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Use Of Linear Programming For Optimal Production In A Production Line In Coca –Cola Bottling Company, Ilorin

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Abstract

Many companies were and are still established to derive financial profit. In this regard the main aim of such establishments is to maximize (optimize) profit. This research is on using Linear programming Technique to derive the maximum profit from production of soft drink for Nigeria Bottling Company Nigeria, Ilorin plant. Linear Programming of the operations of the company was formulated and optimum results derived using Software that employed Simplex method. The result shows that two particular items should be produced even when the company should satisfy demands of the other - not - so profitable items in the surrounding of the plants.

KEYWORDS: Optimization, Linear programming, Objective function, Constraints.

Introduction

Company managers are often faced with decisions relating to the use of limited resources. These resources may include men, materials and money. In other sector, there are insufficient resources available to do as many things as management would wish. The problem is based on how to decide on which resources would be allocated to obtain the best result, which may relate to profit or cost or both. Linear Programming is heavily used in Micro-Economics and Company Management such as Planning, Production, Transportation, Technology and other issues. Although the modern management issues are error changing, most companies would like to maximize profits or minimize cost with limited resources. Therefore, many issues can be characterized as Linear Programming Problems (Sivarethinamohan, 2008).

A linear programming model can be formulated and solutions derived to determine the best course of action within the constraint that exists. The model consists of the objective function and certain constraints. For example, the objective of Nigeria Bottling Company (Coca-cola) is to produce quality products needed by its customers, subject to the amount of resources (raw materials) available to produce the products needed by their respective customers who should also not violate National Agency for Food and Drug Administration Control (NAFDAC) and Standard Organization of Nigeria (SON). The problem then is on how to utilize limited resources to the best advantage, to maximize profit and at the sometime selecting the products to be produced out of the number of products considered for production that will maximize profit.

The research is aimed at deciding how limited resources,raw materials of Nigeria Bottling Company (Coca-cola), Ilorin plant , Kwara State would be allocated to obtain the maximum contribution to profit. It is also aimed at determining the products that contribute to such profit.

The scope of the research is to use Linear Programming on some of the soft drinks produced by Nigeria Bottling Company, Ilorin plant. The data on which this is based are quantity of raw materials available in stock, cost and selling prices and therefore the profit of crate of each product. The profit constitutes the objective function while raw materials available in stock are used as constraints. If demands which must be met are to be available, such can be included in the constraints. The data is secondary data collected in the year 2007 at the Nigeria Bottling Company, Ilorin plant, Kwara State.

The Simplex method, also called Simple technique or Simplex Algorithm, was invented by George Dantzig, an American Mathematician, in 1947. It is the basic workhorse for solving Linear Programming Problems up till today. There have been many refinements to the method, especially to take advantage of computer implementations, but the essentials elements are still the same as they were when the method was introduced (Chinneck, 2000; Gupta and Hira, 2006).

The Simplex method is a Pivot Algorithm that transverses the through Feasible Basic Solutions while Objective Function is improving. The Simplex method is, in practice, one of the most efficient algorithms but it is theoretically a finite algorithm only for non-degenerate problems (Feiring, 1986).

To derive solutions from the LP formulated using the Simplex method, the objective function and the constraints must be standardized.

The characteristics of the standard form are:

(i) All the constraints are expressed in the form of equations except the non-negativity constraints which remain inequalities (≥ 0).

(ii) The right-hand-side of each constraint equation is non-negative.

(iii) All the decision variables are non-negative.

(iv) The Objective function is of maximization or minimization type. Before attempting to obtain the solution of the linear programming problem, it must be expressed in the standard form is then expressed in the "the table form" or "matrix form" as given below:

$$Maximize\ Z = \sum_{j=1}^{r} C_j X_j$$

$$Subject\ to$$

$$\sum_{j=1}^{r} a_{ij} X_j \leq b_i,\ (b_i \geq 0),\ i = 1,2,3,...,m$$

$$X_j \geq 0,\ j = 1,2,3,...,m$$

In standard form (Canonical form), it is

$$Maximize\ Z = \sum_{j=1}^{r} C_j X_j$$

$$Subject\ to\ \sum_{j=1}^{r} a_{ij} X_j + S_i = b_i,\ i = 1,2,3,....,m$$

$$X_j \geq 0,\ j = 1,2,3,....,n$$

$$S_i \geq 0,\ i = 1,2,3,....,m$$

Any vector X satisfying the constraints of the Linear Programming Problems is called Feasible Solution of the problem (Fogiel, 1996; Schulze, 1998; Chinneck, 2000).

