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On πgb-D-sets and Some Low Separation Axioms

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Abstract

This paper introduces and investigates some weak separation axioms by using the notions of πgb-closed sets. Discussions has been carried out on its properties and its various characterizations.

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1.Introduction

Levine [16] introduced the concept of generalized closed sets in topological space and a class of topological spaces called $T_{½}$ spaces. The investigation of generalized closed sets leads to several new separation axioms. Andrijevic [3] introduced a new class of generalized open sets in a topological space, the so-called b-open sets. This type of sets was discussed by Ekici and Caldas [11] under the name of γ-open sets. The class of b-open sets is contained in the class of semi-pre-open sets and contains all semi-open sets and pre-open sets. The class of b-open sets generates the same topology as the class of pre-open sets. Since the advent of these notions, several research paper with interesting results in different respects came to existence([1,3,7,11,12,20,21,22]). Extensive research on generalizing closedness was done in recent years as the notions of a generalized closed, generalized semi-closed, α-generalized closed, generalized semi-pre-open closed sets were investigated in [2,8,16,18,19].

In this paper, we have introduced a new generalized axiom called πgb-separation axioms. We have incorporated πgb-D_i, πgb-R_i spaces and a study has been made to characterize their fundamental properties.

2. Preliminaries

Throughout this paper (X, τ) and (Y, τ) represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ) , cl(A) and int(A) denote the closure of A and the interior of A respectively.(X, τ) will be replaced by X if there is no chance of confusion.

Let us recall the following definitions which we shall require later.

Definition 2.1: A subset A of a space (X, τ) is called
(1) a regular open set if A= int (cl(A)) and a regular closed set if A= cl(int (A));
(2) b-open [3] or sp-open [9], γ –open [11] if A ⊂ cl(int(A))∪int (cl(A)).

The complement of a b-open set is said to be b-closed [3]. The intersection of all b-closed sets of X containing A is called the b-closure of A and is denoted by bCl(A). The union of all b-open sets of X contained in A is called b-interior of A and is denoted by bInt(A). The family of all b-open (resp. α-open, semi-open, preopen, β-open, b-closed, preclosed) subsets of a space X is denoted by bO(X)(resp. αO(X), SO(X), PO(X), βO(X), bC(X), PC(X)) and the collection of all b-open subsets of X containing a fixed point x is denoted by bO(X,x). The sets SO(X, x), αO(X, x), PO(X, x), βO(X, x) are defined analogously.

Lemma 2.2 [3]: Let A be a subset of a space X. Then
(1) bCl(A) = sCl(A) ∩ pCl(A) = A∪[Int(Cl(A)) ∩Cl(Int(A))];
(2) bInt(A) = sInt(A) ∪ pInt(A) = A∩ [Int(Cl(A)) ∪ Cl(Int(A))];

Definition 2.3 : A subset A of a space (X, τ) is called
1) a generalized b-closed (briefly gb-closed)[12] if bcl(A) ⊂ U whenever A ⊂ U and U is open.
2) πg-closed [10] if cl(A)⊂U whenever A⊂ U and U is π-open.
3) πgb -closed [23] if bcl(A)⊂U whenever A ⊂U and U is π-open in (X, τ).
By πGBC(τ) we mean the family of all πgb- closed subsets of the space(X, τ).

Definition 2.4: A function f: (X, τ) → (Y, σ) is called
1) πgb- continuous [23] if every $f^{-1}(V)$ is πgb- closed in (X, τ) for every closed set V of (Y,σ).
2) πgb- irresolute [23] if $f^{-1}(V)$ is πgb- closed in(X, τ) for every πgb -closed set V in (Y, σ).

Definition[24]: (X, τ□) is πgb-T_0 if for each pair of distinct points x, y of X, there exists a πgb -open set containing one of the points but not the other.

Definition[24] : (X, τ) is πgb-T_1 if for any pair of distinct points x, y of X, there is a πgb -open set U in X such that x∈U and y∉U and there is a πgb -open set V in X such that y∈U and x∉V.

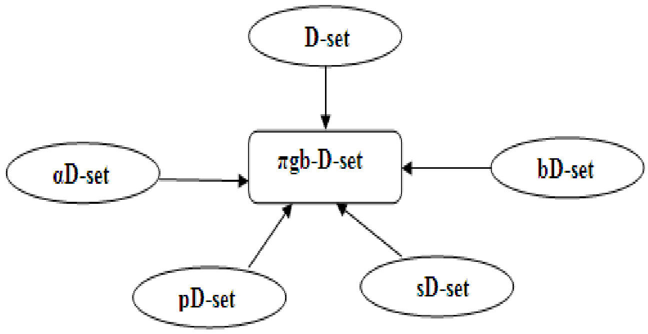
Definition[24] : (X,τ) is πgb-T_2 if for each pair of distinct points x and y in X, there exists a πgb -open set U and a πgb -open set V in X such that x∈U, y∈Vand U∩V =Φ.

