Case Study for Bank ATM Queuing Model

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Abstract:
   Bank ATMs would avoid losing their customers due to a long wait on the line. The bank initially provides one ATM. However, one ATM would not serve a purpose when customers withdraw to use ATM and try to use other bank ATM. Thus, to maintain the customers, the service time needs to be improved. This paper shows that the queuing theory may be used to solve this problem. We obtained the data from a bank ATM in a city. We used Little's Theorem and M/M/1 queuing model. The arrival rate at a bank ATM on Monday during banking time is 1 customer per minute (cpm) while the service rate is 1.66 cpm. The average number of customers in the ATM is 1.5 and the utilization period is 0.60. We discuss the benefits of applying queuing theory to a busy ATM in conclusion.
Keywords: Bank ATM, Little's Theorem, M/M/1 queuing model, Queue, Waiting lines

I.   Introduction

   This paper uses queuing theory to study the waiting lines in Bank ATM in a city. The bank provides one ATM in the main branch.

   In ATM, bank customers arrive randomly and the service time i.e. the time customer takes to do transaction in ATM, is also random. We use M/M/1 queuing model to derive the arrival rate, service rate, utilization rate, waiting time in the queue and the average number of customers in the queue. On average, 500 customers are served on weekdays (monday to Friday) and 300 customers are served on weekends (Saturday and Sunday) monthly. Generally, on Mondays, there are more customers coming to ATM, during $10\ a.m. to\ 5\ p.m.$.

II.   Queuing theory

A. Little's Theorem

   Little's Theorem describes the relationship between throughput rate (i.e. arrival and service rate), cycle time and work in process (i.e. number of customers/jobs in the system). The theorem states that the expected number of customers (N) for a system in steady state can be determined using the following equation:

$$L = \lambda T \qquad\qquad (1)$$

Here, λ is the average customer arrival rate and T is the average service time for a customer.

B.   ATM Model (M/M/1 queuing model)
   M/M/1 queuing model means that the arrival and service time are exponentially distributed (Poisson process). For the analysis of the ATM M/M/1 queuing model, the following variables will be investigated:

- λ: The mean customers arrival rate
- μ: The mean service rate
- $\rho: \frac{\lambda}{\mu}$: utilization factor
- Probability of zero customers in the ATM:
$$P_0 = 1 - \rho \qquad\qquad (2)$$
- P_n: The probability of having n customers in the ATM:
$$P_n = P_0 \rho^n = (1-\rho)\rho^n \qquad (3)$$
- L: The average number of customers in the ATM:
$$L = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu - \lambda} \qquad\qquad (4)$$
- L_q: The average number of customers in the queue:
$$L_q = L \times \rho = \frac{\rho^2}{1-\rho} = \frac{\rho\lambda}{\mu-\lambda} \quad (5)$$
- W_q: The average waiting time in the queue:
$$W_q = \frac{L_q}{\lambda} = \frac{\rho}{\mu-\lambda} \qquad\qquad (6)$$
- W: The average time spent in the ATM, including the waiting time:
$$W = \frac{L}{\lambda} = \frac{1}{\mu-\lambda} \qquad\qquad (7)$$

III.   Observation and Discussion

   We have collected the one month daily customer data by observation during banking time, as shown in Table-1.

