

Denoising of Spectral Data Using Complex Wavelets

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Abstract

Atmospheric signal processing is of interest to many scientists, where there is scope for the development of new and efficient tools for cleaning the spectrum, detection, and estimation of parameters like zonal (U), meridional (V), wind speed (W), etc. This paper deals with a signal processing technique for the cleansing of spectrum, based on the complex wavelets with custom thresholding, by analyzing the Mesosphere–Stratosphere–Troposphere radar data that are backscattered from the atmosphere at high altitudes and severe weather conditions with low signal-to-noise ratio. The proposed algorithm is self-consistent in detecting wind speeds up to a height of 18 km, in contrast to the existing method which estimates the Doppler manually and fails at higher altitudes. The results have been validated using the Global Positioning System sonde data.

Keywords:- Signal Processing, Complex Wavelets, MST RADARS, Denoising.

I. INTRODUCTION

RADAR can be employed, in addition to the detection and characterization of hard targets, to probe the soft or distributed targets such as the Earth's atmosphere. Atmospheric radars, of interest to the current study, are known as clear air radars, and they operate typically in very high frequency (30–300 MHz) and ultrahigh-frequency (300 MHz–3 GHz) bands. The turbulent fluctuations in the refractive index of the atmosphere serve as a target for these radars. The present algorithm used in atmospheric signal processing called “classical” processing can accurately estimate the Doppler frequencies of the backscattered signals up to a certain height. However, the technique fails at higher altitudes and even at lower altitudes when the data are corrupted with noise due to interference, clutter, etc. Bispectral-based estimation algorithm has been tried to eliminate noise [1]. However, this algorithm has the drawback of high computational cost. Multitaper spectral estimation algorithm has been applied for radar data [2]. This method has the advantage of reduced variance at the expense of broadened spectral peak. The fast Fourier transform (FFT) technique for power spectral estimation and “adaptive estimates technique” for estimating the moments of radar data has been proposed in [3]. This method considers a certain number of prominent

peaks of the same range gate and tries to extract the best peak, which satisfies the criteria chosen for the adaptive method of estimation. The method, however, has failed to give consistent results. Hence, there is a need for development of better algorithms for efficient cleaning of spectrum.

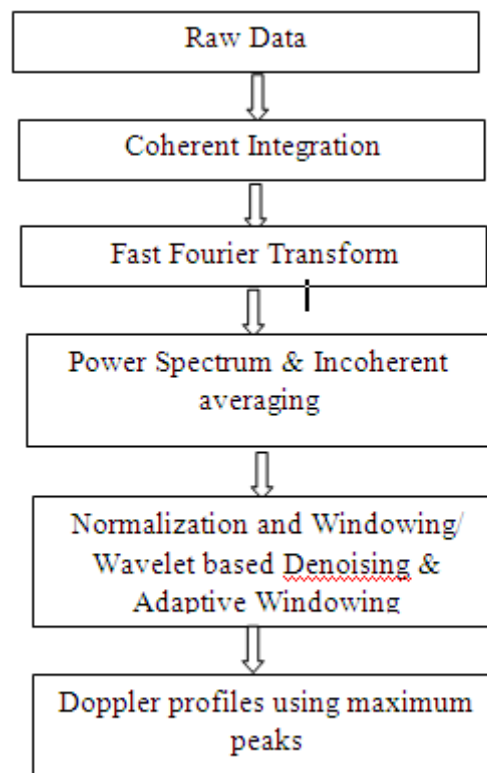


Fig.1. Flow chart for Proposed Method

The National Atmospheric Research Laboratory, Andhra Pradesh, India, has developed a package for processing the atmospheric data. They refer to it as the atmospheric data processor [4]. In this paper, it is named as existing algorithm (EALG). The flowchart of this algorithm is given in Fig. 1. Coherent integration of the raw data (I and Q) collected by radar is performed. It improves the process gain by a factor of number of inter-pulse period. The presence of a quadrature component of the signal improves the signal-to-noise ratio (SNR). The normalization process will reduce the chance of data overflow due to any other succeeding operation. The data are windowed to reduce the leakage and picket fence effects. The spectrum of the signal is computed using FFT. The incoherent integration improves the detectability of the Doppler spectrum.