Definition: A subset A of a topological space (X, τ) is called:
(i)) D-set [25] if there are two open sets U and V such that U ≠X and A=U -V .
(ii) sD-set [5] if there are two semi-open sets U and V such that U ≠X and A=U -V.
(iii) pD-set [14] if there are two preopen sets U and V such that U ≠X and A=U -V .
(iv) αD-set [6] if there are two U,V∈αO(X,τ) such that U≠X and A=U-V.
(v) bD-set [15] if there are two U,V∈BO(X,τ) such that U≠X and A=U-V.
Definition 2.6[17]: A subset A of a topological space X is called an $\tilde{g}_{\alpha}$D-set if there are two $\tilde{g}_{\alpha}$ open sets U,V such that U≠X and A=U-V.
Definition 2.7[4]: X is said to be (i)rgα-R_0 iff rgα-$\overline{\{x\}}$ ⊆G whenever x∈G∈RGαO(X).

(ii) rgα-R_1 iff for x,y ∈X such that rgα-$\overline{\{x\}}$ ≠ rgα-$\overline{\{y\}}$,there exist disjoint U,V∈RGαO(X)such that rgα-$\overline{\{x\}}$ ⊆U and rgα-$\overline{\{y\}}$ ⊆V.
Definition[13]: A topological space (X, τ) is said to be D-compact if every cover of X by D-sets has a finite subcover.
Definition[15]: A topological space (X, τ) is said to be bD-compact if every cover of X by bD-sets has a finite subcover.
Definition[13]: A topological space (X, τ) is said to be D-connected if (X, τ) cannot be expressed as the union of two disjoint non-empty D-sets.
Definition[15]: A topological space (X, τ) is said to be bD-connected if (X, τ) cannot be expressed as the union of two disjoint non-empty bD-sets.

3. πgb-D-sets and associated separation axioms

Definition 3.1: A subset A of a topological space X is called πgb-D-set if there are two U,V∈πGBO(X, τ) such that U≠X and A=U-V.
Clearly every πgb-open set U different from X is a πgb-D set if A=U and V=Φ.
Example 3.2: Let X={a,b,c} and τ={Φ,{a},{b},{a,b},X}.Then {c} is a πgb-D-set but not πgb-open.Since πGBO(X, τ)={ Φ,{a},{b},{b,c},{a,c},{a,b},X}.Then U={b,c}≠X and V={a,b} are πgb-open sets in X. For U and V, since U-V={b,c}-{a,b}={c},then we have S={c} is a πgb-D-set but not πgb-open.
Theorem 3.3: Every D-set,αD-set,pD-set,bD-set,sD-set is πgb-D-set.
Converse of the above statement need not be true as shown in the following example.
Example 3.4: Let X={a,b,c,d} and τ={Φ,{a},{a,d},{a,b,d}{a,c,d}, X}.πGBO(X,τ)=P(X).Hence πgb-D-set=P(X).{b,c,d} is a πgb-D-set but not D-set,αD-set,pD-set,bD-set,sD-set.



Definition 3.5: X is said to be
(i) πgb-D_0 if for any pair of distinct points x and y of X ,there exist a πgb-D-set in X containing x but not y (or) a πgb-D-set in X containing y but not x.
(ii) πgb-D_1 if for any pair of distinct points x and y in X,there exists a πgb-D-set of X containing x but not y and a πgb-D-set in X containing y but not x.
(iii) πgb-D_2 if for any pair of distinct points x and y of X,there exists disjoint πgb-D-sets G and H in X containing x and y respectively.
Example 3.6: Let X={a,b,c,d} and τ={Φ,{a},{a,b},{c,d},{a,c,d},X},then X is πgb-D_i, i=0,1,2.
Remark 3.7
(i) If (X, τ) is πgb-T_i, then (X, τ) is πgb-D_i;i=0,1,2.
(ii) If (X, τ) is πgb-D_i,then it is πgb-D_{i-1};i=1,2.
(ii) If (X, τ) is πgb-T_i,then it is πgb-T_{i-1};i=1,2.
Theorem 3.8: For a topological space (X, τ).the following statements hold.
(i) (X, τ) is πgb-D_0 iff it is πgb-T_0
(ii) (X, τ) is πgb-D_1 iff it is πgb-D_2.
Proof: (1)The sufficiency is stated in remark 3.7 (i)
Let (X, τ) be πgb-D_0.Then for any two distinct points x,y∈X, atleast one of x,y say x belongs to πgbD-set G where y∉G.Let G=U_1-U_2 where U_1≠X and U_1 and U_2∈πGBO(X, τ).Then x∈U_1.For y∉G we have two cases.(a)y ∉ U_1 (b)y∈U_1 and y ∈ U_2.In case (a), x∈U_1 but y∉U_1;In case (b); y ∈ U_2 and x∉U_2.Hence X is πgb-T_0.
(2) Sufficiency: Remark 3.7 (ii).
Necessity: Suppose X is πgb-D_1.Then for each distinct pair x,y∈X ,we have πgbD-sets G_1 and G_2 such that x∈G_1 and y∉G_1; x∉G_2 and y∈G_2. Let G_1 =U_1-U_2 andG_2 =U_3 –U_4. By x∉G_2, it follows that either x∉U_3 or x∈ U_3 and x ∈ U_4
Now we have two cases(i)x ∉U_3.By y ∉G, we have two subcases (a)y ∉ U_1.By x ∈ U_1-U_2,it follows that x ∈U_1-(U_2∪ U_3) and by y ∈ U_3-U_4,we have y ∈ U_3-