Table-1 Monthly Customer counts

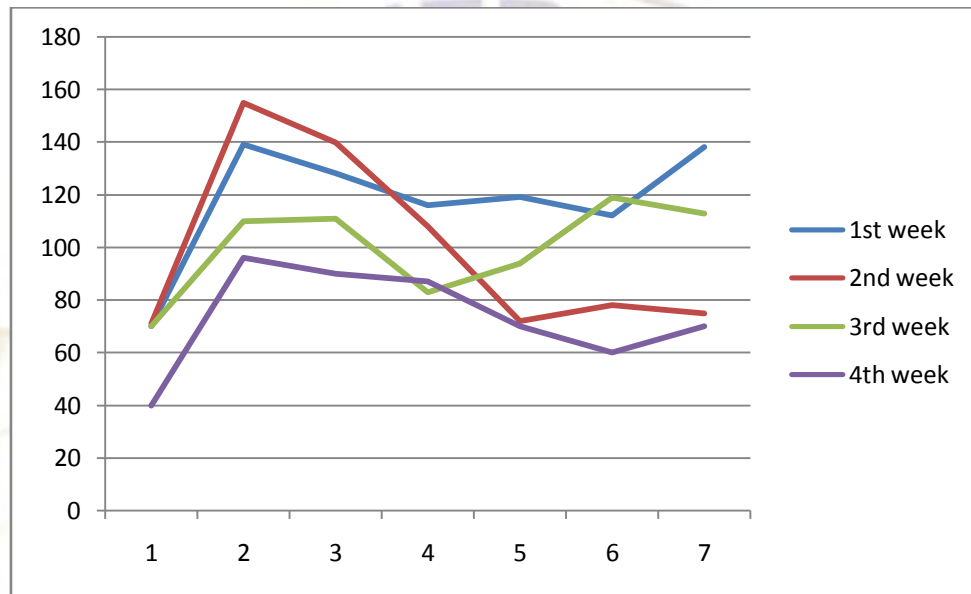|	Sun	Mon	Tue	Wed	Thu	Fri	Sat
1st week	70	139	128	116	119	112	138
2nd week	71	155	140	108	72	78	75
3rd week	70	110	111	83	94	119	113
4th week	40	96	90	87	70	60	70
Total	**251**	**500**	**469**	**394**	**355**	**369**	**396**

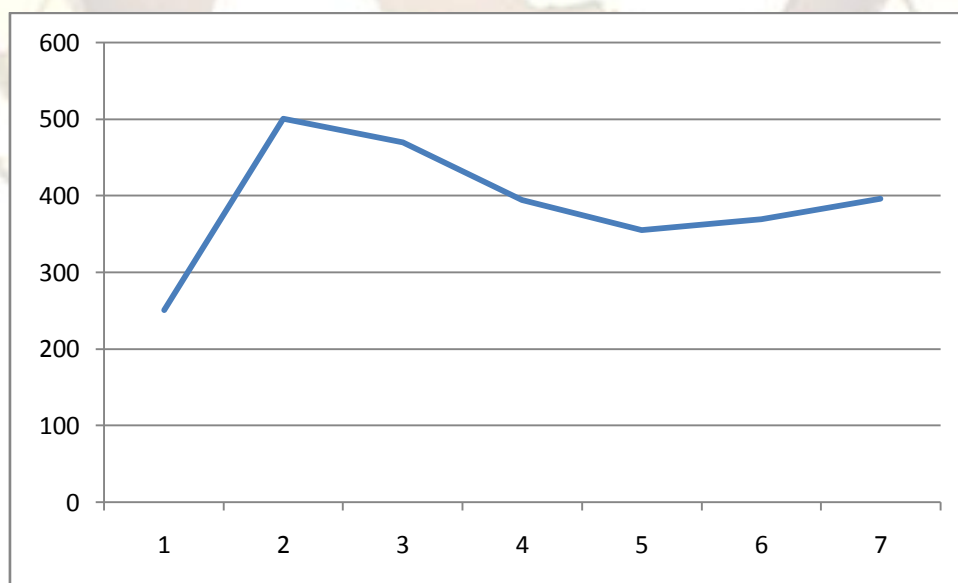Figure-1   one month daily customer counts

Figure-2   one month total customer counts

From the above figure-1, we can say that, the number of customers on Mondays is double the number of customers on Sundays during a month. The busiest period for the bank ATM is on Mondays and Tuesdays during banking time ($10\ a.m.\ to\ 5\ p.m.$). Hence, we will focus our analysis in this time period. Also, we can observe from figure-2 that, after Monday, the number of customers start decreasing slowly as the week progresses. On Thursdays, it is least and on Fridays and Saturdays, it stays slightly more than Thursdays. This is because the next day will be a holiday.

A. Calculation

We have observed that, after Sunday, during first two days of a week, there are, on average 60 people coming to the ATM in one hour time period of banking time. From this we can derive the arrival rate as:

$$\lambda = \frac{60}{60} = 1\,\text{customer/minute (cpm)}$$

We also found out from observation that each customer spends $3/2$ minutes on average in the ATM (W), the queue length is around 1 people (L_q) on average and the average waiting time is around $1/2$ minutes i.e. 30 seconds.

