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Method For Solving The Transportation Problems Using Trapezoridal Fuzzy Numbers

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ABSTRACT

The fuzzy set theory has been applied in many fields such as operation research, management science and control theory etc. The fuzzy numbers and fuzzy values are widely used in engineering applications because of their suitability for representing uncertain information. In standard fuzzy arithmetic operations we have some problem in subtraction and multiplication operations. In this paper, we investigate fuzzy transportation, Problem with the aid of trapezoidal fuzzy numbers. Fuzzy U-V distribution method is proposed to find the optimal solution in terms of fuzzy numbers. A new relevant numerical example is also included.

KEYWORDS

Fuzzy numbers, trapezoidal fuzzy numbers, fuzzy Vogel's approximation method, fuzzy U-V distribution method, ranking function.

INTRODUCTION

The transportation problem (TP) refers to a special class of linear programming problems. In a typical problem a product is to transported from m sources to n designations and their capacities are $a_1, a_2, \dots a_m$ and $b_1, b_2 \dots . b_n$ respectively. In additions there is a penalty c_{ij} associated with transporting unit of product from source i to destination j. This penalty may be cost or delivery time or safety of delivery etc. A variable x_{ij} represents the unknown quantity to be shipped from some i to destination j.

Iserman (1) introduced algorithm for solving this problem which provides effective solutions. Lai and Hwang(5), Bellman and Zadeh (3) proposed the concept of decision making in fuzzy environment. The Ringuest and Rinks (2) proposed two iterative algorithms for solving linear, multicriteria transportation problem. Similar solution in Bit A.K (8). In works by S.Chanas and D.Kuchta (10) the approach introduced in work (2).

In this paper, the fuzzy transportation problems using trapezoidal fuzzy numbers are discussed. Here after, We have to propose the method of fuzzy U-V distribution method to be finding out the optimal solution for the total fuzzy transportation minimum cost.

2. FUZZY CONCEPTS

L.A.Zadeh advanced the fuzzy theory in 1965. The theory proposes a mathematical technique for dealing with imprecise concept and problems that have many possible solutions.

The concept of fuzzy mathematical programming on a general level was first proposed by Tanaka et al (1947) in the frame work of the fuzzy decision of bellman and Zadeh(3) now, we present some necessary definitions.

2.1. Definition:-

A real fuzzy number $\tilde{a}$ is a fuzzy subset of the real number R. with membership function $\mu_{\tilde{a}}$ satisfying the following conditions.

1. $\mu_{\tilde{a}}$ is continuous from R to the closed interval [0,1]
2. $\mu_{\tilde{a}}$ is strictly increasing and continuous on $[a_1, a_2]$
3. $\mu_{\tilde{a}}$ is strictly decreasing and continuous on $[a_3, a_4]$

Where a_1, a_2, a_3 and a_4 are real numbers, and the fuzzy number denoted by $\tilde{a}= [a_1, a_2, a_3, a_4]$ is called a trapezoidal fuzzy number.

2.2. Definition:-

The fuzzy number $\tilde{a}= [a_1, a_2, a_3, a_4]$ is a trapezoidal number, denoted by $[a_1, a_2, a_3, a_4]$ its membership function $\mu_{\tilde{a}}$ is given below the figure.

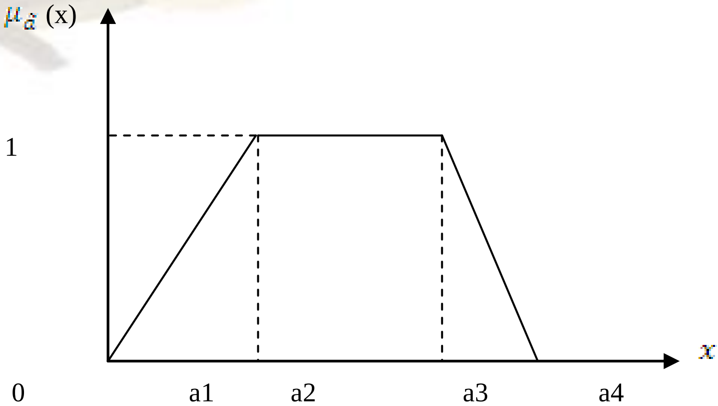

Fig: Membership function of a fuzzy number $\tilde{a}$

2.3 Definition:-

We define a ranking a function R: F(R)→R, Which maps each fuzzy number into the real line, F(R) represents the set of all trapezoidal fuzzy numbers, If R be any ranking function, then R ($\tilde{a}$) = $(a_1 + a_2 + a_3 + a_4)$ / 4.