$(U_1 \cup U_4)$.Hence $(U_1-(U_3 \cup U_4)) \cap U_3-(U_1 \cup U_4)=\Phi$.(b)$y \in U_1$ and $y \in U_2$,we have $x \in U_1-U_2$;$y \in U_2 \Rightarrow (U_1-U_2) \cap U_2=\Phi$.

(ii)$x \in U_3$ and $x \in U_4$.We have $y \in U_3-U_4$; $x \in U_4 \Rightarrow (U_3-U_4) \cap U_4=\Phi$.Thus X is πgb-$D_2$.

Theorem 3.9: If (X, τ) is πgb-D_1, then it is πgb-T_0.

Proof: Remark 3.7 and theorem 3.8

Definition 3.10: Let (X,τ) be a topological space. Let x be a point of X and G be a subset of X. Then G is called an πgb-neighbourhood of x (briefly πgb-nhd of x) if there exists an πgb-open set U of X such that $x \in U \subset G$.

Definition 3.11: A point $x \in X$ which has X as a πgb-neighbourhood is called πgb-neat point.

Example 3.12: LetX={a,b,c}.τ={Φ,{a},{b},{a,b},X}.πGBO(X, τ)={Φ,{a},{b},{a,b},{b,c},{a,c}, X}. The point {c} is a πgb-neat point.

Theorem 3.13: For a πgb-T_0 topological space (X, τ), the following are equivalent.

(i)(X, τ) is a πgb-D_1

(ii)(X, τ) has no πgb-neat point.

Proof: (i) ⇒(ii).Since X is a πgb-D_1,then each point x of X is contained in an πgb-D-set O=U-V and hence in U.By definition,U≠X.This implies x is not a πgb-neat point.

(ii)⇒(i) If X is πgb-T_0,then for each distinct points x, $y \in X$, atleast one of them say(x) has a πgb-neighbourhood U containing x and not y. Thus U≠X is a πgbD-set. If X has no πgb-neat point, then y is not a πgb-neat point. That is there exists πgb-neighbourhood V of y such that V ≠X. Thus y ∈ (V-U) but not x and V-U is a πgb-D-set. Hence X is πgb-D_1.

Remark 3.14 : It is clear that an πgb-T_0 topological space (X, τ)is not a πgb-D_1 iff there is a πgb-neat point in X. It is unique because x and y are both πgb-neat point in X, then atleast one of them say x has an πgb- neighbourhood U containing x but not y.This is a contradiction since U ≠X.

Definition 3.15: A topological space (X, τ) is πgb-symmetric if for x and y in X, x ∈ πgb-cl({y}) ⇒y ∈ πgb-cl({x}).

Theorem 3.16: X is πgb-symmetric iff {x} is πgb-closed for $x \in X$.

Proof: Assume that x ∈ πgb-cl({y}) but y∉ πgb-cl({x}).This implies $(\pi gb\text{-}cl(\{x\}))^c$ contains y.Hence the set {y} is a subset of $(\pi gb\text{-}cl(\{x\}))^c$ .This implies πgb-cl({y}) is a subset of $(\pi gb\text{-}cl(\{x\}))^c$. Now $(\pi gb\text{-}cl(\{x\}))^c$ contains x which is a contradiction.

Conversely, Suppose that {x}⊂E∈πGBO(X, τ) but πgb-cl({y}) which is a subset of E^c and x ∉E. But this is a contradiction.

Theorem 3.17 : A topological space (X, τ) is a πgb-T_1 iff the singletons are πgb-closed sets.

Proof: Let (X, τ) be πgb-T_1 and x be any point of X. Suppose $y \in \{X\}^c$. Then x ≠y and so there exists a πgb-open set U such that y∈U but x ∉U. Consequently, $y \in U \subset (\{x\})^c$.That is $(\{x\})^c=\cup\{U/y \in (\{x\})^c\}$ which is πgb-open.

Conversely suppose {x} is πgb-closed for every x∈X. Let x, y∈X with x≠y.Then x≠y ⇒y ∈ $(\{x\})^c$.Hence $(\{x\})^c$ is a πgb-open set containing y but not x. Similarly $(\{y\})^c$ is a πgb-open set containing x but not y. Hence X is πgb-T_1-space.

Corollary 3.18 : If X is πgb-T_1, then it is πgb-symmetric.