Theoretically, the average waiting time is

$$W_q = \frac{L_q}{\lambda} = \frac{1\ customer}{1\ cpm} = 1\ minute$$

From this calculation, we can see that, the observed actual waiting time does not differ by much when it is compared with the theoretical waiting time.

Next, we will calculate the average number of people in the ATM using (1),

$$L = 1\ cpm \times \frac{3}{2}\ minutes = \frac{3}{2} = 1.5\ customers$$

Using (4), we can also derive the utilization rate and the service rate.

$$\mu = \frac{\lambda(1+L)}{L} = \frac{1(1+\frac{3}{2})}{\frac{3}{2}} = \frac{5/2}{3/2} = \frac{5}{3} = 1.66\ cpm$$

Hence, $\rho = \frac{\lambda}{\mu} = \frac{1\ cpm}{\frac{5}{3}\ cpm} = \frac{3}{5} = 0.60$

This is the probability that, the server, in this case ATM, is busy to serve the customers, during banking time. So, during banking time, the probability of zero customers in the ATM is

$$P_0 = 1 - \rho = 1 - 0.60 = 0.40$$

The queuing theory provides the formula to calculate the probability of having n customers in the ATM as follows:

$$P_n = (1 - \rho)\rho^n = (1 - 0.60)(0.60)^n = (0.40)(0.60)^n$$

We assume that impatient customers will start to balk when they see more than 3 people are already queuing for the ATM. We also assume that the maximum queue length that a patient customer can tolerate is 10 people. As the capacity of the ATM is 1 people, we can calculate the probability of 4 people in the system (i.e in the ATM).

Therefore, the probability of customers going away = P(more than 3 people in the queue) = P(more than 4 people in the ATM) is

$$P_{5-11} = \sum_{n=5}^{11} P_n = 0.07558 = 7.55\ \%$$

B. Evaluation

- The utilization is directly proportional with the mean number of customers. It means that the mean number of customers will increase as the utilization increases.

- The utilization rate at the ATM is at 0.60. However, this is the utilization rate during banking time on Mondays and Tuesdays. On weekend, the utilization rate is almost half of it. This is because the number of people on weekends is only half of the number of people on weekdays.

- In case of the customers waiting time is lower or in other words, we waited for less than 30 seconds, the number of customers that are able to be served per minute will increase. When the service rate is higher the utilization will be lower, which makes the probability of the customers going away decreases.

C. Benefits

- This research can help bank ATM to increase its QoS (Quality of Service), by anticipating, if there are many customers in the queue.

- The result of this paper is helpful to analyse the current system and improve the next system. Because the bank can now estimate the number of customers waiting in the queue and the number of customers going away each day.

- By estimating the number of customers coming and going in a day, the bank can set a target that, how many ATMs are required to serve people in the main branch or any other branch of the bank.

IV. Conclusion

This paper has discussed the application of queuing theory to the Bank ATM. From the result,

