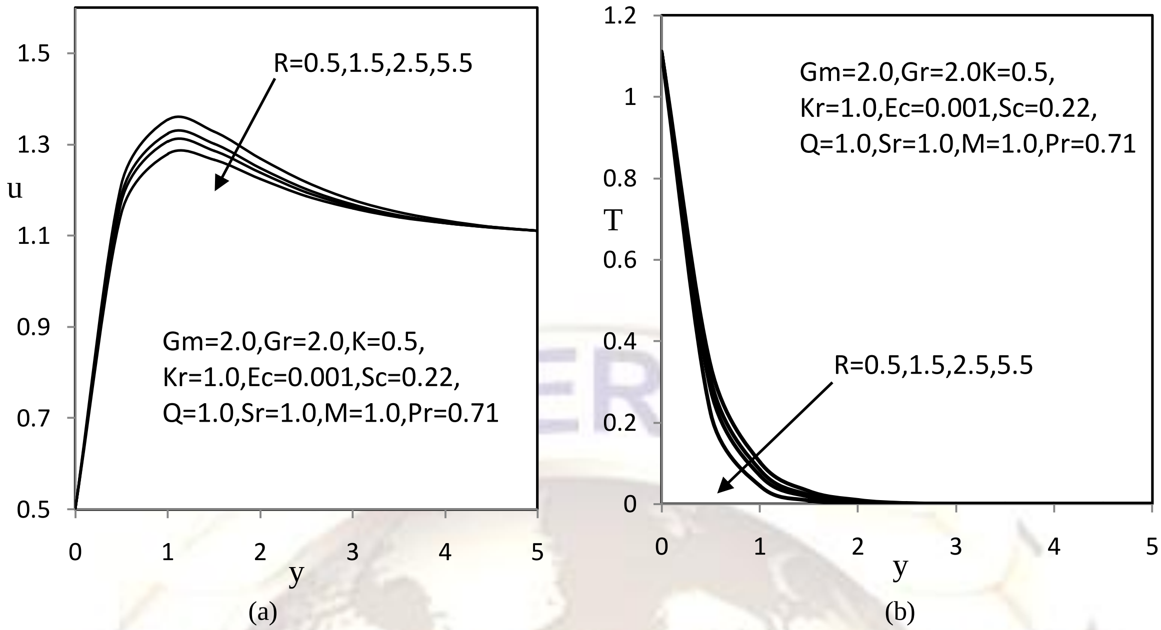



Fig. 6: Effects of $R$ on a) Velocity and b) Temperature profile

For different values of the Heat source parameter $Q$, the Velocity and Temperature profiles are plotted in Figs 7(a) and 7(b) respectively. The numerical results show that the effect of increasing values of Heat source, result in a decreasing velocity and temperature.