Algorithm to solve linear programming problem:

(i) See that all $b_i's$ are positive, if a constraint has negative b_i multiply it by -1 to make b_i positive.

(ii) Convert all the inequalities by the addition of slack or by subtraction of surplus variable as the case may be.

(iii) Find the starting Basic Feasible Solution.

(iv) Construct the Simplex table as follows:

Basic Variable	E_j	X_1	X_2	$X_3....$	X_n	y_1	$y_2...$	y_m	X_b
y_1	0	a_{11}	a_{12}	$a_{13}...$	a_{1m}	1	0	0	b_1
y_2	0	a_{21}	a_{22}	$a_{23}...$	a_{2m}	0	1	0	b_2
y_3	0	a_{31}	a_{32}	$a_{33}...$	a_{3m}	0	0	1	b_m
	Z_j	Z_1	Z_2	Z_3	Z_4	0	0	0	$Z = Z_0$
	E_j	E_1	E_2	E_3	E_4	0	0	0	
	.	.	.	.	.	.	.	.	
$\Delta b_j = Z_j - E_j$		ΔX_1	ΔX_2	$\Delta X_3...$	ΔX_n	Δy_1	$\Delta y_2..$	Δy_m	

(v) Testing for optimality of Basic Feasible Solution by computing $\Delta Z_j - E_j$. If $Z_j - E_j \geq 0$, the solution is optimal; otherwise, we proceed to the next step.

(vi) To improve on the Basic Feasible Solution, we find the basic matrix. The variable that corresponds to the most negative of $Z_j - E_j$ is the INCOMING VECTOR while the variable that corresponds to the minimum ratio b_i / a_{ij} for a particular j, and $a_{ij} \geq 0,\ i = 1,2,3,...,m$ is the OUTGOING VECTOR.

(vii) The key element or the pivot element is determined by considering the intersection between the arrows that corresponds to both incoming and outgoing vectors. The key element is used to generate the next table. In the next table, pivot element is replaced by UNITY, while all other elements of the pivot column are replaced by zero. To calculate the new values for all other elements in the remaining rows of that first column, we use the relation.

New row = Former element in old rows − (intersection element in the old row) × (Corresponding element of replacing row).

$(viii)$ Test of this new Basic Feasible Solution for optimality as (6) it is not optimal; repeat the process till optimal solution is obtained. This was implemented by the software Management Scientist Version 6.0.

Table 1: Quantity of raw materials available in stock

Raw materials	Quantity available
Concentrates	$4332(units)$
Sugar	$467012(kg)$
Water (H_2O)	$16376630(litres)$
Carbon $(iv)oxide(C_2O)$	$8796(Vol.per\ \ pressure)$

Table 2: Quantity of raw materials needed to produce a crate of each product listed

Flavors	$Concentrate$	$Sugar$	$Water$	$Cabon(iv)oxide$
Coke 35cl	0.00359	0.54	6.822	0.0135
Fanta Orange 35cl	0.00419	1.12	7.552	0.007
Fanta Orange 35cl	0.00217	0.803	4.824	0.005
Fanta Lemon 35cl	0.0042	1.044	7.671	0.0126
Schweppes	0.00359	0.86	6.539	0.0125
Sprite 50cl	0.00359	0.73	7.602	0.0133
Fanta Tonic 35cl	0.00359	0.63	6.12	0.0133
Krest soda 35cl	0.0036	0.89	7.055	0.0146
Coke 50cl	0.00438	0.23	7.508	0.0156

Table 3: Average Cost and Selling of a crate of each product

Product	Average Cost price(₦)	Average Selling price(₦)	Profit(₦)
Coke 35cl	358.51	690	331.49
Fanta Orange 35cl	370.91	690	319.09
Fanta Orange 50cl	489.98	790	300.02
Fanta Lemon 35cl	368.67	690	321.33
Schweppes	341.85	690	348.15
Sprite 50cl	486.04	790	303.96
Fanta Tonic 35cl	322.09	690	367.91
Krest soda 35cl	266.72	690	423.28
Coke 50cl	489.89	790	300.11

