Computation of Stress Intensity Factor of Cracked Aluminium Plate Using Virtual Crack Closure Technique

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ABSTRACT

The stress intensity factor (SIF) is important parameter for estimating the life of cracked structure. There are several techniques for calculating SIF like displacement extrapolation method, j –integral technique and virtual crack closure technique. In this paper SIF of an Aluminium plate with central crack is computed using virtual crack closure technique (VCCT) with the help pf Finite element software ANSYS .The values obtained are compared with empirical values and displacement extrapolation method values and found that they are in order. Using Paris law the life of the Aluminium plate with fatigue load is also predicted.

Key words: stress intensity factor, strain energy release rate, displacement extrapolation technique, and virtual crack closure technique.

1. INTRODUCTION:

Fracture mechanics is a field of solid mechanics that deals with the mechanical behavior of cracked bodies.

Stress intensity factor is important parameter linear elastic fracture mechanics

For the structure contains crack and singular stress fields. By using the SIF the stress intensity at the crack tip is measured. The stress intensity factor calculated using the stress and strain analysis or using the parameter of the strain energy release rate during the crack growth. The stress intensity factor performed in analytical or numerical techniques. Analytical technique is difficult to calculate but analytical solution can be applied for a range of crack Lengths. In numerical technique stress and strain field calculation are required for each crack length. For complex structures, it is difficult to perform an analysis taking into account all boundary effects near the crack tip, so the numerical calculation of SIF has some advantages for these structures. The Evolution of computers (hardware and software) permits the use of more complex numerical techniques and to obtain solutions with smaller calculation time. Hence, the numerical techniques for estimating stress intensity factors are nowadays more popular than the analytical techniques

Some numerical methods for SIF's – J-integral and virtual crack closure technique, displacement extrapolation, Singularity subtraction method[5], virtual crack closure technique and J-integral techniques are base on energetic assumption. In this paper worked on virtual crack closure technique (VCCT).

STRAIN-ENERGY RELEASE RATE:

The traditional-materials strength stress analysis of a cracked component may be hardly tackled. Although the stress discretisation may be improved by using crack tip elements, the meaningful analysis is generally that performed using the energy release rate (G)[7].

It is well-known that plastic deformation occurs in engineering metal, alloys and some polymers. Due to this fact, Irwin[1] and Orowan[2] modified Griffith's elastic surface energy expression, $2\gamma_S = \frac{\pi\beta a\sigma^2}{E}$, by adding a plastic deformation energy or plastic strain work γ_p in the fracture process. For tension loading, the total elastic-plastic strain-energy is known as the strain energy release rate $G_{1=}$which is the energy per unit crack surface area available for infinitesimal crack extension [3], thus.

$$G_{1=} 2(\gamma_S + \gamma_P) \text{ --- } (1)$$

$$G_1 = \frac{\pi a\sigma^2}{E'} --- (2)$$

Here, E' =E/B Rearranging eq. (2) gives the stress equation as $\sigma = \sqrt{\frac{E^1 \ G1}{\pi a}}$ This is one of the most important relations in the field of linear fracture mechanics. Hence, eq (3) suggests that G_1represents the material's resistance(R) to crack extension and it is known as the crack driving force. On the other hand, $K1$ is the intensity of the stress field at the crack tip. The condition of eq. (2.) implies that

$G_1 = R$ before relatively slow crack growth occurs. However, rapid crack growth(propagation) takes place when $G_1 \rightarrow G_{1c}$ which is the critical strain energy release rate known as the crack driving force or fracture toughness of a material under tension loading. Consequently, the fracture criterion by G_{1c} establishes crack propagation when $G_1 \geq G_{1c}$ In this case, the critical stress or fracture stress σ_c and the critical crack driving force G_{1c} can be predicted using eq. (2) when the crack is unstable. Hence,

$$\sigma = \sqrt{\frac{E^1 \ G1c}{\pi a}}$$

Griffith assumed that the crack resistance R consisted of surface energy only for brittle materials. This implies that $R = 2\gamma_S$ but most engineering materials undergo, to an extent, plastic deformation so that $R = (2\gamma_S + \gamma_p)$ Figure2.5 shows a plastic zone at the crack tip representing plasticity or localized yielding, induced by an external nominal stress. This implies that γ_p the energy is manifested due to this small plastic zone in the vicinity of the crack tip.

