An Approach For Interval Discrete-Time Systems Reduction Using Least Squares Method

K.Kiran Kumar* and Dr.G.V.K.R.Sastry*
* Department of Electrical Engineering, A.U.College of Engg.(A), Andhra University, Visakhapatnam, India-530003

Abstract

This paper presents a new approach for the reduction of high order discrete time Interval systems based on least squares method and Time moments matching technique. The proposed method is computationally simple and overcomes some of the limitations and drawbacks of the existing methods. A numerical example illustrates the proposed algorithm.

Keywords: Discrete time Interval systems, Least squares, Moment matching, Order Reduction.

1. Introduction

In general, the practical systems have uncertainties about its parameters. Thus practical systems will have coefficients that may vary and it is represented by in Very few methods are available in the literature for the model reduction of high order discrete time Interval systems.

The present work deals with the extending of the concept of least squares method about a general point 'a' to discrete time Interval systems combined with bilinear and inverse bilinear transformations.

2. Proposed Reduction Procedure

Let the n^{th} order discrete time Interval system be defined as

$$H(z) = \frac{[u_0^-, u_0^+] + [u_1^-, u_1^+]z + \cdots + [u_{n-1}^-, u_{n-1}^+]z^{n-1}}{[v_0^-, v_0^+] + [v_1^-, v_1^+]z + \cdots + [v_n^-, v_n^+]z^n} \quad \ldots (1)$$

where, $[u_i^-, u_i^+]$ for i = 0 to n-1 and $[v_i^-, v_i^+]$ for i = 0 to n are the interval parameters.

Let the corresponding r^{th} order reduced interval discrete time system (model) be defined as

$$R(Z) = \frac{[x_0^-, x_0^+] + [x_1^-, x_1^+]Z + \cdots + [x_{r-1}^-, x_{r-1}^+]Z^{r-1}}{[y_0^-, y_0^+] + [y_1^-, y_1^+]Z + \cdots + [y_r^-, y_r^+]Z^r}$$

Where r = 1,2,3,……, n-1

Applying the Bilinear transformation $z = \frac{1+w}{1-w}$ to the eqn. (1), the n^{th} order original discrete time Interval system is transformed to w- plane is represented as:

$$H(w) = \frac{[a_0^-, a_0^+] + [a_1^-, a_1^+]w + \cdots + [a_{n-1}^-, a_{n-1}^+]w^{n-1}}{[b_0^-, b_0^+] + [b_1^-, b_1^+]w + \cdots + [b_n^-, b_n^+]w^n} \quad \text{---}(2)$$

where $[a_i^-, a_i^+]$ for i = 0 to n-1 and $[b_i^-, b_i^+]$ for i = 0 to n are the uncertain coefficients of numerator and denominator respectively.

The n^{th} order original interval system in w-plane, H(w) is transformed to four fixed n^{th} order transfer functions using Kharitonov's theorem defined as

$$G_p(w) = \frac{A_{p0} + A_{p1}w + A_{p2}w^2 + \cdots + A_{pn-1}w^{n-1}}{B_{p0} + B_{p1}w + B_{p2}w^2 + \cdots + B_{pn}w^n} \quad \ldots (3)$$

where p=1, 2, 3, 4 and n = order of the original system.

Replace the $G_p(w)$ by $G_p(w+a)$ where the value of 'a' obtained by the harmonic mean of the real parts of the roots of $G_p(w)$ defined as:

$$\frac{1}{a} = \sum_{i=1}^{n}\left[\frac{1}{|P_i|}\right] \Big/ n \quad \ldots (4)$$

Where P_i are the poles of $G_p(w)$

Let the corresponding r^{th} order reduced model be synthesized as $R_p(w) = \frac{N_p(w)}{D_p(w)}$

$$R_p(w+a) = \frac{d_{p0} + d_{p1}w + d_{p2}w^2 + \cdots + d_{pr-1}w^{r-1}}{e_{p0} + e_{p1}w + e_{p2}w^2 + \cdots + e_{pr}w^r} \quad \ldots (5)$$

$$R_p(w) = \frac{D_{p0} + D_{p1}w + D_{p2}w^2 + \cdots + D_{pr-1}w^{r-1}}{E_{p0} + E_{p1}w + E_{p2}w^2 + \cdots + E_{pr}w^r} \quad \ldots (5.1)$$