2.4 Arithmetic Operations:-

Let $\tilde{a}= (a_1, a_2, a_3, a_4)$ and $\tilde{b}= (b_1, b_2, b_3, b_4)$ two trapezoidal fuzzy numbers them the arithmetic operation on $\tilde{a}$ and $\tilde{b}$ as follows:

Additions:

$$\tilde{a}+\tilde{b}= (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$$

Subtraction:

$$\tilde{a}-\tilde{b}= (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$$

Multiplication:

$$\tilde{a}.\tilde{b}=[\tfrac{a_1}{4}(b_1 + b_2 + b_3 + b_4), \tfrac{a_2}{4}(b_1 + b_2 + b_3 + b_4), \tfrac{a_3}{4}(b_1 + b_2 + b_3 + b_4), \tfrac{a_4}{4}(b_1 + b_2 + b_3 + b_4)]$$

$$\tilde{a}.\tilde{b} = \left[\frac{a_4}{4}(b_1+b_2+b_3+b_4), \frac{a_3}{4}(b_1+b_2+b_3+b_4), \frac{a_2}{4}(b_1+b_2+b_3+b_4), \frac{a_1}{4}(b_1+b_2+b_3+b_4)\right]$$

if R($\tilde{a}$) < 0

3. FUZZY TRANSPORTATION PROBLEM

Consider transportation with m fuzzy origins (rows) and n fuzzy destinations (Columns). Let $C_{ij} = \left[c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}\right]$ be the cost of transporting one unit of the product from i[th] fuzzy origin to j[th] fuzzy destination $a_i = \left[a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}\right]$ be the quantity of commodity available at fuzzy origin i, $b_j = \left[b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}\right]$ be the quantity of commodity requirement at fuzzy destination j. $x_{ij} = \left[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}\right]$ is quantity transported from i[th] fuzzy origin to j[th] fuzzy destination. The above fuzzy transportation problem can be stated in the below tabular form.

	FD_1	FD_2 ----	FD_n	Fuzzy available
$F'O_1$	x_{11} c	x_{12} ---- c_{12} ----	x_{1n} c	a_1
FO_2 ⋮	x_{21} ⋮	x_{22} ⋮	x_{2n} ⋮	a_2 ⋮
FO_m	x_{m1} c	x_{m2} ---- c_m	x_{2n} c	a_m
Fuzzy Requirement	b_1	b_2 ----	b_n	$\sum_{i=1}^{m} a_i = \sum_{j}$

Where

$$C_{ij} = \left[c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}\right], x_{ij} = \left[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}\right]$$

$$a_i = \left[a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}\right]$$ and

$$b_j = \left[b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}\right]$$

The linear programming model representing the fuzzy transportation is given by

Minimize

$$z = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}\right]\left[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}\right]$$

if R($\tilde{a}$) ≥ 0

Subject to the constraints

$$\sum_{j=1}^{n}\left[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}\right] = \left[a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}\right] \; for\ i = 1,2, \dots\dots m$$

$$\sum_{i=1}^{n}\left[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}\right] = \left[b_i^{(1)}, b_i^{(2)}, b_i^{(3)}, b_i^{(4)}\right] \; for\ j = 1,2, \dots\dots n$$

$$\left[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}\right] \geq 0$$

The given fuzzy transportation problem is said to be balanced if

$$\sum_{i=1}^{m}\left[a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}\right] = \sum_{j=1}^{n}\left[b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}\right]$$

if the total fuzzy available is equal to the total fuzzy requirement.

4. THE COMPUTATIONAL PROCEDURE FOR FUZZY U-V DISTRIBUTION METHODS

4.1.Fuzzy Vogel's Approximathical Methods

There are numerous methods for finding the fuzzy initial basic feasible solution of a transportation problem. In this paper we are to follow fuzzy Vogel's approximation method.

Step 1. Find the fuzzy penalty cost, namely the fuzzy difference between the smallest and smallest fuzzy costs in each row and column.

Step 2. Among the fuzzy penalties as found in step1 choose the fuzzy maximum penalty, by ranking method. If this maximum penalty is more than one, choose any one arbitrarily.