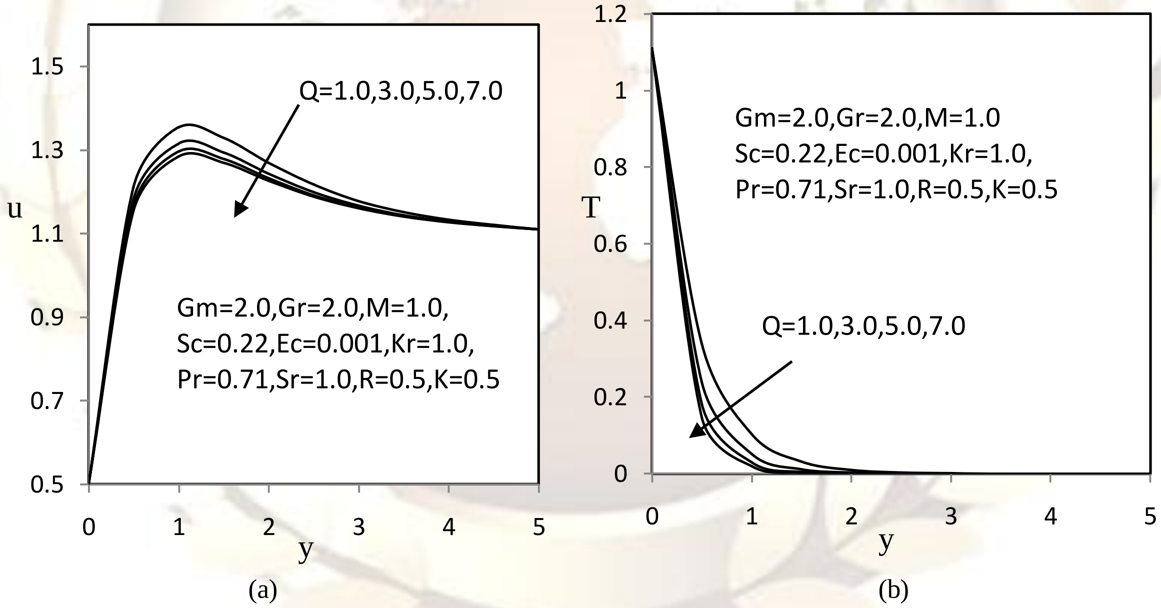
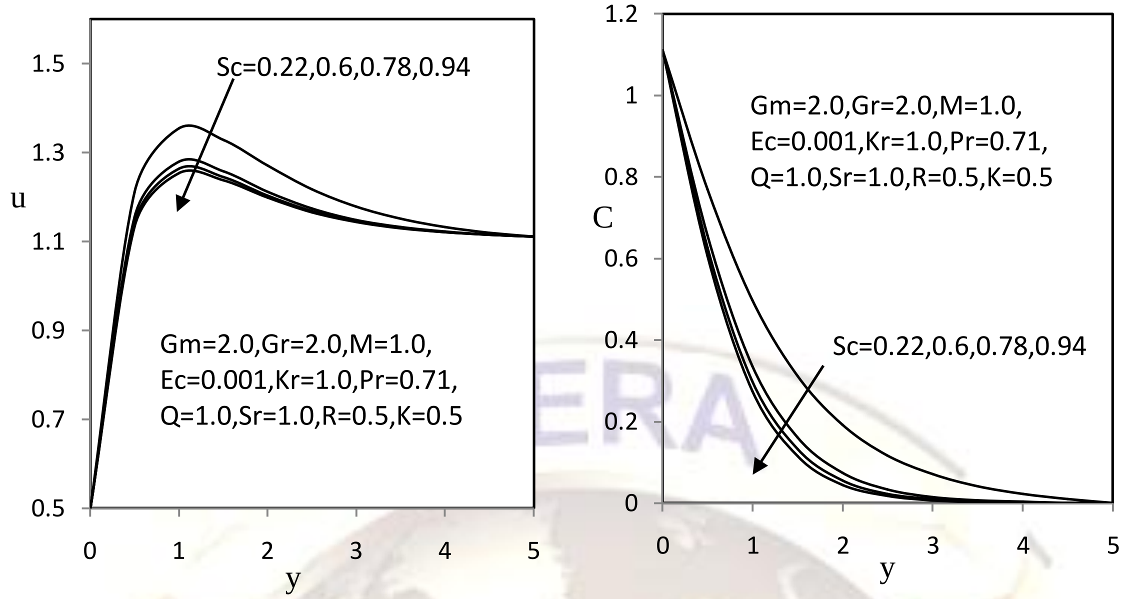