Step1: Determination of the time moments and Markov parameters from the shifted original system:

the time moments (c_i) can be obtained by expanding $G_p(w+a)$ about w = 0, as

$$G_p(w+a) = \sum_{i=0}^{\infty} c_i w^i \quad \ldots (6)$$

Similarly, if $G_p(w + a)$ is expanded about $s=\infty$, then the Markov parameters m_j are obtained by:

$$G_p(w+a) = \sum_{j=1}^{\infty} m_j w^{-j} \quad \ldots (6.1)$$

Equating the equation (5) and (6) to retain the time moments of the original system which generates the following set of equations:

$$\begin{bmatrix} c_r & c_{r-1} & \cdots & c_1 \\ c_{r+1} & c_r & \cdots & c_2 \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & & \cdots \\ c_{2r-1} & c_{2r-2} & \cdots & c_r \end{bmatrix} * \begin{bmatrix} e_{p0} \\ e_{p1} \\ \cdot \\ \cdot \\ e_{pr-1} \end{bmatrix} = \begin{bmatrix} -c_0 \\ -c_1 \\ \cdot \\ \cdot \\ -c_{r-1} \end{bmatrix} \quad \ldots(7)$$

or ,H e = c, in matrix vector form where the coefficients of 'e_{pr}' , the parameters of the reduced denominator obtained by least squares sense using the generalized inverse method
$e = (H^T\ H)^{-1}\ H^T\ c$ …(8)

If the coefficients e_i given by the equation (8) do not constitute a stable denominator, then another row is to be added to the existing equation set so that the model assumes a matching of the next time moment of the original system

[c_{2r} c_{2r-1} …. c_{r+1}] and [- c_r] …(9)

If the denominator polynomial is still not stable, then the H matrix and c vector are again to be extended by matching next time moments.

Step2: The above step can also be generalized by including Markov parameters in the least square fitting process, as follows:

$$\left.\begin{aligned} d_{p0} &= e_{p0}c_0 \\ d_{p1} &= e_{p1}c_0 + e_{p0}c_1 \\ d_{p2} &= e_{p2}c_0 + e_{p1}c_1 + e_{p0}c_2 \\ &\ \ \vdots \\ d_{pr-1} &= e_{pr-1}c_0 + \cdots + e_{p0}c_{r-1} \\ 0 &= e_{pr-1}c_1 + \cdots + e_{p0}c_r \\ 0 &= e_{pr-1}c_2 + \cdots + e_{p0}c_{r+1} \\ &\ \ \vdots \\ 0 &= e_{pr-1}c_t + \cdots + e_{p0}c_{r+t-1} \end{aligned}\right\} \quad ..(10)$$

and

$$\left.\begin{aligned} d_{pr-1} &= m_1 \\ d_{pr-2} &= m_1 e_{pr-1} + m_2 \\ &\ \ \vdots \\ d_{pt} &= m_1 e_{pt+1} + m_2 e_{pt+2} + \cdots + m_{r-t} \end{aligned}\right\} \quad ..(11)$$

where the c_i and m_j are the time moments proportional and Markov parameters of the system respectively .Elimination of d_j(j= t, t+1, …,r-1) in equation (11) by substituting into (10) gives the reduced denominator coefficients as the solution of:

$$\begin{bmatrix} c_{r+t-1} & c_{r+t-2} & \cdots & \cdots & \cdots & \cdots & c_t \\ c_{r+t-2} & c_{r+t-3} & \cdots & \cdots & \cdots & c_t & c_{t-1} \\ \cdot & \cdot & \cdots & \cdots & \cdots & \cdot & \cdot \\ c_{r-1} & c_{r-2} & \cdots & \cdots & \cdots & c_1 & c_0 \\ c_{r-2} & c_{r-3} & \cdots & \cdots & \cdots & c_0 & -m_1 \\ c_{r-3} & c_{r-4} & \cdots & \cdots & c_0 & -m_1 & -m_2 \\ \cdot & \cdot & & & \cdot & \cdot & \cdot \\ \cdot & \cdot & & & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdots & \cdot & \cdot & \cdot \\ c_t & c_{t-1} & \cdots & c_0 & -m_1 & \cdots & -m_{r-t-1} \end{bmatrix}$$