The radar echoes may be corrupted by ground clutter, system bias, interference, etc.

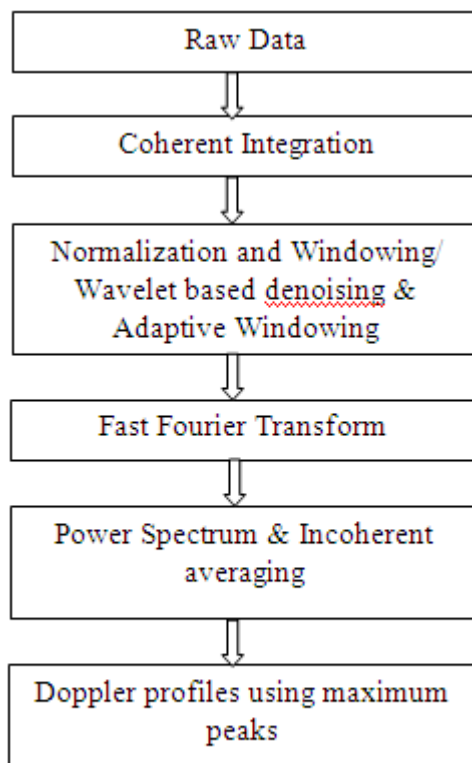


Fig.2. Flow chart for Existing Method

The data is to be cleaned from these problems before going for analysis. After performing power spectrum cleaning, one has to manually select a proper window size depending upon the wind shear [6], etc., from which the Doppler profile is estimated by using a maximum peak detection method [3]. The EXISTING METHOD is able to detect the Doppler clearly up to 11 km as the noise level is very low. Above 11 km, noise is dominating, and, hence, the accuracy of the Doppler estimated using the EXISTING METHOD is doubtful as is discussed in the subsequent sections. To overcome the effect of noise at high altitudes, a wavelet-based denoising algorithm is applied to the radar data before computing its spectrum [5]. This paper gives better results compared to [4], but this method fails to extract the exact frequency components after denoising at higher altitudes. To overcome this effect we proposed a new method, where spectrum is estimated prior to denoising and then denoised using Complex Wavelet Transform (CWT) with the help of Custom thresholding method.

III. Complex Wavelet Transform (CWT)

Complex wavelet transforms (CWT) uses complex-valued filtering (analytic filter) that decomposes the real/complex signals into real and imaginary parts in transform domain. The real and imaginary coefficients are used to compute amplitude and phase information, just the type of information

needed to accurately describe the energy localization of oscillating functions (wavelet basis). The Fourier transform is based on complex-valued oscillating sinusoids

$$e^{j\Omega t} = \cos(\Omega t) + j \sin(\Omega t)$$

The corresponding complex-valued scaling function and complex-valued wavelet is given as

$$\psi_c(t) = \psi_r(t) + j \psi_i(t)$$

where $\psi_r(t)$ is real and even,

$$j \psi_i(t) \text{ is imaginary and odd.}$$

Gabor introduced the Hilbert transform into signal theory in [9], by defining a complex extension of a real signal $f(t)$ as:

$$x(t) = f(t) + j g(t)$$

where, $g(t)$ is the Hilbert transform of $f(t)$ and denoted as $H\{f(t)\}$ and $j = (-1)^{1/2}$.

The signal $g(t)$ is the 90° shifted version of $f(t)$ as shown in figure (3.1 a). The real part $f(t)$ and imaginary part $g(t)$ of the analytic signal $x(t)$ are also termed as the 'Hardy Space' projections of original real signal $f(t)$ in Hilbert space. Signal $g(t)$ is orthogonal to $f(t)$. In the time domain, $g(t)$ can be represented as [7]

$$g(t) = H\{f(t)\} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{f(\tau)}{t - \tau} d\tau = f(t) * \frac{1}{\pi t}$$

If $F(\omega)$ is the Fourier transform of signal $f(t)$ and $G(\omega)$ is the Fourier transform of signal $g(t)$, then the Hilbert transform relation between $f(t)$ and $g(t)$ in the frequency domain is given by

$$G(\omega) = F\{H\{f(t)\}\} = -j \operatorname{sgn}(\omega) F(\omega)$$

where, $-j \operatorname{sgn}(\omega)$ is a modified 'signum' function.