Fig. 7: Effects of $Q$ on a) Velocity and b) Temperature profile

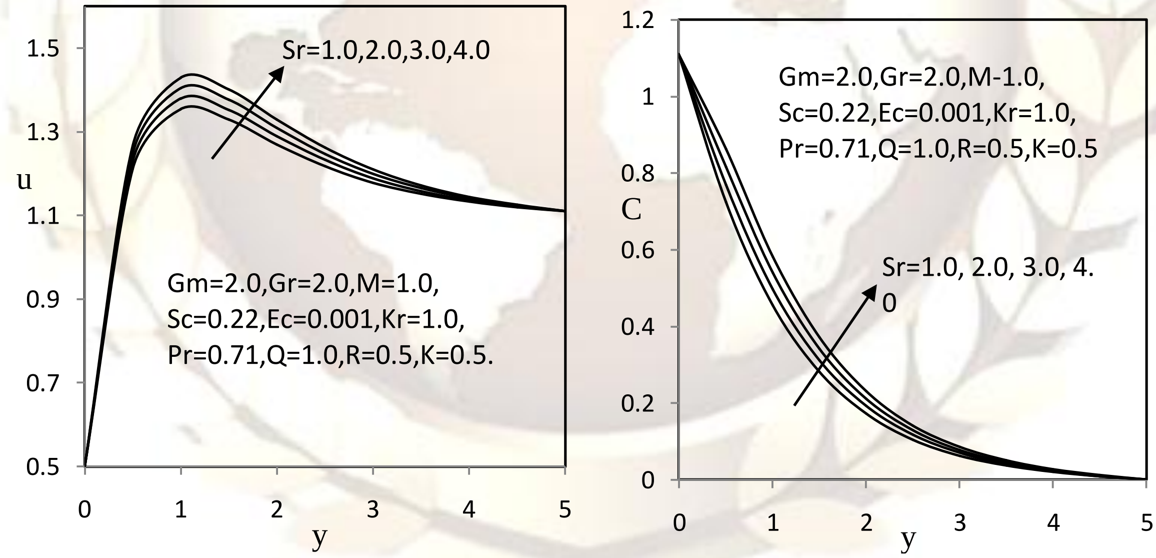
The influence of the Schmidt number $S_C$ on the Velocity and Concentration profiles are plotted in Figs 8(a) and 8(b) respectively. The Schmidt number embodies the ratio of the momentum to the mass diffusivity. The Schmidt number therefore quantifies the relative effectiveness of momentum and mass transport by diffusion inthe hydrodynamic (velocity) and concentration (species) boundary layers. As the Schmidt number increases the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the velocity and concentration boundary layers. These behaviours are clear from Figs 8(a) and 8(b).




(a) (b)

Fig. 8: Effects of $S_C$ on a) Velocity and b) Concentration profile

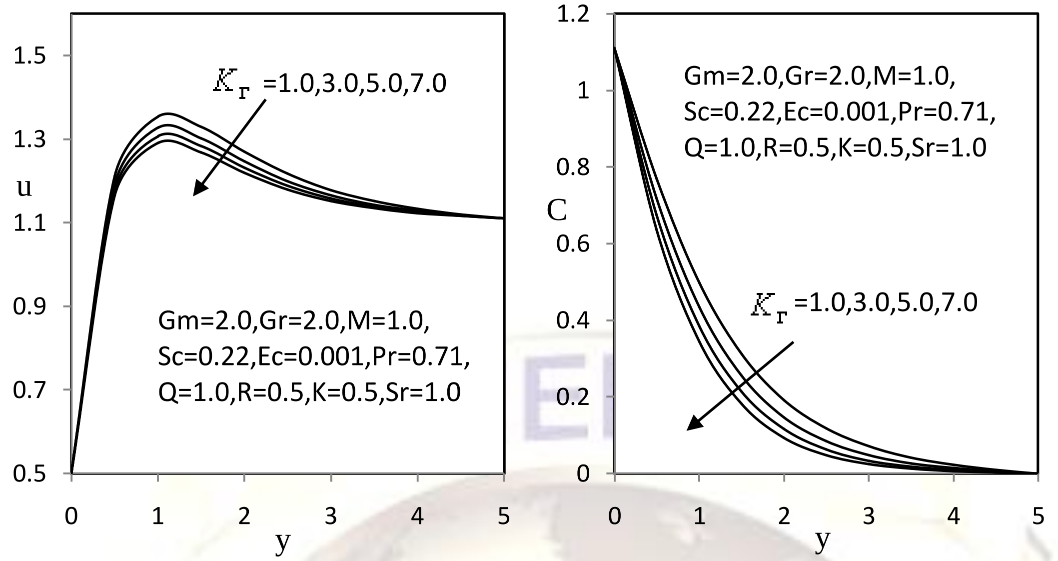
Figs 9(a) and 9(b) depict the Velocity and Concentration profiles for different values of the Soret number $S_\Gamma$. The Soret number $S_\Gamma$ defines the effect of the temperature gradients inducing Significant mass diffusion effects. It is noticed that an increase in the Soret number $S_\Gamma$ results in an increase in the velocity andconcentration within the boundary layer.