2. Materials and Methods:

ALUMINIUM:

Aluminium is the third most abundant element (after oxygen and silicon), and the most abundant metal, in the Earth's crust. It makes up about 8% by weight of the Earth's solid surface. Aluminium metal is so chemically reactive that native specimens are rare and limited to extreme reducing environments. Instead, it is found combined in over 270 different minerals

Table 1 Mechanical properties

Aluminium properties	
Electrical resistivity	(20 °C)28.2nΩ·m
Thermal conductivity	237 W·m^{-1}·K^{-1}
Thermal expansion	(25 °C) 23.1 µm·m^{-1}·K^{-1}
Speed of sound (thin rod)	(r.t.) (rolled) 5,000 m·s^{-1}
Young's modulus	69 GPa
Shear modulus	26 GPa
Bulk modulus	76 GPa
Poisson ratio	0.35
Vickers hardness	167 MPa
Brinell hardness	245 MPa
fracture toughness	28K_{Ic} (MPam$^{1/2}$)

Applications of aluminium in Transportation (automobiles, aircrafts) Packaging(cans, foil, etc), Electrical transmission lines for power distribution, Heat sinks for electronic appliances such as transistors and CPUs.

VIRTUAL CRACK CLOSURE TECHNIQUE:

The approach for evaluating the energy-release rate is based on the virtual crack-closure technique (VCCT). The energy-release rate calculation occurs during the solution phase of the analysis and the results are saved for post processing

Using VCCT for Energy-Release Rate Calculation [6] VCCT is based on the assumption that the energy needed to separate a surface is the same as the energy needed to close the same surface. The implementation described here uses the modified crack-closure method (a VCCT-based method) and assumes further that stress states around the crack tip do not change significantly when the crack grows by a small amount (Δa). 2-D Crack Geometry For 2-D crack geometry with a low-order element mesh, the energy-release rate is defined as:

$$G1 = -\frac{1}{2\Delta a} R_Y \, \Delta V$$

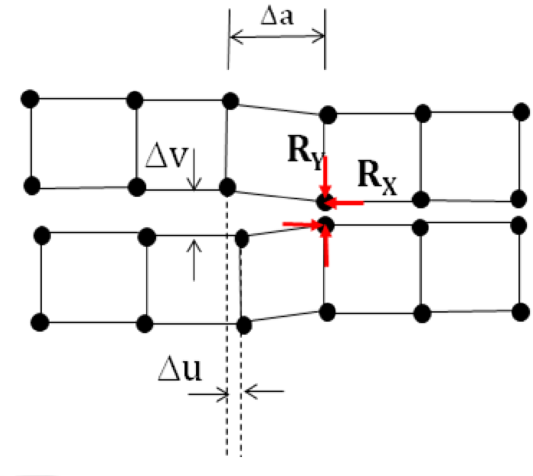
$$G1 = -\frac{1}{2\Delta a} R_Y \, \Delta U$$

G_I and G_{II} = mode I and II energy-release rate, respectively

Δu and Δv = relative displacement between the top and bottom nodes of the crack face in local coordinates x and y, respectively

R_x and R_y = reaction forces at the crack-tip node

Fig: 1 2-D Crack Geometry Schematic diagram



$$K = \sqrt{\frac{GE}{(1-\vartheta^2)}}$$

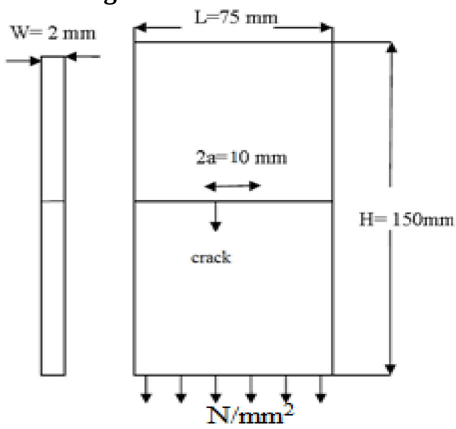
Where,

KI = Stress intensity factor
G = Energy release rate value
E = Young's modulus

ϑ = Poisson's Ratio

CALCULATION OF SIF USING VCCT :