Step 3. In the selected row or column as by step 2, find out the cell having the least fuzzy cost. Allocate to this cell as much as possible depending on the fuzzy available and fuzzy requirements.

Step 4. Delete the row or column which is fully exhausted. Again compute column and row fuzzy penalties for the reduced fuzzy transportation table and then go to step 2, repeat the procedure until all the rim requirement are satisfied.

Once the fuzzy initial fuzzy feasible solution can be computed, the next step in the problem is to determine whether the solution obtained is fuzzy optimal or not. Fuzzy optimality test can be conducted to any fuzzy initial basic feasible solution of a fuzzy transportation provided such allocations has exact m+n-1 non-negative allocations where m is the number of fuzzy origins and n is the number of fuzzy destinations. Also these allocations must be independent positions, which fuzzy optimality finding procedure is given below.

4.2. Fuzzy U-V distribution method

This proposed method is used for finding the optimal basic feasible solution in fuzzy transportation problem and the following step by step procedure is utilized to find out the same.

Step 1. Find out a set of numbers $\left[u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}\right]$ and $\left[v_j^{(1)}, v_j^{(2)}, u_j^{(3)}, v_j^{(4)}\right]$ for each row and column satisfying $\left[u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}\right]+\left[v_j^{(1)}, v_j^{(2)}, u_j^{(3)}, v_j^{(4)}\right] = \left[c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}\right]$ for each occupied cell. To start with we assign a fuzzy zero to any row or column having maximum number zero to any row or column having maximum number of allocation, if this maximum numbers of allocation is more than one, choose any one arbitrary.

Step 2. For each empty (un occupied) cell, we find fuzzy sum $\left[u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}\right]$ and $\left[v_j^{(1)}, v_j^{(2)}, u_j^{(3)}, v_j^{(4)}\right]$.

Step 3. Find out for each empty cell the net evaluation value $\left[z_{ij}^{(1)}, z_{ij}^{(2)}, z_{ij}^{(3)}, z_{ij}^{(4)}\right]$ = $\left[c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}\right] - \left\{\left[u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}\right] + \left[v_j^{(1)}, v_j^{(2)}, u_j^{(3)}, v_j^{(4)}\right]\right\}$ this step gives the optimality conclusion.

Case(i). If all $R\left[z_{ij}^{(1)}, z_{ij}^{(2)}, z_{ij}^{(3)}, z_{ij}^{(4)}\right] > 0$ then the solution is fuzzy optimal, but an alternative solution exists.

Case (ii). If $R\left[z_{ij}^{(1)}, z_{ij}^{(2)}, z_{ij}^{(3)}, z_{ij}^{(4)}\right] \geq 0$ then the solution is fuzzy optimal, but an alternative solution exists.

Case (iii). If at least one $R\left[z_{ij}^{(1)}, z_{ij}^{(2)}, z_{ij}^{(3)}, z_{ij}^{(4)}\right] < 0$ then the solution is not fuzzy optimal. In this case we go to next step, to improve the total fuzzy transportation minimum cost.

Step.4. Select the empty cell having the most negative value of $R\left[z_{ij}^{(1)}, z_{ij}^{(2)}, z_{ij}^{(3)}, z_{ij}^{(4)}\right]$ from this cell we draw a closed path drawing horizontal and vertical lines with corner cell occupied. Assign Sign + and – alternately and find the fuzzy minimum allocation from the cell having negative sign. This allocation should be added to the allocation having negative sign. This allocation should be added to the allocation having negative sign.

Step.5.
The above step yield a better solution by making one (or more) occupied cell as empty and one empty cell as occupied. For this new set of fuzzy basic feasible allocation repeat from the step 1, till an fuzzy optimal basic feasible solution is obtained.

5. Example:

To solve the following fuzzy transportation problem stating with the fuzzy initial fuzzy basic feasible solution obtained by fuzzy Vogel's approximation.

	FD_1	FD_2	FD_3	FD_4	Fuzzy available
FO_1	(-4,0,4,16)	(-4,0,4,16)	(-4,0,4,16)	(-2,0,2,8)	(0,4,8,12)
FO_2	(8,16,24,32)	(8,14,18,24)	(4,8,12,16)	(2,6,10,14)	(4,8,18,26)
Fo_3	(4,8,18,26)	(0,12,16,20)	(0,12,16,20)	(8,14,18,24)	(4,8,12,16)
Fuzzy Requirement	(2,6,10,14)	(2,2,8,12)	(2,6,10,14)	(2,6,10,14)	(8,20,38,54)

Solution:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = (8,20,38,54)$$

The problem is balanced fuzzy transportation problem. There exists a fuzzy initial basic feasible solution.