$$X \begin{bmatrix} e_{p0} \\ e_{p1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ e_{pr-1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ m_1 \\ m_2 \\ m_3 \\ \cdot \\ \cdot \\ m_{r-t} \end{bmatrix} \quad \ldots..(12)$$

or H e = m in matrix vector form and e can be calculated $e = (H^T\ H)^{-1}\ H^T\ m$ …..(13) are the coefficients of the reduced model denominator. If this estimate still does not yield a stable reduced denominator, then H and m in (12) are extended by another row, which corresponds to using the next Markov parameter from the full system in Least Squares match.

Step3: Determination of the coefficients of the denominator of the reduced model $R_p(w)$.

Once the coefficients of vector 'e' obtained from equation (13) apply inverse shift of w→(w-a) and finally the r[th] order reduced denominator obtained as:

$$D_p(w) = E_{p0} + E_{p1}w + E_{p2}w^2 + \cdots + E_{pr}w^r$$

Step4: Determination of the coefficients of the Numerator $N_p(w)$ of the reduced model by moment matching

Later calculate the reduced numerator as before by matching proper number of Time moments of $G_p(w)$ to that of reduced model.

Step5: Apply inverse bilinear transformation $\mathbf{w} = \frac{z-1}{z+1}$ to R (w) and determine R(z).

3. ILLUSTRATIVE EXAMPLE

Consider the 3[rd] order discrete interval system given by its transfer function

$$G(z) = \frac{[3.25, 3.35]z^2 + [3.5, 3.65]z + [2.8, 3]}{[5.4, 5.5]\ z^3 + [1, 1.1]\ z^2 + [1.5, 1.6]z + [2.1, 2.15]}$$

On Applying Bilinear transformation, the above transfer function is obtained as:

$$G(w)=\frac{[2.4,2.85]\,w^2+[0.5,1.1]w+[9.55,10]}{[3.65,4]\,w^3+[19.8,20.45]\,w^2+[9.15,9.8]w+[10,10.7]}$$

Applying Kharitonov's theorem, the four fixed coefficient transfer functions are:

$$G_1(w)=\frac{2.85w^2+0.5w+9.55}{4w^3+20.45w^2+9.15w+10}$$

$$G_2(w)=\frac{2.85w^2+1.1w+9.55}{3.65w^3+20.45w^2+9.8w+10}$$

$$G_3(w)=\frac{2.4w^2+0.5w+10}{4w^3+19.8w^2+9.15w+10.7}$$

$$G_4(w)=\frac{2.4w^2+1.1w+10}{3.65w^3+19.8w^2+9.8w+10.7}$$

Applying the proposed reduction procedure, the corresponding 2nd order reduced models retaining 6 Time moments with Min. ISE are obtained as:

$$R_1(w)=\frac{0.11269w+0.460399}{w^2+1.01301w+0.655786}\quad \text{ISE}=0.003844,\ \text{HM}=0.273055$$

$$R_2(w)=\frac{0.185314w+0.440269}{w^2+1.103648w+0.651078}\quad \text{ISE}=0.002840,\ \text{HM}=0.305723$$

$$R_3(w)=\frac{0.085543w+0.507296}{w^2+1.021272w+0.651078}\quad \text{ISE}=0.00310,\ \text{HM}=0.273445$$

$$R_4(w)=\frac{0.152554w+0.491192}{w^2+1.119290\,w+0.730050}\quad \text{ISE}=0.002192,\ \text{HM}=0.308096$$

Thus the reduced interval model in w- plane is

$$R(w)=\frac{[0.085543,0.185314]w+[0.440269,0.507296]}{[1,1]\,w^2+[1.017301,1.119290]w+[0.651075,0.730050]}$$