Proof: In a πgb-T_1 space, singleton sets are πgb-closed. By theorem 3.17 , and by theorem 3.16 , the space is πgb-symmetric.

Corollary 3.19: The following statements are equivalent

(i)X is πgb-symmetric and πgb-T_0

(ii)X is πgb-T_1.

Proof: By corollary 3.18 and remark 3.7 ,it suffices to prove (1) ⇒(2).Let x≠y and by πgb-T_0 ,assume that $x \in G_1 \subset (\{y\})^c$ for some $G_1 \in$πGBO(X).Then x∉ πgb-cl({y}) and hence y ∉ πgb-cl({x}).There exists a $G_2 \in$ πGBO(X, τ) such that $y \in G_2 \subset (\{x\})^c$. Hence (X, τ) is a πgb-$T_1$ space.

Theorem 3.15: For a πgb-symmetric topological space(X, τ), the following are equivalent.

(1) X is πgb-T_0
(2) X is πgb-D_1
(3) X is πgb-T_1.

Proof: (1)⇒(3):Corollary 3.19 and (3)⇒(2)⇒(1):Remark 3.7 **.**

4. Applications

Theorem 4.1: If f : (X, τ)→(Y, σ) is a πgb-continuous surjective function and S is a D-set of (Y,σ), then the inverse image of S is a πgb-D-set of (X, τ)

Proof: Let U_1 and U_2 be two open sets of (Y, σ).Let $S=U_1-U_2$ be a D-set and $U_1 \neq Y$.We have $f^{-1}(U_1) \in$πGBO(X,τ) and $f^{-1}(U_2) \in$πGBO(X, τ) and $f^{-1}(U_1) \neq X$.Hence$f^{-1}(S)=f^{-1}(U_1-U_2)=f^{-1}(U_1)-f^{-1}(U_2)$.Hence $f^{-1}(S)$ is a πgb-D-set.

Theorem 4.2 J: If f: (X, τ) → (Y, σ) is a πgb-irresolute surjection and E is a πgb-D-set in Y,then the inverse image of E is an πgb-D-set in X.

Proof: Let E be a πgbD-set in Y.Then there are πgb-open sets U_1 and U_2 in Y such that E= U_1-U_2 and $U_1 \neq Y$.Since f is πgb-irresolute,$f^{-1}(U_1)$ and $f^{-1}(U_2)$ πgb-open in X.Since $U_1 \neq Y$,we have $f^{-1}(U_1) \neq X$.Hence $f^{-1}(E) = f^{-1}(U_1-U_2) = f^{-1}(U_1)- f^{-1}(U_2)$ is a πgbD-set.

Theorem 4.3: If (Y, σ) is a D_1 space and f:(X, τ) → (Y, σ) is a πgb -continuous bijective function ,then (X, τ) is a πgb-D_1-space.

Proof: Suppose Y is a D_1space.Let x and y be any pair of distinct points in X,Since f is injective and Y is a D_1space,yhen there exists D-sets S_x and S_y of Y containing f(x) and f(y) respectively that $f(x) \notin S_y$ and $f(y) \notin S_x$. By theorem 4.1 $f^{-1}(S_x)$ and $f^{-1}(S_y)$ are πgb-D-sets in X containing x and y respectively such that $x \notin f^{-1}(S_y)$ and $y \notin f^{-1}(S_x)$.Hence X is a πgb-D_1-space.

Theorem 4.4: If Y is πgb-D_1 and f: (X,τ)→ (Y,σ) is πgb-irresolute and bijective, then (X,τ) is πgb-D_1.

Proof: Suppose Y is πgb-D_1 and f is bijective, πgb-irresolute.Let x,y be any pair of distinct points of X.Since f is injective and Y is πgb-D_1,there exists πgb-D-sets G_x and G_y of Y containing f(x) and f(y) respectively such that f(y) ∉G_z and f(x) ∉G_y. By theorem 4.2 **,** $f^{-1}(G_z)$ and $f^{-1}(G_y)$ are πgbD-sets in X containing x and y respectively.Hence X is πgb-D_1.

Theorem 4.5: A topological space (X, τ) is a πgb-D_1 if for each pair of distinct points x,y∊X,there exists a πgb-continuous surjective function f: (X,τ) → (Y,σ) where (Y,σ) is a D_1 space such that f(x) and f(y) are distinct.

Proof: Let x and y be any pair of distinct points in X, By hypothesis,there exists a πgb-continuous surjective function f of a space (X, τ) onto a D_1-space(Y, σ)such that f(x)≠f(y).Hence there exists disjoint D-sets $S_x$and S_y in Y such that f(x)∊S_x and f(y)∊S_y.Since f is πgb-continuous and surjective,by theorem 4.1 $f^{-1}(S_x)$ and $f^{-1}(S_y)$are disjoint πgb-D-sets in X containing x and y respectively.Hence (X, τ) is a πgb-D_1-set.