(a) (b)

Fig. 9: Effects of $S_\Gamma$ on a) Velocity and b) Concentration profile

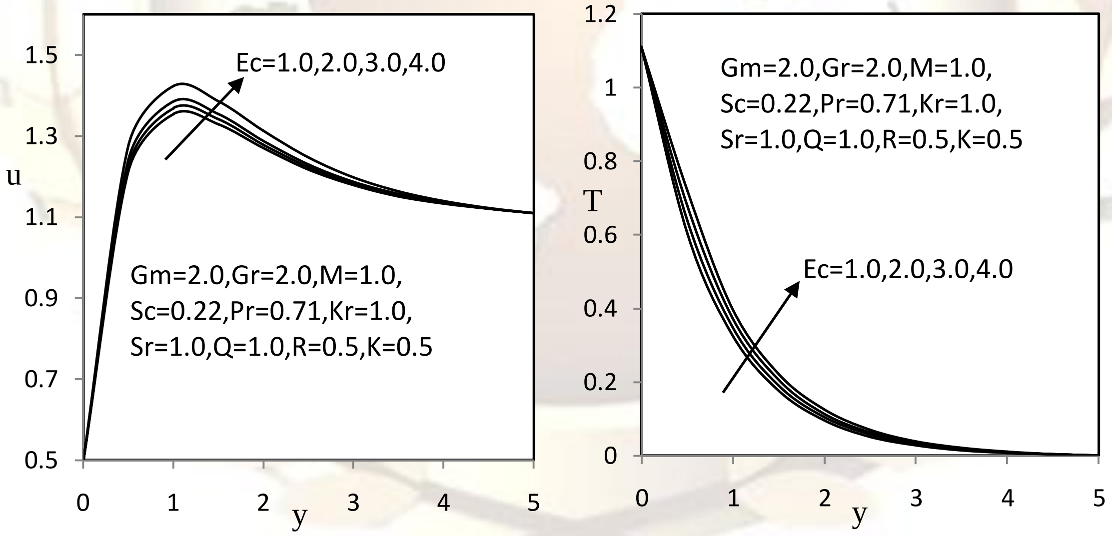
Figs 10(a) and 10(b) illustrates the behaviourVelocity and Concentration for different values of chemical reaction parameter $K_\Gamma$. It is observed that an increase in leads to a decrease in both the values of velocity and concentration. A distinct velocity escalation occurs near the wall after which profiles decay smoothly to the stationary value in free stream. Chemical reaction therefore boosts momentum transfer i.e. accelerates the flow.




(a) (b)

Fig. 10: Effects of $K_r$ on a) Velocity and b) Concentration profile

The effect of the viscous dissipation parameter i.e., the Eckert number $E_C$ on the velocity and temperature are shown in figures 11(a) and 11 (b) respectively. The Eckert number $E_C$ expresses the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. The positive Eckert number implies cooling of the plate i.e., loss of heat from the plate to the fluid. Hence, greater viscous dissipative heat causes a rise in the temperature as well as the velocity, which is evident from Figures 11(a) and 11(b).




(a) (b)

Fig. 11: Effects of $Ec$ on a) Velocity and b) Temperature profile

In order to ascertain the accuracy of the numerical results, the present results are compared with the existed results of SatyaNarayana et al [21].The results of these comparisons are found to be in very good agreement [See table 1]. Tables 2 and 3 show the effects of the Eckert number and Soret number on the skin friction $C_f$, Nusselt number $Nu$, and Sherwood number $Sh$. From table 2, it is observed that as Eckert number increases the skin-friction decreases, and the Nusselt number decreases. From Table 3, it can be seen that as the Soret number increases, the skin-friction increases and the Sherwood number decreases.

Table 1. Effects of $R$ on $C_f$ and $Nu$ { $G_r = G_m = 2.0$, $M = 1.0$, $K = 0.5$, $Ec = 0.001$, $Q = 4.0$, $P_r = 0.71$, $Sc = 0.6$, $K_r = 1.0$, $S_r = 0.0$, $\varepsilon = 0.001$, $n = 0.1$, $t = 1.0$, $A = 0.5$, $U_p = 0.5$ }

| $R$ | P.V SatyaNarayana[21] $C_f$ | Present Results $C_f$ | P.V SatyaNarayana [21] $Nu$ | Present Results $Nu$ |
|---|---|---|---|---|
| 0.00 | 2.743 | 2.74301 | -3.854 | -3.85401 |
| 0.25 | 2.062 | 2.06202 | -3.932 | -3.93201 |
| 0.50 | 1.650 | 1.65001 | -3.966 | -3.96602 |
| 0.75 | 1.451 | 1.45101 | -3.993 | -3.99301 |