Fig2. CENTRAL CRACK

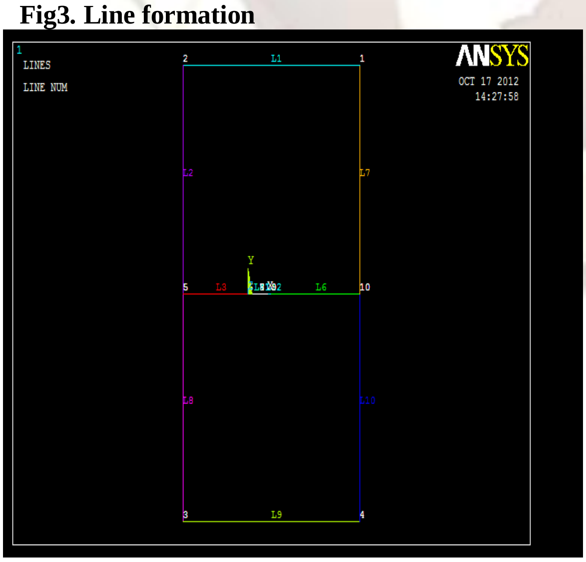


Enter the material type and properties and then enter the key points to Create the lines. Form the areas using line .Mesh the areas using smart size .Apply the boundary conditions and loads. Then
Give the program using CINT commands in the ANSYS command prompt .To obtain the g value use the PRCINT command

Table2. Key points Data

Key point #	x	Y
1	47.5	75
2	-27.5	75
3	-27.5	-75
4	47.5	-75
5	-27.5	0
6	0	0
7	10	0
8	10	0
9	20	0
10	47.5	0