Theorem 4.6: X is πgb-D_1 iff for each pair of distinct points x, y∊X ,there exists a πgb-irresolute surjective function f: (X,τ) → (Y,σ),where Y is πgb-D_1 space such that f(x) and f(y) are distinct.

Proof: Necessity: For every pair of distinct points x, y ∊X, it suffices to take the identity function on X.

Sufficiency: Let x ≠y ∊X.By hypothesis ,there exists a πgb-irresolute, surjective function from X onto a πgb-D_1 space such that f(x) ≠f(y).Hence there exists disjoint πgb-D sets G_x, G_y ⊂Y such that f(x) ∊G_x and f(y) ∊G_y. Since f is πgb-irresolute and surjective, by theorem 4.2 , $f^{-1}(G_x)$ and $f^{-1}(G_y)$ are disjoint πgb-D-sets in X containing x and y respectively. Therefore X is πgb-D_1 space.

Definition 4.7: A topological space (X, τ) is said to be πgb-D-connected if (X, τ) cannot be expressed as the union of two disjoint non-empty πgb-D-sets.

Theorem 4.8: If (X, τ) → (Y, σ) is πgb-continuous surjection and (X, τ) is πgb-D-connected, then (Y, σ) is D-connected.

Proof: Suppose Y is not D-connected. Let Y=A∪B where A and B are two disjoint non empty D sets in Y. Since f is πgb-continuous and onto, X=$f^{-1}(A)$∪$f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty πgb-D-sets in X. This contradicts the fact that X is πgb-D-connected. Hence Y is D-connected.

Theorem 4.9: If (X,τ) → (Y,σ) is πgb-irresolute surjection and (X, τ) is πgb-D-connected, then (Y, σ) is πgb-D-connected.

Proof: Suppose Y is not πgb-D-connected.Let Y=A∪B where A and B are two disjoint non empty πgb-D- sets in Y.Since f is πgb-irreesolute and onto,X=$f^{-1}(A)$∪$f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty πgb-D-sets in X.This contradicts the fact that X is πgb-D-connected. Hence Y is πgb-D-connected.

Definition 4.10: A topological space (X,τ) is said to be πgb- D-compact if every cover of X by πgb-D-**sets** has a finite subcover.

Theorem 4.11: If a function f: (X,τ) → (Y,σ) is πgb-continuous surjection and (X,τ) is πgb-D-compact then (Y, σ) is D-compact.

Proof: Let f: (X, τ) → (Y,σ) is πgb-continuous surjection.Let {A_i:i∊Λ} be a cover of Y by D-set.Then {$f^{-1}(A_i)$:i∊Λ} is a cover of X by πgb-D-set.Since X is πgb-D-compact,every cover of X by πgb-D set has a finite subcover ,say{$f^{-1}(A_1)$,$f^{-1}(A_2)$….$f^{-1}(A_n)$}.Since f is onto,{A_1,A_2…….,A_n) is a cover of Y by D-set has a finite subcover.Therefore Y is D-compact.

Theorem 4.12: If a function f: (X,τ) → (Y,σ) is πgb-irresolute surjection and (X, τ) is πgb-D-compact then (Y, σ) is πgb-D-compact.

Proof: Let f: (X, τ) → (Y, σ) is πgb-irresolutesurjection.Let {A_i:i∊Λ} be a cover of Y by πgb-D-set.Hence Y=$\bigcup_i A_i$Then X=f^{-1}(Y)=$f^{-1}(\bigcup_i A_i)$= $\bigcup_i f^{-1}(A_i)$.Since f is πgb-irresolute,for each i∊Λ,{$f^{-1}(A_i)$:i∊Λ} is a cover of X by πgb-D-set.Since X is πgb-D-compact,every cover of X by πgb-D set has a finite subcover ,say{ $f^{-1}(A_1)$, $f^{-1}(A_2)$…. $f^{-1}(A_n)$}.Since f is onto,{A_1,A_2…….,A_n) is a cover of Y by πgb-D-set has a finite subcover.Therefore Y is πgb-D-compact.

5.πgb-R_0 spaces and πgb-R_1 spaces

Definition 5.1: Let (X, τ) be a topological space then the πgb-closure of A denoted by πgb-cl (A) is defined by πgb-cl(A) = ∩ { F | F ∈ πGBC (X, τ) and F⊃A}.

Definition 5.2: Let x be a point of topological space X.Then πgb-Kernel of x is defined and denoted by $Ker_{\pi gb}${x}=∩{U:U∈πGBO(X) and x∈U}.

Definition 5.3: Let F be a subset of a topological space X .Then πgb-Kernel of F is defined and denoted by $Ker_{\pi gb}$(F)=∩{U:U∈πGBO(X) and F⊂U}.

Lemma 5.4: Let (X, τ) be a topological space and x ∈ X.Then $Ker_{\pi gb}$(A) = {x ∊X| πgb-cl({x})∩A ≠Φ}.