Model Formulation

Maximize Z

$$Z = 331.49X_1 + 319.09X_2 + 300.02X_3 + 321.33X_4 + 348.15X_5 + 303.96X_6 + 367.91X_7 + 323.28X_8 + 300.11X_9$$

Subject to:

$$0.00359X_1 + 0.00419X_2 + 0.0021X_3 + 0.0042X_4 + 0.00359X_5 + 0.00359X_6 + 0.00359X_7 + 0.0036X_8 + 0.00438X_9 \le 4332$$

$$0.54X_1 + 1.12X_2 + 0.803X_3 + 1.044X_4 + 0.86X_5 + 0.73X_6 + 0.63X_7 + 0.89X_8 + 0.23X_9 \le 467012$$

$$6.822X_1 + 7.552X_2 + 4.824X_3 + 7.671X_4 + 6.539X_5 + 7.602X_6 + 6.12X_7 + 7.055X_8 + 7.508X_9 \le 16376630$$

$$0.0135X_1 + 0.007X_2 + 0.005X_3 + 0.0126X_4 + 0.0125X_5 + 0.0133X_6 + 0.0133X_7 + 0.0149X_8 + 0.0156X_9 \le 8796$$

$$X_i \ge 0, \quad for\ i = 1, 2, 3, \ldots, 9$$

Now, introducing the slack variable to convert inequalities to equations, it gives:

$$Z = 331.49X_1 + 319.09X_2 + 300.02X_3 + 321.33X_4 + 348.15X_5 + 303.96X_6 + 367.91X_7 + 323.28X_8 + 300.11X_9$$

Subject to:

$$0.00359X_1 + 0.00419X_2 + 0.00217X_3 + 0.0042X_4 + 0.00359X_5 + 0.00359X_6 + 0.00359X_7 + 0.0036X_8 + 0.00438X_9 + X_{10} = 4332$$

$$0.54X_1 + 1.12X_2 + 0.803X_3 + 1.044X_4 + 0.86X_5 + 0.73X_6 + 0.63X_7 + 0.89X_8 + 0.23X_9 + X_{11} = 467012$$

$$6.822X_1 + 7.522X_2 + 4.834X_3 + 7.671X_4 + 6.539X_5 + 7.602X_6 + 6.12X_7 + 7.055X_8 + 7.058X_9 + X_{12} = 16376630$$

$$0.00135X_1 + 0.007X_2 + 0.005X_3 + 0.0126X_4 + 0.0125X_5 + 0.0133X_6 + 0.0133X_7 + 0.0149X_8 + 0.0156X_9 + X_{13} = 8796$$

$$X_i \ge 0, \quad for\ i = 1, 2, 3, \ldots, 13$$

Where,
X_1 = Coke 35cl
X_2 = Fanta Orange 35cl
X_3 = Fanta Orange 50cl
X_4 = Fanta Lemon 35cl
X_5 = Schweppes
X_6 = Sprite 50cl
X_7 = Fanta Tonic 35cl
X_8 = Krest soda 35cl
X_9 = Coke 50cl

Analysis and Result

Optimal Solution
Objective function value = 263,497,283

Variables	Value
X_1	0.00000
X_2	0.00000
X_3	462547.21890
X_4	0.00000
X_5	0.00000
X_6	0.00000
X_7	0.00000
X_8	0.00000
X_9	415593.00000

The Management Scientist Version 6.0 gives:

$$Z = 263,497,283 \quad X_3 = 462,547 \ and \quad X_9 = 415,593$$

Interpretation of Result

Based on the data collected the optimum results derived from the model indicate that two products should be produced 50cl Coke and 50cl Fanta Orange. Their production quantities should be 462,547 and 415,593 crates respectively. This will produce a maximum profit of ₦263,497,283.

Conclusion

Based on the analysis carried out in this research and the result shown, Nigeria Bottling Company, Ilorin plant should produce Fanta Orange 50cl and 35cl,Coke 50cl and 35cl, Fanta lemon 35cl, Sprite 50cl,Schweppes,Krest soda 35cl but more of Coke 50cl and Fanta Orange 50cl in order to satisfy their customers. Also, more of Coke 50cl and Fanta Orange 50cl should be produced in order to attain maximum profit because they contribute mostly to the profit earned.

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