This analytic extension provides the estimate of instantaneous frequency and amplitude of the given signal $x(t)$ as:

$$\text{Magnitude of } x(t) = \sqrt{(f(t))^2 + g(t)^2}$$

$$\text{Angle of } x(t) = \tan^{-1}[g(t) / f(t)]$$

The other unique benefit of this quadrature representation is the non-negative spectral representation in Fourier domain [7] and [8], which leads toward half the bandwidth utilization. The reduced bandwidth consumption is helpful to avoid aliasing of filter bands especially in multirate signal processing applications. The reduced aliasing of filter bands is the key for shift-invariant property of CWT. In one dimension, the so-called dual-tree complex wavelet transform provides a representation of a signal $x(n)$ in terms of complex wavelets, composed of real and imaginary parts which are in turn wavelets

themselves. Figure 3 shows the Analysis and Synthesis of Dual tree complex wavelet transform for three levels.

customized thresholding function is introduced. The Custom thresholding function is continuous around the threshold, and which can be adapted to the characteristics of the input signal. Based on extensive experiments, we could see that soft-thresholding

III. CUSTOM THRESHOLDING

To overcome the pulse broadening effect in improved thresholding function a new function called

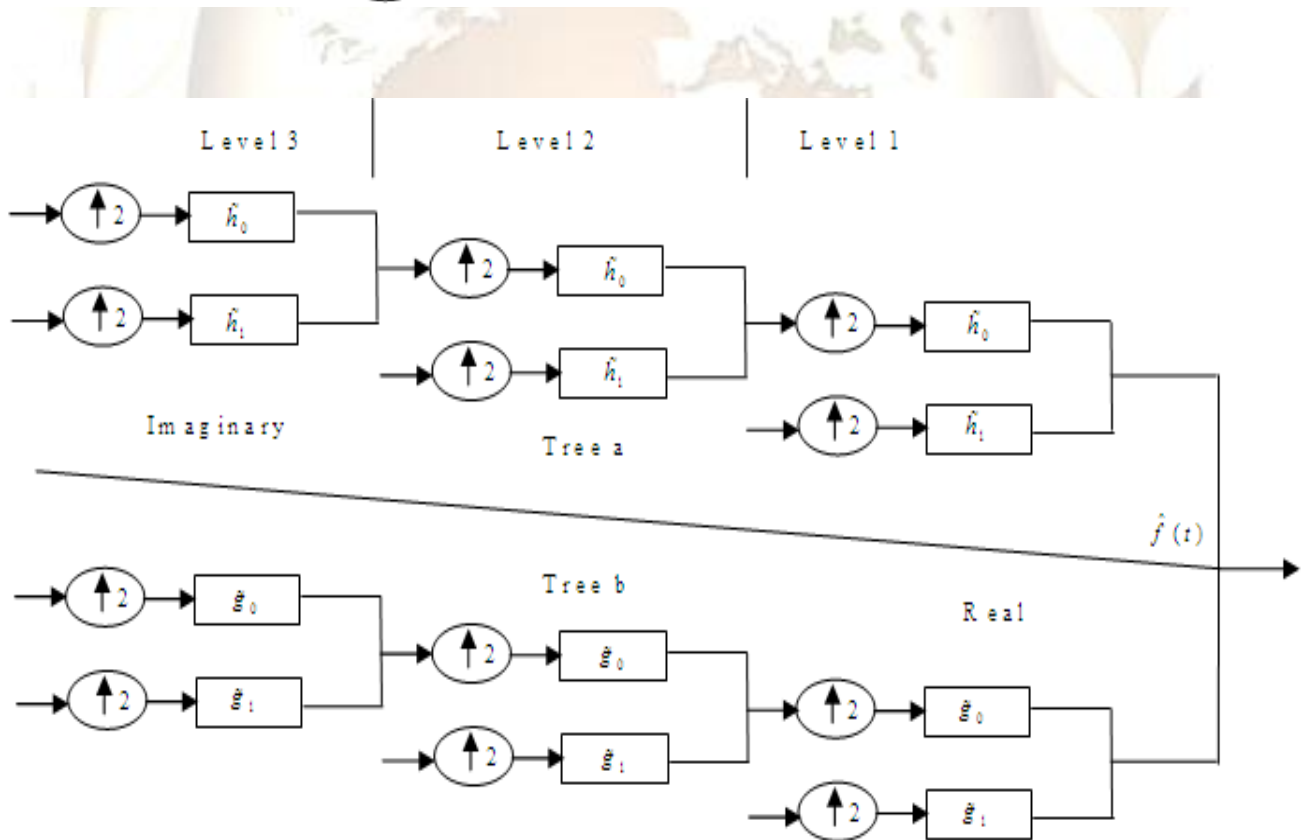
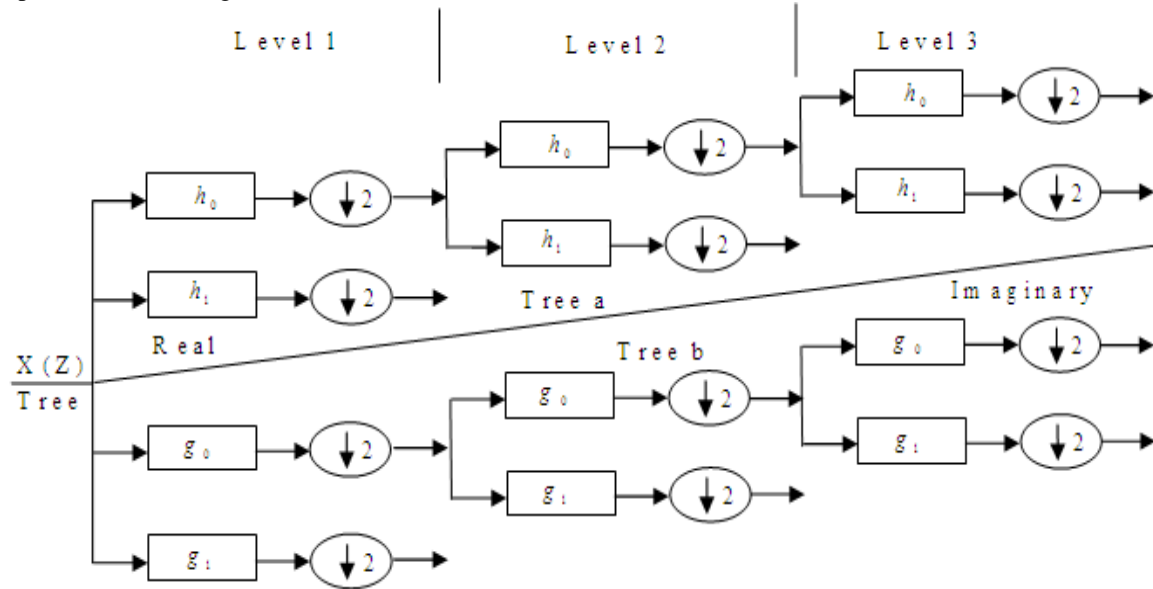


Fig. 3 (a) Analysis filter bank (b) Synthesis filter bank for Dual Tree-DWT for three levels.