Proof: Let x ∈ $Ker_{\pi gb}$(A) and πgb-cl({x})∩A = Φ. Hence x ∉X−πgb-cl({x}) which is an πgb-open set containing A. This is impossible, since x ∈ $Ker_{\pi gb}$(A).

Consequently, πgb- cl({x})∩A ≠Φ Let πgb- cl({x})∩ A ≠Φ and x ∉$Ker_{\pi gb}$ (A). Then there exists an πgb-open set G containing A and x ∉G. Let y ∈ πgb-cl({x})∩A. Hence G is an πgb- neighbourhood of y where x ∉G. By this contradiction, x ∈ $Ker_{\pi gb}$(A).

Lemma 5.5: Let (X, τ) be a topological space and x ∈ X. Then y ∈ $Ker_{\pi gb}$({x}) if and only if x ∈ πgb-Cl({y}).

Proof: Suppose that y ∉ Ker $_{\pi gb}$({x}). Then there exists an πgb-open set V containing x such that y ∉V. Therefore we have x ∉πgb-cl({y}). Converse part is similar.

Lemma 5.6: The following statements are equivalent for any two points x and y in a topological space (X, τ) :
(1) Ker $_{\pi gb}$ ({x}) ≠Ker $_{\pi gb}$({y});
(2) πgb- cl({x}) ≠ πgb-cl({y}).

Proof: (1) ⇒ (2): Suppose that Ker $_{\pi gb}$({x}) ≠ Ker $_{\pi gb}$({y}) then there exists a point z in X such that z ∈X such that z∈Ker $_{\pi gb}$({x}) and z ∉ Ker $_{\pi gb}$({y}). It follows from z ∈ Ker $_{\pi gb}$({x}) that {x} ∩ πgb-cl({z}) ≠Φ .This implies that x ∈ πgb-cl({z}). By z ∉Ker $_{\pi gb}$({y}), we have {y}∩πgb- cl({z}) = Φ. Since x ∈ πgb-cl({z}) , πgb-cl({x}) ⊂ πgb-cl({z}) and {y} ∩ πgb-cl({z}) = Φ. Therefore, πgb-cl({x}) ≠ πgb-cl({y}). Now Ker $_{\pi gb}$({x})≠ Ker $_{\pi gb}$({y}) implies that πgb-cl({x}) ≠ πgb-cl({y}).

(2) ⇒ (1): Suppose that πgb-cl({x}) ≠ πgb-cl({y}). Then there exists a point z ∈ X such that z ∈ πgb-cl({x}) and z ∉ πgb-cl({y}). Then, there exists an πgb-open set containing z and hence containing x but not y, i.e., y ∉ Ker({x}). Hence Ker({x}) ≠Ker({y}).

Definition5.7: A topological space X is said to be πgb-R_0 iff πgb-cl{x}⊆G whenever x∈G∈ πGBO(X).

Definition5.8: A topological space (X,τ) is said to be πgb-R_1 if for any x,y in X with πgb-cl({x})≠πgb-cl({y}),there exists disjoint πgb-open sets U and V such that πgb-cl({x})⊆U and πgb-cl({y})⊆V

Example 5.9: Let X={a,b,c,d}.τ={Φ,{b},{a,b},{b,c},{a,b,c},X}.πGBO (X,τ)=P(X)Then X is πgb-R_0 and πgb-R_1.

Theorem 5.10 : X is πgb-R_0 iff given x≠y∈; πgb-cl{x}≠ πgb-cl{y}.

Proof: Let X be πgb-R_0 and let x≠y∈X.Suppose U is a πgb-open set containing x but not y, then y∈ πgb-cl{y}⊂X-U and hence x∉ πgb-cl{y}.Hence πgb-cl{x}≠ πgb-cl{y}.

Conversely,let x≠y∈X such that πgb-cl{x}≠ πgb-cl{y},.This implies πgb-cl{x}⊂X- πgb-cl{y}=U(say),a πgb-open set in X. This is true for every πgb-cl{x}.Thus ∩πgb-cl{x}⊆U where x∈ πgb-cl{x}⊂U∈ πGBO(X).This implies ∩πgb-cl{x}⊆U where x∈ U∈ πGBO(X).Hence X is πgb-R_0.

Theorem 5.11 : The following statements are equivalent
(i)X is πgb-R_0-space
(ii)For each x∈X,πgb-cl{x}⊂Ker$_{\pi gb}${x}
(iii)For any πgb-closed set F and a point x∉F,there exists U∈πGBO(X) such that x∉U and F⊂U,
(iv) Each πgb-closed F can be expressed as F=∩{G:G is πgb-open and F⊂G}
(v)) Each πgb-open G can be expressed as G=∪{A:A is πgb-closed and A⊂G}
(vi) For each πgb-closed set,x∉F implies πgb-cl{x}∩F=Φ.