	FD_1	FD_2	FD_3	FD_4	Fuzzy available
FO_1	(0,4, 8,12) (-4,0,4, 16)	(-4,0,4, 16)	(-4,0,4, 16)	(-2,0,2, 8)	(0,4, 8,12)
FO_2	(8,16 ,24,32)	(8,14 ,18,24)	(-10,-2,12, 24) (4,8, 12,16)	(2,6, 10,14) (2,6, 10,14)	(4,8, 18,26)
Fo_3	(-10,-2,6,14) (4,8, 18,26)	(2,2, 8,12) (0,12 ,16,20)	(-22,-6,12, 24) (0,12 ,16,20)	(8,14 ,18,24)	(4,8, 12,16)
Fuzzy Requirement	(2,6, 10,14)	(2,2, 8,12)	(2,6, 10,14)	(2,6, 10,14)	(8,20 ,38,54)

Since the number of occupied cell having m+n-1 = 6 and are also independent, there exists a non-degenerate fuzzy basic feasible solution.

Therefore the initial transportation minimum cost is,

$$[z^{(1)},z^{(2)},z^{(3)},z^{(4)}] = (0,4,8,12)(-4,0,4,16) + (-10,-2,12,24)(4,8,12,16) +$$
$$+(2,6,10,14)(2,6,10,14) + (-10,,-2,6,14)(4,8,18,26)$$
$$+(2,2,8,12)(0,12,16,20) + (-22,-6,12,24)(0,12,16,20)$$
$$[z^{(1)},z^{(2)},z^{(3)},z^{(4)}] = [-464,-22,556,1028]$$

To find the optimal solution

Applying the fuzzy U-V distribution methods, we determine a set of numbers $[u_i^{(1)},u_i^{(2)},u_i^{(3)},u_i^{(4)}]$ and $[v_j^{(1)},v_j^{(2)},v_j^{(3)},v_j^{(4)}]$ each row and column such that

$$[c_{ij}^{(1)},c_{ij}^{(2)},c_{ij}^{(3)},c_{ij}^{(4)}] = [u_i^{(1)},u_i^{(2)},u_i^{(3)},u_i^{(4)}] + [v_j^{(1)},v_j^{(2)},v_j^{(3)},v_j^{(4)}]$$

for each occupied cell. Since 3 rd row has maximum numbers of allocations, we give fuzzy number $[u_3^{(1)},u_3^{(2)},u_3^{(3)},u_3^{(4)}] = [0,0,0,0]$. The remaining numbers can be obtained as given below $[c_{31}^{(1)},c_{31}^{(2)},c_{31}^{(3)},c_{31}^{(4)}] =$

$$[u_3^{(1)},u_3^{(2)},u_3^{(3)},u_3^{(4)}] + [v_1^{(1)},v_1^{(2)},v_1^{(3)},v_1^{(4)}]$$

$$[v_1^{(1)},v_1^{(2)},v_1^{(3)},v_1^{(4)}] = (4,8,18,26)$$

$$[v_2^{(1)},v_2^{(2)},v_2^{(3)},v_2^{(4)}] = (0,12,16,20)$$

$$[v_3^{(1)},v_3^{(2)},v_3^{(3)},v_3^{(4)}] = (0,12,16,20)$$

$$[c_{11}^{(1)},c_{11}^{(2)},c_{11}^{(3)},c_{11}^{(4)}] = [u_1^{(1)},u_1^{(2)},u_1^{(3)},u_1^{(4)}] + [v_1^{(1)},v_1^{(2)},v_1^{(3)},v_1^{(4)}]$$

$$[u_1^{(1)},u_1^{(2)},u_1^{(3)},u_1^{(4)}] = (-30,-18,-8,12)$$

$$[u_2^{(1)},u_2^{(2)},u_2^{(3)},u_2^{(4)}] = [-16,-8,0,16]$$

$$[v_4^{(1)},v_4^{(2)},v_4^{(3)},v_4^{(4)}] = (-14,6,18,30)$$

We find, for each empty cell of the sum $[u_i^{(1)},u_i^{(2)},u_i^{(3)},u_i^{(4)}]$ and $[v_j^{(1)},v_j^{(2)},v_j^{(3)},v_j^{(4)}]$. Next we find net evaluation $[z_{ij}^{(1)},z_{ij}^{(2)},z_{ij}^{(3)},z_{ij}^{(4)}]$ is given by