Applying inverse bilinear transformation

$$R(z)=\frac{[0.525812,0.69261]z+[0.254955,0.421753]}{[2.668379,2.84934]\,z^2+[-1.817134,-1.557201]z+[1.651078,1.730050]}$$

To achieve the steady state matching of the responses of the original and reduced order models, the coefficients are multiplied with the gain correction factor,

$$K=\frac{G(z)|_{z=1}}{R(z)|_{z=1}}$$

Then the 2nd order reduced model in z- domain is

$$R(z)=\frac{[1.609369944,1.814859835]z+[0.780349086,1.105128127]}{[2.668379,2.84934]\,z^2+[-1.817134,-1.557201]z+[1.651078,1.730050]}$$

Using the Multi point Pade Approximation Method of O.Ismail [the 2nd order reduced model is obtained as:

$$R(z)=\frac{[0.4717,\ 0.5998]z+[0.9966,\ 1.0075]}{[1.4148,\ 1.4589]\,z^2+[0.8054,\ 0.8674]\,z+[1,\ 1]}$$

The step responses of the proposed reduced model and that of O.Ismail are compared with the original interval system for lower bound and upper bound in Fig. 1 and Fig.2 respectively.

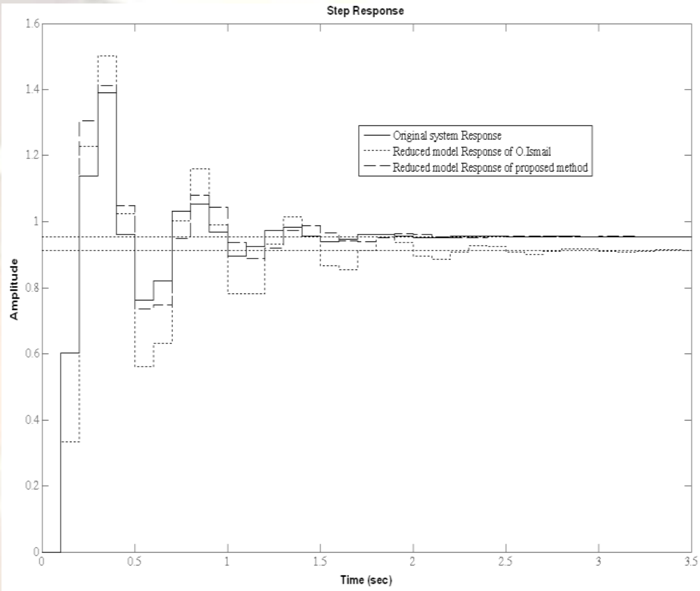

Fig:1

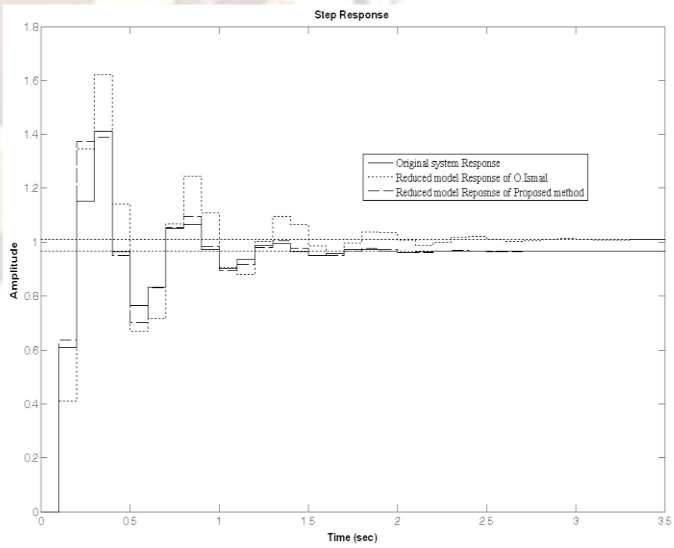

Fig:2

Conclusion

A novel method is suggested for the order reduction of high order discrete interval systems based on Least – Squares method.

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