outperforms hard-thresholding in general. However, there were also cases where hard-thresholding yielded a much superior result, and in those cases the quality of the estimate could be improved by using a custom thresholding function which is similar to the hard-thresholding function but with a smooth transition around the threshold λ . Based on these observations, we defined a new custom thresholding function as follows:

$$f_c(x) = \begin{cases} x - \text{sgn}(x)(1-\alpha)\lambda & \text{if } |x| \geq \lambda \\ 0 & \text{if } |x| \leq \lambda \\ \alpha\lambda \left(\frac{|x|-\gamma}{\lambda-\gamma} \right)^2 \left\{ (\alpha-3) \left(\frac{|x|-\gamma}{\lambda-\gamma} \right) + 4 - \alpha \right\} & \text{otherwise} \end{cases}$$

where $0 < \gamma < \lambda$ and $0 \leq \alpha \leq 1$. This idea is similar to that of the *semisoft* or *firm shrinkage* proposed by Gao and Bruce [10], and the *non-negative garrote* thresholding function suggested by Gao [10], in the sense that they are continuous at λ and can be adapted to the signal characteristics. In our definition of $f_c(x)$, γ is the cut-off value, below which the wavelet coefficients are set zero, and α is the parameter that decides the shape of the thresholding function $f_c(x)$. This function can be viewed as the linear combination of the hard-thresholding function and the soft-thresholding function $\alpha \cdot f_h(x) + (1-\alpha) \cdot f_s(x)$ that is made continuous around the threshold λ .

Note that,

$$\lim_{\alpha \rightarrow 0} f_c(x) = f_s(x) \text{ and } \lim_{\alpha \rightarrow 1, \gamma \rightarrow \gamma} f_c(x) = f_h(x)$$

This shows that the custom thresholding function can be adapted to both the soft- and hard-thresholding functions.

IV. PROPOSED METHOD

The algorithm for cleansing the spectrum using CWT with custom thresholding is as follows.

- 1) Let $f(n)$ be the noisy spectral data, for $n = 0, 1, \dots, N-1$.
- 2) Generate noisy signal $y(n)$ using $x(n) = f(n) + \sigma z(n)$, $n = 1, 2, \dots, N$
- 3) Spectrum is calculated for the above signal $y(n)$ as $Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y[n] e^{-j\omega n}$.
- 4) Input $x(n)$ to the two DWT trees with one tree uses the filters h_0, h_1 and the other tree with filters g_0, g_1 .
- 5) Apply custom thresholding to wavelet coefficients in the two trees.
- 6) Compute IDWT using these thresholded wavelet coefficients.
- 7) The coefficients from the two trees are then averaged to obtain the denoised original signal.

8) Then variance is calculated using

$$\frac{1}{N} \sum_{k=0}^{N-1} \left(Y(k) - \sum_{k=0}^{N-1} Y(k) \right)^2$$

V. RESULTS

In this section, we present the results for the cleansing of spectrum for complex test data based on the proposed Method. The data are generated by applying a Gaussian random input to a complex system. As the true spectrum is not available variance is taken as the performance measure, instead of Mean-square error.

The variance is calculated as

$$\frac{1}{N} \sum_{k=0}^{N-1} \left(Y(k) - \sum_{k=0}^{N-1} Y(k) \right)^2$$

A complex signal is generated and Fourier transform is calculated for that signal. To test the performance of the algorithm in the presence of additive white Gaussian noise, a new signal $y_1(n)$ is generated as

$$y_1(n) = y(n) + \alpha v(n) \text{ for } n = 0, 1, 2, \dots, N-1;$$

where $v(n)$ is a complex Gaussian noise with zero mean and unit variance and α is the amplitude associated with $v(n)$. The number of samples in each realization is assumed to be 512, i.e., $N = 512$. The result based on the Monte Carlo simulation using proposed method is shown in fig.2. The same using existing method is shown in the fig.3. Table 1 gives the improved SNR and Variance of denoised signals by processing time domain data and frequency domain data using complex wavelets. From the table 6.1 it is clear that the denoised data obtained by processing

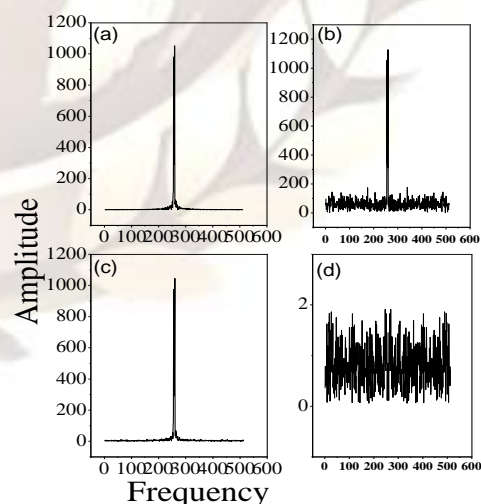


Fig. 2 (a) original Spectrum (b) Noisy Spectrum (c) Denoised Spectrum (d) variance

Now I processed same test signal and MST radar signal using complex wavelet transform with improved and custom thresholding techniques.