Fig3. Line formation



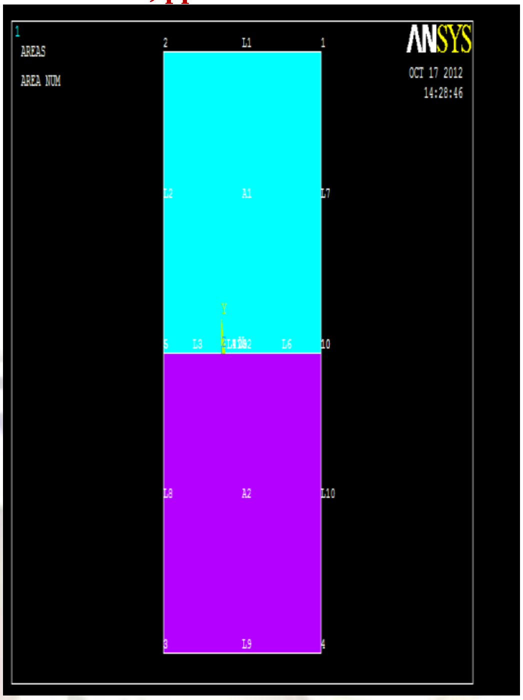


Fig4.Area formation

Fig5. Meshing

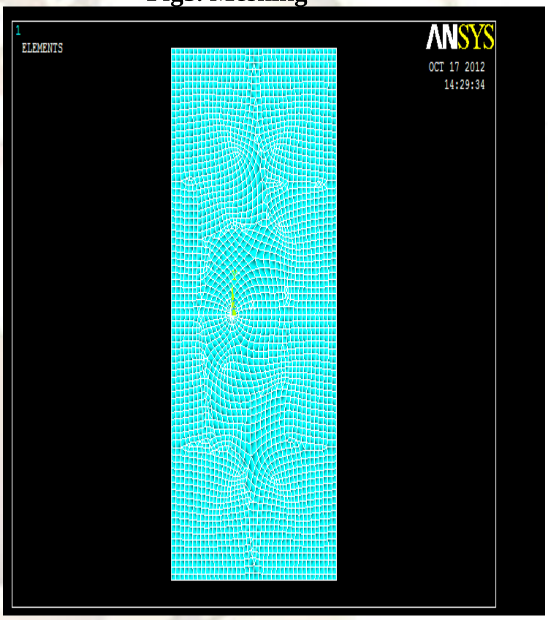
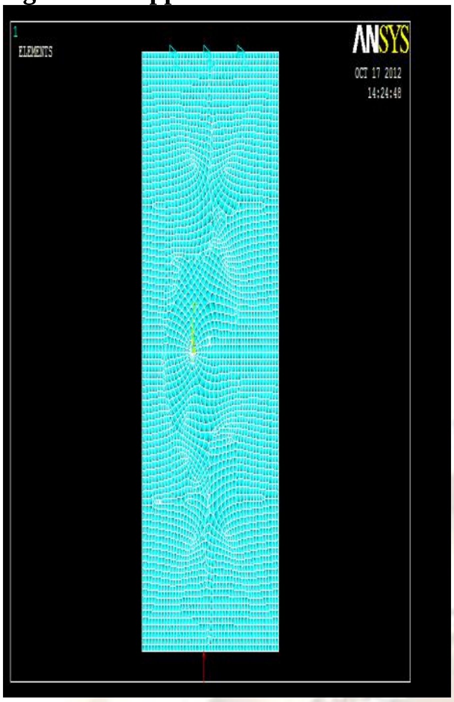
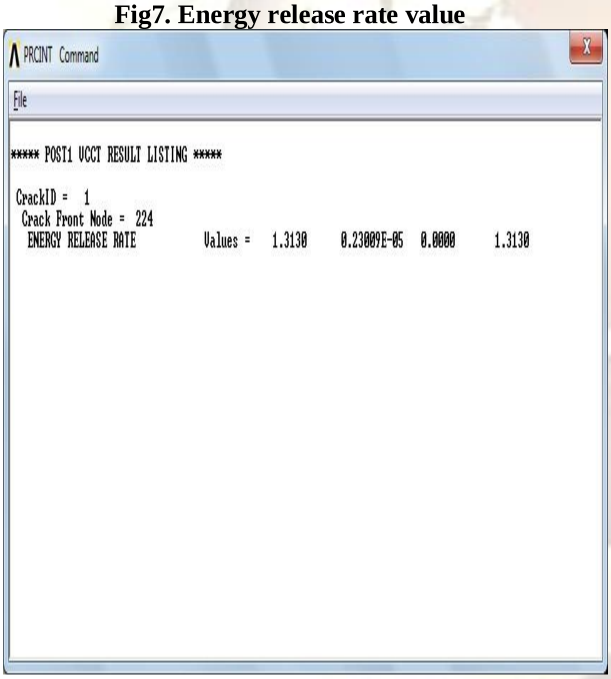


Fig6. Load applied



3. Results and Discussion :

Fig7. Energy release rate value



Calculation k value using energy released rate at crack length 10mm and at 80N/mm

$$K = \sqrt{\frac{GE}{(1-\vartheta^2)}}$$

$$K = \sqrt{\frac{1.31*69000}{(1-0.35^2)}}$$

$K_{(vcct)}$ = 321.36 MN(mm)$^{1/2}$

Calculations of $k_{(empirical)}$ value using empirical formula

At crack length 10mm and load at 80N/mm

$$K = \alpha\sigma\sqrt{\pi a}$$

$$\alpha = [1 + 0.50(\frac{a}{w})^2 + 20.46(a/w)^4 + 81.72(a/w)^6]^{1/2}$$

$$\alpha = [1 + 0.50(5/75)^2 + 20.46(5/75)^4 + 81.72(5/75)^6]^{1/2}$$

$\alpha = 1.00132$

K= 1.00132*80*$\sqrt{\pi * 5}$

K=318.08 MN(mm)$^{1/2}$

Table. 3 comparison OF $K_{(Energy)}$ &$K_{(theoretical)}$ values for crack length 10mm at different loads

S.NO	LOAD N/mm^2	G VALUE	K(VCCT) Mpa(mm)$^{1/2}$	K(empirical) Mpa(mm)$^{1/2}$	%ERROR
1	80	1.31	321.36	318.08	1.02
2.	100	2.05	401.65	397.60	1
3.	120	2.95	481.97	475.59	1.3
4.	140	4.02	562.31	554.86	1.3
5.	160	5.25	642.64	634.13	1.3

Displacement Extrapolation method :

For obtaining the k value zoom the model and select nodes for path at crack tip and at front of the crack and give the path name. Then in ANSYS model tree general post processor in nodal calc select the stress int.