we have obtained that, the rate at which customers arrive in the queuing system is 1 customer per minute and the service rate is 1.66 customers per minute. The probability of buffer flow if there are 3 or more customers in the queue is 7 out of 100 customers. The probability of buffer overflow is the probability that, customers will run away, because may be they are impatient to wait in the queue. This theory is also applicable for the bank, if they want to calculate all the data daily and this can be applied to other branch ATM also. In this way, this research can contribute to the betterment of a bank in terms of its functioning through ATM.
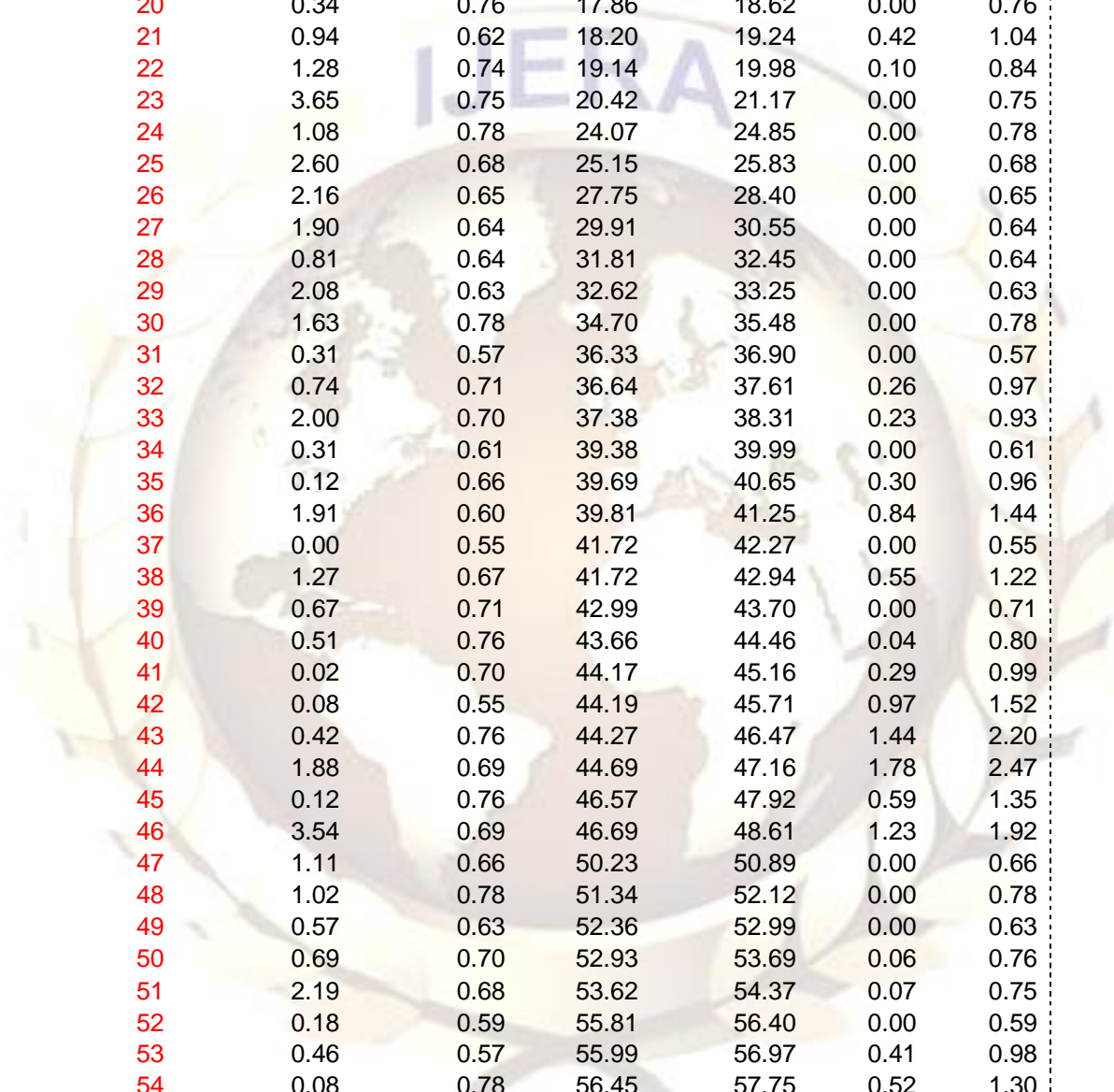
Now, we will develop a simulation model for the ATM. By developing a simulation model, we will be able to confirm the results of the analytical model that we develop in this paper. By this model, it can mirror the actual operation of the ATM more closely.

We discuss the simulation of ATM model as follows:

Simulation of a Single-Server Queuing Model

Nbr of arrivals =	125	<<Maximum 500			
Enter x in column A to select interarrival pdf:					
	Constant =				
x	**Exponential:**	$\lambda =$	1		
	Uniform:	a =		b =	
	Triangular:	a =		b =	c =
Enter x in column A to select service time pdf:					
	Constant =				
	Exponential:	$\mu =$	1.66		
x	**Uniform:**	a =	0.5	b =	0.8
	Triangular:	a =		b =	c =

Output Summary		
Av. facility utilization =	0.63	
Percent idleness (%) =	36.82	
Av. queue length, Lq =	0.44	
Av. nbr in system, Ls =	1.07	Press F9 to trigger a new simulation run.
Av. queue time, Wq =	0.46	
Av. system time, Ws =	1.13	
Sum(ServiceTime) =	83.49	
Sum(Wq) =	57.58	
Sum(Ws) =	141.07	