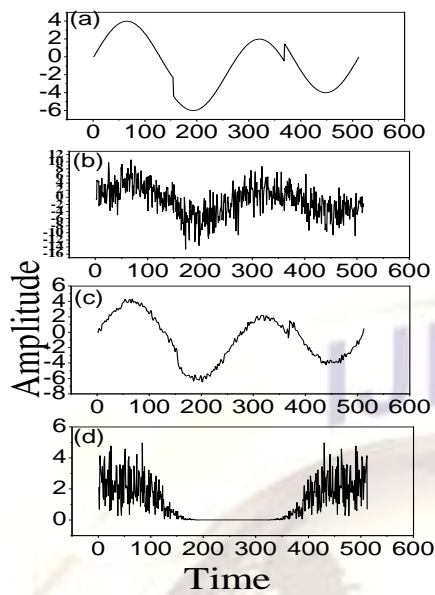


Fig. 3 (a) original Signal (b) Noisy Signal (c) Denoised Signal (d) variance frequency data using CWT has got better performance than processing the time-domain data. The same is represented graphically in figure 4.

Table 1 Performance of Proposed Method With respect to input SNR.

Domain Type	Input SNR	Output SNR	Variance
Frequency Domain	-10	12.9608	2.8997e+003
	-5	21.5316	800.6487
	0	24.3023	231.3500
	5	26.6103	78.7414
	10	30.9121	25.2987
Time Domain	-10	10.3287	3.9843e+003
	-5	17.6794	1.7855e+003
	0	22.1051	498.1208
	5	24.9066	101.4611
	10	28.9177	57.5245

VI. CONCLUSION

The wavelet transform allows processing of non-stationary signals such as MST radar signal. This is possible by using the multi resolution decomposing into sub signals. This assists greatly to remove the noise in the certain pass band of frequency. At first I have processed test signal (time and frequency domains) with -10 dB SNR using complex wavelet transform (CWT) by designing sub band filters using Hilbert transform maintaining perfect-reconstruction property, with the help of custom thresholding. The main idea in implementing this technique is to overcome the limitations like shift sensitivity, poor directionality, and absence of phase information which occurs in DWT. Using this technique we can extract the signal even-though input SNR is -15 dB.

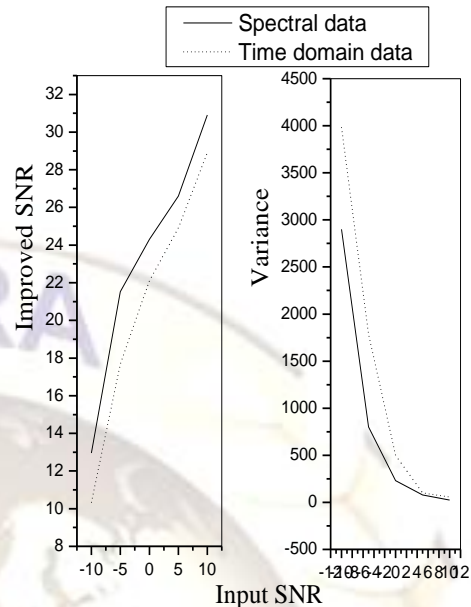


Fig. 4 (a) Input SNR vs. Output SNR (b) Input SNR vs. Variance

Among these two methods with different thresholding techniques, the processing of spectral data using complex wavelet transform with custom thresholding method is giving better results than processing the time domain signal. The proposed method may be used effectively in detection, image compression and image denoising and also it is giving better results at low signal-to-noise ratio cases (even at -15 dB)

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