Fig8.DEFORMEDSHAPE

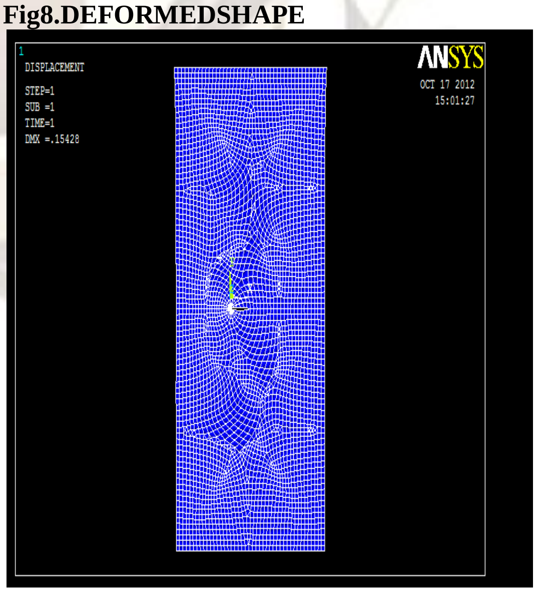


Fig9. Selection of nodes for Extrapolate method

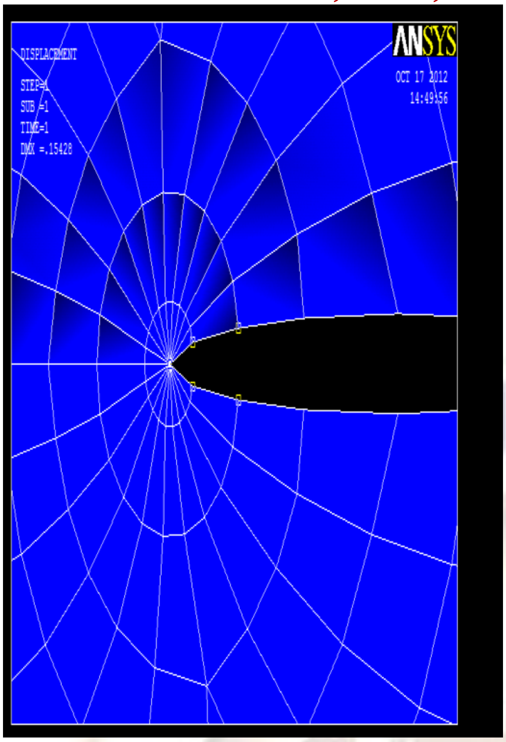


Fig 10. K value

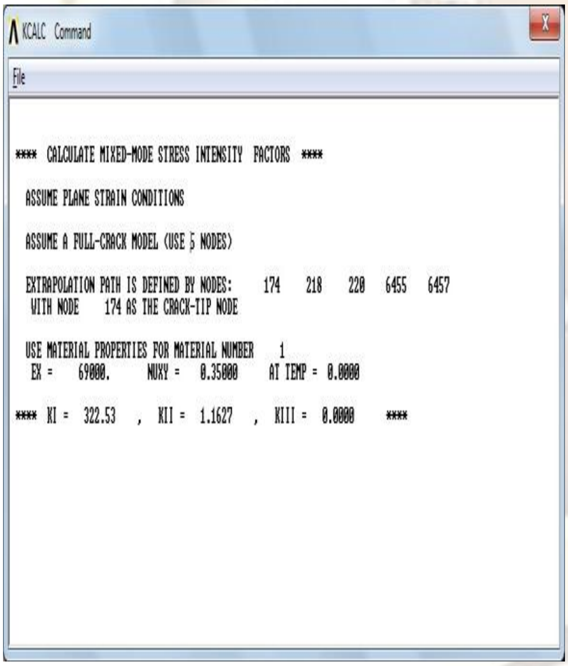


Table 4. Comparison of K $_{(VCCT)}$ and K$_{(disp\ extrapolation)}$ values for crack length 10mm at different loads

S.NO	LOAD 80N/mm^2	K(EXTRAPOLATE) Mpa(mm)$^{1/2}$	K(VCCT) Mpa(mm)$^{1/2}$	%ERROR
1.	80	322.53	321.36	0.36
2.	100	403.17	401.65	0.38
3.	120	483.80	481.97	0.39
4.	140	564.44	562.31	0.38
5.	160	645.07	642.64	0.38

Table5. Comparison of k$_{(disp\ extrapolation)}$ and k$_{(theoretical)}$ values

S.NO	LOAD 80N/mm^2	K(EXTRAPOLATE) Mpa(mm)$^{1/2}$	K(empirical) Mpa(mm)$^{1/2}$	%Error
1	80	322.53	318.08	1.4%
2	100	403.17	397.60	1.4%
3	120	483.80	475.59	1.4%
4	140	564.44	554.86	1.7%
5	160	645.07	634.13	1.7%