Proof: (i)⇒(ii): For any x ∈X, we have Ker $_{\pi gb}${x}= ∩{U:U∈πGBO(X). Since X is πgb-R_0there exists πgb-open set containing x contains πgb-cl{x}.Hence πgb- cl{x}⊂Ker $_{\pi gb}${x}.
(ii) ⇒ (iii): Let x ∉F∈πGBC(X). Then for any y ∈F, πgb-cl{y}⊂F and so x∉πgb- cl{y}⇒y∉πgb-cl{x}.That is there exists U_y∈πGBO(X) such that y∈ U_y and x∉U_y for all y∈F. Let U=∪{U_y∈πGBO(X) such that y∈ U_y and x ∉ U_y}.Then U is πgb-open such that x∉U and F⊂U.
(iii)⇒(iv): Let F be any πgb-closed set and N=∩{G:G is πgb-open and F⊂G}.Then F⊂N---(1).Let x∉F,then by (iii) there exists G∈πGBO(X) such that x∉G and F⊂G, hence x∉N which implies x∈N ⇒x∈F. Hence N⊂F.---(2).From (1) and (2),each πgb-closed F=∩{G:G is πgb-open and F⊂G}.
(iv)⇒(v) Obvious.
(v) ⇒(vi) Let x∉F∈πGBC(X).Then X-F=G is a πgb-open set containing x.Then by (v),G can be expressed as the union of πgb-closed sets A ⊆G and so there is an M∈πGBC(X) such that x∈M⊂G and hence πgb-cl{x}⊂G implies πgb- cl{x}∩F=Φ.
(vi)⇒(i) Let x ∈G∈πGBO(X).Then x∉(X-G) which is πgb-closed set. By (vi) πgb- cl{x}.∩(X-G)=Φ.⇒πgb-cl{x}⊂G.Thus X is πgb-R_0-space.

Theorem 5.12 :A topological space (X, τ) is an πgb-R_0 space if and only if for any x and y in X, πgb-cl({x})≠πgb-cl({y}) implies πgb-cl({x}) ∩πgb-cl({y}) =Φ.

Proof: Necessity. Suppose that (X, τ) is πgb-R and x, y ∈ X such that πgb-cl({x}) ≠πgb-cl({y}). Then, there exist z ∈πgb-cl({x}) such that z∉πgb-cl({y}) (or z ∈cl({y})) such that z ∉πgb-cl({x}). There exists V ∈πGBO(X) such that y ∉ V and z ∈V .Hence x ∈ V . Therefore, we have x ∉πgb-cl({y}). Thus x ∈ (πgb-cl({y}))^c∈πGBO(X), which implies πgb-cl({x}) ⊂(πgb-cl({y}))^{c}and πgb-cl({x}) ∩πgb-cl({y}) =Φ.

Sufficiency. Let V ∈πGBO(X) and let x ∈V .To show that πgb-cl_({x}) ⊂V. Let y ∉V , i.e., y ∈V^c. Then x ≠y and x ∉πgb-cl({y}). This showsthat πgb-cl({x}) ≠πgb-cl({y}). By assumption ,πgb-cl({x}) ∩πgb-cl({y}) =Φ. Hencey ∉πgb-cl({x}) and therefore πgb-cl({x}) ⊂ V .

Theorem 5.13 : A topological space (X, τ) is an πgb- R_0 space if and only if for anypoints x and y in X , Ker $_{\pi gb}$({x}) ≠Ker $_{\pi gb}$({y}) implies Ker $_{\pi gb}$({x}) ∩Ker $_{\pi gb}$({y})=Φ.

Proof. Suppose that (X, τ) is an πgb- R_0 space. Thus by Lemma 5.6, for any points x and y in X if Ker$_{\pi gb}$({x})≠Ker$_{\pi gb}$({y}) then πgb-cl({x})≠πgb-cl({y}).Now to prove that Ker$_{\pi gb}$({x})∩Ker$_{\pi gb}$({y})

=Φ. Assume that z ∈Ker $_{\pi gb}$({x})∩Ker $_{\pi gb}$({y}). By z ∈ Ker$_{\pi gb}$({x}) and Lemma 5.5, it follows that x∈πgb-cl({z}). Since x∈πgb-cl({z}); πgb-cl({x})=πgb-cl({z}). Similarly, we have πgb-cl({y}) = πgb-cl({z})=πgb-cl({x}). This is a contradiction. Therefore, we have Ker $_{\pi gb}$({x}) ∩Ker $_{\pi gb}$({y})=Φ

Conversely, let (X, τ) be a topological space such that for any points x and y inX such that πgb-cl{x}≠πgb-cl{y}.Ker$_{\pi gb}$({x}) ≠Ker$_{\pi gb}$({y}) implies Ker $_{\pi gb}$({x}) ∩Ker $_{\pi gb}$({y})=Φ.Since z∈πgb-cl{x}⇒x∈Ker $_{\pi gb}$({z}) and therefore Ker $_{\pi gb}$({x}) ∩Ker $_{\pi gb}$({y})≠Φ.By hypothesis,we have Ker$_{\pi gb}$({x})=Ker $_{\pi gb}$({z}). Then z ∈πgb-cl({x}) ∩πgb-cl({y}) implies that Ker$_{\pi gb}$({x}) = Ker$_{\pi gb}$({z})=Ker$_{\pi gb}$({y}). This is a contradiction. Hence πgb-cl({x})∩πgb-cl({y})=Φ;By theorem 5.12, (X, τ) is an πgb-R_0 space.