Nbr	InterArvlTime	ServiceTime	ArrvlTime	DepartTime	Wq	Ws
1	1.79	0.55	0.00	0.55	0.00	0.55
2	0.68	0.56	1.79	2.35	0.00	0.56
3	0.51	0.79	2.47	3.26	0.00	0.79
4	1.05	0.70	2.98	3.96	0.28	0.98
5	0.90	0.61	4.03	4.64	0.00	0.61
6	1.42	0.57	4.93	5.50	0.00	0.57
7	0.58	0.63	6.35	6.98	0.00	0.63
8	0.68	0.74	6.93	7.72	0.05	0.79
9	0.42	0.60	7.61	8.32	0.11	0.71
10	2.76	0.52	8.03	8.84	0.29	0.81
11	0.57	0.54	10.79	11.33	0.00	0.54
12	0.93	0.62	11.36	11.98	0.00	0.62

13	0.30	0.54	12.29	12.83	0.00	0.54
14	1.29	0.51	12.59	13.34	0.24	0.75
15	0.44	0.78	13.88	14.66	0.00	0.78
16	1.32	0.53	14.32	15.19	0.34	0.87
17	0.43	0.63	15.64	16.27	0.00	0.63
18	0.64	0.65	16.07	16.92	0.20	0.85
19	1.15	0.72	16.71	17.64	0.21	0.93
20	0.34	0.76	17.86	18.62	0.00	0.76
21	0.94	0.62	18.20	19.24	0.42	1.04
22	1.28	0.74	19.14	19.98	0.10	0.84
23	3.65	0.75	20.42	21.17	0.00	0.75
24	1.08	0.78	24.07	24.85	0.00	0.78
25	2.60	0.68	25.15	25.83	0.00	0.68
26	2.16	0.65	27.75	28.40	0.00	0.65
27	1.90	0.64	29.91	30.55	0.00	0.64
28	0.81	0.64	31.81	32.45	0.00	0.64
29	2.08	0.63	32.62	33.25	0.00	0.63
30	1.63	0.78	34.70	35.48	0.00	0.78
31	0.31	0.57	36.33	36.90	0.00	0.57
32	0.74	0.71	36.64	37.61	0.26	0.97
33	2.00	0.70	37.38	38.31	0.23	0.93
34	0.31	0.61	39.38	39.99	0.00	0.61
35	0.12	0.66	39.69	40.65	0.30	0.96
36	1.91	0.60	39.81	41.25	0.84	1.44
37	0.00	0.55	41.72	42.27	0.00	0.55
38	1.27	0.67	41.72	42.94	0.55	1.22
39	0.67	0.71	42.99	43.70	0.00	0.71
40	0.51	0.76	43.66	44.46	0.04	0.80
41	0.02	0.70	44.17	45.16	0.29	0.99
42	0.08	0.55	44.19	45.71	0.97	1.52
43	0.42	0.76	44.27	46.47	1.44	2.20
44	1.88	0.69	44.69	47.16	1.78	2.47
45	0.12	0.76	46.57	47.92	0.59	1.35
46	3.54	0.69	46.69	48.61	1.23	1.92
47	1.11	0.66	50.23	50.89	0.00	0.66
48	1.02	0.78	51.34	52.12	0.00	0.78
49	0.57	0.63	52.36	52.99	0.00	0.63
50	0.69	0.70	52.93	53.69	0.06	0.76
51	2.19	0.68	53.62	54.37	0.07	0.75
52	0.18	0.59	55.81	56.40	0.00	0.59
53	0.46	0.57	55.99	56.97	0.41	0.98
54	0.08	0.78	56.45	57.75	0.52	1.30
55	1.60	0.74	56.53	58.49	1.22	1.96
56	0.55	0.57	58.13	59.06	0.36	0.93
57	0.80	0.75	58.68	59.81	0.38	1.13
58	0.22	0.55	59.48	60.36	0.33	0.88
59	1.76	0.72	59.70	61.08	0.66	1.38
60	0.72	0.72	61.46	62.18	0.00	0.72
61	1.15	0.58	62.18	62.76	0.00	0.58
62	4.52	0.59	63.33	63.92	0.00	0.59
63	2.04	0.71	67.85	68.56	0.00	0.71