Fatigue life:

Fatigue in materials subjected to repeated cyclic loading can be defined as a progressive failure due to crack initiation (stage I), crack growth (stage II), and crack propagation (stage III) or instability stage.[4]

In general, fatigue is a form of failure caused by fluctuating or cyclic loads over a short or prolong period of time. Therefore, fatigue is a time-dependent failure mechanism related to micro structural features. The fluctuating loading condition is not a continuous failure process as opposed to cyclic loading. The former is manifested in bridges, aircrafts and machine components, while the latter requires a continuous constant or variable stress amplitude until fracture occurs. It is also important for the reader to know that fatigue failure or fracture can occur at a maximum stress below the static yield strength of a particular material. Obviously, temperature effects must be considered in fatigue failure characterization. From an engineering point of view, predicting fatigue life is
Major a requirement [8].

$$\frac{da}{dN} = C(\Delta K)^m$$

Where $\Delta K = K_{max} - K_{min}$ with K max and K min referring to the maximum and Minimum values of the stress intensity factor in the load cycle C and m are empirical constants for aluminium $c=10^{-12}$ and m=3. For calculation of fatigue life of the plate consider $\sigma_{min} = 80\ N/mm^2$ and $\sigma_{max} = 160\ N/mm^2$.

$$K = \alpha\sigma\sqrt{\pi a}$$

$$\alpha = [1 + 0.50(\frac{a}{w})^2 + 20.46(a/w)^4 + 81.72(a/w)^6]^{1/2}$$

$$\alpha = [1 + 0.50(5/75)^2 + 20.46(5/75)^4 + 81.72(5/75)^6]^{1/2}$$

$$\alpha = 1.00132$$

$(\Delta K)_{10} = 1.00132 * 80 * \sqrt{\pi * 0.005}$ $= 10.23$ $(\Delta K)_{12} = 1.00201 * 80 * \sqrt{\pi * 0.006}$ $= 11$

$$\frac{da}{dN} = C(\Delta K)^m$$

$(da/dN) = 10^{-12}(10.23)^3$

$(da/dN)_{10} = (1070.59)\,10^{-12}$

$(da/dN)_{12} = (1331)10^{-12}$

Meanof$(da/dN)_{10}$and$(da/dN)_{12}$ $=(1198.98)10^{-12}$

$dN = 0.001/(1198.98)\,10^{-12}$

dN=832781.61

Table 6. Calculations of cycles

S.NO	CRACK LENGTH(a0) mm	CRACK LENGTH(a) mm	ΔN(Cycles)	N(cycles)
1	10	12	832781.61	832781.61
2	12	14	669191.54	1495973.15
3	14	16	533665.85	2029639
4	16	18	440553.19	2470192.19

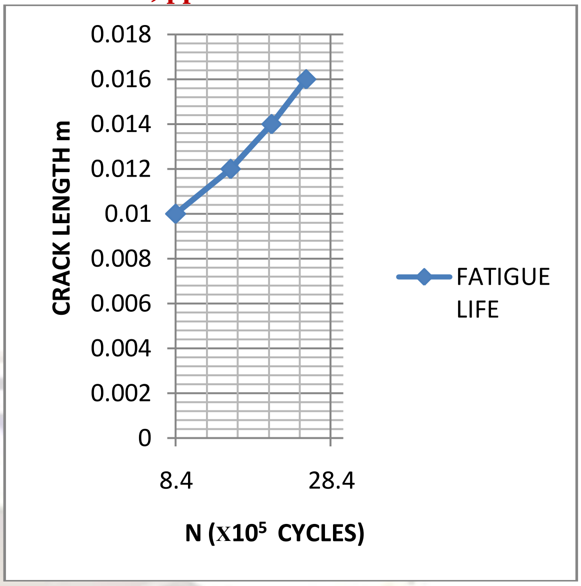


Fig.11 graph for fatigue life

CONCLUSION: a comparison for calculation of stress intensity factors s for cracked aluminium plate subjected to in-plane loading the virtual crack closure technique. VCCT proved and accurate tool for calculating SIF the life plate is calculated for a crack growth 10 to 18 mm

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