$$*\ [z_{ij}^{(1)},z_{ij}^{(2)},z_{ij}^{(3)},z_{ij}^{(4)}] = [c_{ij}^{(1)},c_{ij}^{(2)},c_{ij}^{(3)},c_{ij}^{(4)}] - \{[u_i^{(1)},u_i^{(2)},u_i^{(3)},u_i^{(4)}] + [v_j^{(1)},v_j^{(2)},u_j^{(3)},v_j^{(4)}]\}$$

Where $U_i = [u_i^{(1)},u_i^{(2)},u_i^{(3)},u_i^{(4)}]$

$$V_j = [v_j^{(1)},v_j^{(2)},u_j^{(3)},v_j^{(4)}]$$

U-V.Distribution Methods Table

	FD_1	FD_2	FD_3	FD_4	U_i
FO_1	(0,4,8,12) (-4,0,4,16)	*(-36,-8,10,46) (-4,0,4,16)	*(-36,-8,10,46) (-4,0,4,16)	*(-44,-10,14,52) (-2,0,2,8)	(-30,-18,-8,12)
FO_2	*(-34,-2,24,44) (8,16,24,32)	*(-28,-2,14,40) (8,14,18,24)	(-10,-2,12,24) (4,8,12,16)	(2,6,10,14) (2,6,10,14)	(-16,-8,0,16)
Fo_3	(-10,-2,6,14) (4,8,18,26)	(2,2,8,12) (0,12,16,20)	(-22,-6,12,24) (0,12,16,20)	*(-22,-4,12,38) (8,14,18,24)	(0,0,0,0)
V_j	(4,8,18,26)	(0,12,16,20)	(0,12,16,20)	(-14,6,18,30)	

All $* \left[z_{ij}^{(1)}, z_{ij}^{(2)}, z_{ij}^{(3)}, z_{ij}^{(4)}\right] \geq 0,$ the solution is fuzzy optimal and unique.

The Fuzzy optimal solution in terms of trapezoidal fuzzy numbers.

$$\left[x_{11}^{(1)}, x_{11}^{(2)}, x_{11}^{(3)}, x_{11}^{(4)}\right] = [0,4,8,12]; \left[x_{23}^{(1)}, x_{23}^{(2)}, x_{23}^{(3)}, x_{23}^{(4)}\right] = [-10,-2,12,24]$$

$$\left[x_{24}^{(1)}, x_{24}^{(2)}, x_{24}^{(3)}, x_{24}^{(4)}\right] = [2,6,10,14]; \left[x_{31}^{(1)}, x_{31}^{(2)}, x_{31}^{(3)}, x_{31}^{(4)}\right] = [-10,-2,6,14]$$

$$\left[x_{32}^{(1)}, x_{32}^{(2)}, x_{32}^{(3)}, x_{32}^{(4)}\right] = [2,2,8,12]; \left[x_{33}^{(1)}, x_{33}^{(2)}, x_{33}^{(3)}, x_{33}^{(4)}\right] = [-22,-6,12,24]$$

Hence the total fuzzy transportation minimum cost is

$$\left[z^{(1)}, z^{(2)}, z^{(3)}, z^{(4)}\right] = (0,4,8,12)(-4,0,4,16) + (-10,-2,12,24)(4,8,12,16) + (2,6,10,14)(2,6,10,14) + (-10,,-2,6,14)(4,8,18,26) + (2,2,8,12)(0,12,16,20) + (-22,-6,12,24)(0,12,16,20)$$

$$\left[z^{(1)}, z^{(2)}, z^{(3)}, z^{(4)}\right] = [-464,-22,556,1028]$$

6. CONCLUSION

We have thus obtained an optimal solution for a fuzzy transportation problem using trapezoidal fuzzy number. A new approach called fuzzy U-V Computational procedure to find the optimal solution is also discussed. The new arithmetic operations of trapezoidal fuzzy numbers are employed to get the fuzzy optimal solutions. The same approach of solving the fuzzy problems may also be utilized in future studies of operational research.

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