64	0.63	0.72	69.89	70.61	0.00	0.72
65	0.12	0.77	70.52	71.38	0.09	0.86
66	0.74	0.55	70.64	71.93	0.74	1.29
67	0.14	0.65	71.38	72.58	0.55	1.20
68	0.40	0.70	71.52	73.28	1.06	1.76
69	0.31	0.65	71.92	73.93	1.36	2.01
70	0.30	0.70	72.23	74.63	1.70	2.40
71	0.55	0.70	72.53	75.33	2.10	2.80
72	0.20	0.52	73.08	75.85	2.25	2.77
73	1.43	0.74	73.28	76.59	2.57	3.31
74	0.95	0.57	74.71	77.16	1.88	2.45
75	1.05	0.61	75.66	77.77	1.50	2.11
76	2.47	0.67	76.71	78.44	1.06	1.73
77	0.28	0.56	79.18	79.74	0.00	0.56
78	1.35	0.78	79.46	80.52	0.28	1.06
79	1.14	0.76	80.81	81.57	0.00	0.76
80	4.62	0.56	81.95	82.51	0.00	0.56
81	0.05	0.71	86.57	87.28	0.00	0.71
82	0.02	0.63	86.62	87.91	0.66	1.29
83	0.22	0.72	86.64	88.63	1.27	1.99
84	0.47	0.70	86.86	89.33	1.77	2.47
85	0.52	0.71	87.33	90.04	2.00	2.71
86	1.05	0.57	87.85	90.61	2.19	2.76
87	0.28	0.50	88.90	91.11	1.71	2.21
88	3.08	0.54	89.18	91.65	1.93	2.47
89	1.10	0.70	92.26	92.96	0.00	0.70
90	0.49	0.71	93.36	94.07	0.00	0.71
91	3.37	0.59	93.85	94.66	0.22	0.81
92	0.40	0.68	97.22	97.90	0.00	0.68
93	2.77	0.64	97.62	98.54	0.28	0.92
94	0.00	0.71	100.39	101.10	0.00	0.71
95	0.83	0.77	100.39	101.87	0.71	1.48
96	1.05	0.65	101.22	102.52	0.65	1.30
97	0.13	0.68	102.27	103.20	0.25	0.93
98	0.14	0.52	102.40	103.72	0.80	1.32
99	1.45	0.80	102.54	104.52	1.18	1.98
100	0.52	0.78	103.99	105.30	0.53	1.31
101	1.34	0.65	104.51	105.95	0.79	1.44
102	2.19	0.78	105.85	106.73	0.10	0.88
103	0.84	0.57	108.04	108.61	0.00	0.57
104	2.18	0.66	108.88	109.54	0.00	0.66
105	2.63	0.75	111.06	111.81	0.00	0.75
106	0.17	0.78	113.69	114.47	0.00	0.78
107	2.02	0.76	113.86	115.23	0.61	1.37
108	1.16	0.74	115.88	116.62	0.00	0.74
109	0.66	0.68	117.04	117.72	0.00	0.68
110	0.55	0.63	117.70	118.35	0.02	0.65
111	0.51	0.67	118.25	119.02	0.10	0.77
112	0.43	0.63	118.76	119.65	0.26	0.89
113	1.27	0.77	119.19	120.42	0.46	1.23
114	0.54	0.75	120.46	121.21	0.00	0.75
115	0.08	0.77	121.00	121.98	0.21	0.98
116	0.54	0.74	121.08	122.72	0.90	1.64
117	1.36	0.73	121.62	123.45	1.10	1.83

118	4.13	0.62	122.98	124.07	0.47	1.09
119	0.81	0.73	127.11	127.84	0.00	0.73
120	0.64	0.55	127.92	128.47	0.00	0.55
121	0.00	0.77	128.56	129.33	0.00	0.77
122	0.03	0.73	128.56	130.06	0.77	1.50
123	0.31	0.53	128.59	130.59	1.47	2.00
124	1.86	0.74	128.90	131.33	1.69	2.43
125	0.01	0.77	130.76	132.10	0.57	1.34

By the simulation model, we can say that, the sum of the service time for 125 customers is 83.48 minutes. Hence, the service time for one customer is 0.67 minutes.

Therefore, the service rate $\mu = \frac{1}{0.67} = 1.49\ cpm$.

The actual service rate is $\mu = 1.66\ cpm$. So, we can say that, there is no much difference between the actual service rate and the estimated service rate.

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