Table 2. Effects of $Ec$ on $C_f$ and $Nu$ { $G_r = G_m = 2.0$, $M = 1.0$, $K = 0.5$, $R = 0.5$, $Q = 5.0$, $P_r = 0.71$, $Sc = 0.6$, $K_r = 0.5$, $S_r = 0.5$, $\varepsilon = 0.001$, $n = 0.1$, $t = 1.0$, $A = 0.5$, $U_p = 0.5$ }

| $Ec$ | $C_f$ | $Nu$ |
|---|---|---|
| 0.01 | 2.19601 | -2.78201 |
| 0.03 | 2.27301 | -2.83001 |
| 0.05 | 2.35102 | -2.87802 |
| 0.07 | 2.42901 | -2.97201 |

Table 3. Effects of $K_r$ on $C_f$ and $Nu$ { $G_r = G_m = 2.0$, $M = 1.0$, $K = 0.5$, $R = 0.5$, $Q = 5.0$, $P_r = 0.71$, $Ec = 0.001$, $Sc = 0.6$, $K_r = 0.5$, $\varepsilon = 0.001$, $n = 0.1$, $t = 1.0$, $A = 0.5$, $U_p = 0.5$ }

| $S_r$ | $C_f$ | $Sh$ |
|---|---|---|
| 1.0 | 3.12546 | -1.10801 |
| 2.0 | 3.25621 | -1.32001 |
| 3.0 | 3.37125 | -1.68002 |
| 4.0 | 3.49678 | -1.82501 |

## CONCLUSIONS

In this article a mathematical model has been presented for the viscous dissipation and Soret effects on unsteady MHD flowpast a semi-infinite vertical permeable moving porous plate with thermal radiation. The non- dimensional governing equations aresolved with the help of finite element method. The conclusions of the study are as follows:

1. The velocity increases with the increase thermal Grashof number and solutalGrashof number.
2. The velocity decreases with an increase in the magnetic parameter.
3. The velocity increases with an increase in the permeability parameter.
4. Increasing the Prandtl number substantially decreases the translational velocity and the temperature function.
5. Increasing the Radiation parameter reduces both velocity and temperature.
6. The velocity and temperature increases with an increase in the Eckert number.
7. The velocity as well as concentration decreases with an increase in the Schmidt number.
8. An increase in the Soret number leads to increase in the velocity and concentration.
9. The velocity as well as concentration decreases with an increase in the chemical reactionparameter.

## NOMENCLATURE

$B_o$, Magnetic induction; $C$, Dimensionless concentration; $C'$, Species concentration in thefluid; $C'_w$, Concentration of the plate; $C'_\infty$, Concentration in the fluid far away from the plate; $C_p$, Specific heat at constant pressure; $D$, Mass diffusivity; $E_C$, Eckert number; $g$, Acceleration due to gravity; $G_r$, Thermal Grashof number; $G_m$, solutalGrashof number; $k$, Thermal conductivity of the fluid; $K_r$, Chemical reaction parameter; $K$, Permeability parameter; $M$, Magnetic field parameter; $P_r$, Prandtl number; $q'$, Radiative heat flux; $R$, Radiation Parameter; $S_C$, Schmidth number; $S_r$, Soret number; $t'$, Time; $t$

, Dimensionless time; $T'_w$, Temperature of the plate; $T'_\infty$, Temperature of the fluid far away from the plate; $u'$, Velocity of the fluid in the $x$ – Direction; $u$, Dimensionless velocity; $v$, Kinematic viscosity; $u'_p$, Plate velocity; $U'_\infty$, Free stream velocity; $y$, Dimensionless coordinate axis normal to the plate; $y'$, Coordinate axis normal to the plate

**Greek symbols:** $\beta$, Volumetric coefficient of thermal expansion; $\beta^*$, Volumetric coefficient of expansion with Concentration; $\mu$, Coefficient of viscosity; $\rho$, Density of the fluid; $\sigma^*$, Estefan Boltzmann constant; $\theta$, Dimensionless temperature; $\eta$, Similarity parameter;

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