Theorem 5.14 : For a topological space (X, τ), the following properties are equivalent.

(1) (X, τ) is an πgb-R_0 space

(2) x∈πgb-cl({y}) if and only if y∈πgb-cl({x}), for any points x and y in X.

Proof: (1) ⇒ (2): Assume that X is πgb- R_0. Let x ∈πgb-cl({y}) and G be any πgb- open setsuch that y∈G. Now by hypothesis, x ∈G. Therefore, every πgb- openset containing y contains x. Hence y ∈πgb-Cl({x}).

(2) ⇒ (1) : Let U be an πgb- open set and x∈U.If y ∉U, then x ∉πgb-cl({y}) and hence y∉πgb-cl({x}). This implies that πgb-cl({x}) ⊂U. Hence (X, τ) is πgb-R_0.

Theorem 5.15 : For a topological space (X, τ), the following properties are equivalent:

(1) (X, τ) is an πgb-R_0 space;

(2) πgb-cl({x}) = Ker$_{\pi gb}$({x}) for all x ∈ X.

Proof: (1) ⇒(2) : Suppose that (X,τ) is an πgb-R_0 space. By theorem 5.11,πgb-cl({x}) ⊂ Ker$_{\pi gb}$({x}) for each x∈X. Let y ∈Ker$_{\pi gb}$({x}), then x∈πgb-cl({y})and so πgb-cl({x}) =πgb-cl({y}). Therefore, y ∈πgb-cl({x}) and hence Ker$_{\pi gb}$({x}) ⊂πgb-cl({x}). This shows that πgb- cl({x}) = Ker$_{\pi gb}$({x}).

(ii) ⇒ (i) Obvious from 5.13E**.**

Theorem 5.16: For a topological space (X,τ),the following are equivalent.

(i) (X,τ) is a πgb-R_0 space.

(ii) If F is πgb-closed, then F=Ker$_{\pi gb}$(F).

(iii) If F is πgb-closed, and x∈F, then Ker({x})⊂F.

(iv) If x∈X, then Ker$_{\pi gb}$({x})⊂πgb-cl({x}).

Proof :(i)⇒(ii) Let F be a πgb-closed and x∉F.Then X-F is πgb-open and contains x. Since (X,τ) is a πgb-R_0,πgb-cl({x})⊆X-F. Thus πgb-cl({x})∩F=Φ. And by lemma 5.4, x∉πgb-Ker(F).Therefore πgb-Ker(F)=F.

(ii)⇒(iii) If A⊆B, then Ker$_{\pi gb}$(A)⊆Ker$_{\pi gb}$(B).

From (ii),it follows that Ker$_{\pi gb}$({x})⊆Ker $_{\pi gb}$(F).

(iii)⇒(iv) Since x∈πgb-cl({x}) and πgb-cl({x}) is πgb-closed.By(iii),Ker$_{\pi gb}$({x})⊂πgb-cl({x}).

(iv)⇒(i)We prove the result using theorem 5.11**.**Let x ∈ πgb-cl({y}) and by theorem B,y ∈ Ker$_{\pi gb}$({x}).Since x ∈ πgb-cl({x}) and πgb-cl({x}) is πgb-closed,then by (iv) we get y ∈ Ker$_{\pi gb}$({x})⊆ πgb-cl({x}),Therefore x ∈ πgb-cl({y}) ⇒y ∈ πgb-cl({x}). Conversely, let y∈πgb-cl({x}).By lemma 5.5**,** x ∈Ker$_{\pi gb}$({y}).Since y∈πgb-cl({y})and πgb-cl({y}) is πgb-closed,then by (iv) we get x∈Ker$_{\pi gb}$({y})⊆πgb-cl({y}).Thus y∈πgb-cl({x})⇒x∈πgb-cl({y}).By theorem 5.14, we prove that (X,τ) is πgb-R_0 space.

Remark 5.17: Every πgb-R_1 space is πgb-R_0 space.

Let U be a πgb-open set such that x∈U. If y∉U,then since x∉πgb-cl({y}),πgb-cl({x})≠πgb-cl({y}).Hence there exists an πgb-open set V such that y∈V such that πgb-cl({y})⊂V and x∉V⇒y∉πgb-cl({x}).Hence πgb-cl({x})⊆U. Hence (X,τ) is πgb-R_0.

Theorem 5.18: A topological space (X, τ) is πgb-R_1 iff for x,y∈X, Ker$_{\pi gb}$({x})≠πgb-cl({y}),there exists disjoint πgb-open sets U and V such that πgb-cl({x})⊂ andπgb-cl({y}) ⊂V.

Proof: It follows from lemma 5.5.

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