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A Relative Study Of Kalman Filtering For Control Of A Permanent-Magnet Synchronous Motor Drive

Munmun Dutta*, Balram Timande**
*(Department of Electrical & Electronic Engineering, DIMAT, Raipur, India)
**(Department of Electronic & Telecommunication Engineering, DIMAT, Raipur, India)

ABSTRACT
There is increasing demand for dynamical systems to become more realizable and more cost-effective. These requirements extend new method of control and operation. This paper presents a comparative study of the Extended Kalman Filter (EKF) for estimation of the rotor speed and position of a permanent-magnet synchronous motor (PMSM) drive[1]. The system is highly nonlinear and one therefore cannot directly use any linear system tools for estimation. However, if one can linearize the system around a nominal (possibly time-varying) operating point then linear system tools could be used for control and estimation. Extended Kalman Filter is generalized algorithm, which can be used for non-linear systems such as PMSM. The estimation is done upon undisturbed input signals from overriding controller and disturbed output signals of a real non-linear plant, which are measured [2]. The entire state estimated system has been modeled using MATLAB.

Keywords- Two-Phase Permanent Magnet Synchronous Motor, Kalman Filter, Extended Kalman Filter, Matlab function.

1. INTRODUCTION

High torque to inertia ratio, superior power density, high efficiency and many other advantages made PMSM the most widely acceptable electrical motor in industrial applications. Compared with the inverter-fed induction motor drive, the PMSM has no rotor loss and hence it is more efficient and a larger torque-to-weight ratio is achievable. To reduce the cost and to improve the reliability, sensor less PMSM control strategies have been developed [2]. In these strategies, the motor position and speed is estimated and used as a feedback signal for closed-loop speed control.

In this paper, we propose to estimate the motor quantities like speed, flux vector position, currents in direct and quadrature axes, from the measurements of three phase stator currents using an Extended Kalman filter (EKF)[3]. The Kalman filter is a special kind of observer which provides optimal estimation of the system states based on least-square techniques. The extended Kalman filter (EKF) is widely used for nonlinear filter problems. It is derived from the Kalman filter based on the successive linearization of the signal process and observation map[3]. EKF is insensitive to parameter changes and used for stochastic systems where measurement and modeling noise is taken into account.

2. MATHEMATICAL MODEL OF PMSM

The Permanent Magnet Synchronous Motor is rotating electric machine where the stator is a classic three phase coils like that of an induction motor and the permanent magnets are located on the rotor. A PMSM can be mathematically represented by the following equation in the d-q axis synchronously rotating rotor reference frame for assumed sinusoidal stator excitation [4].

$$i_a = -(R_a/L)i_a + (\lambda/L)\omega_r \sin\theta_r + (\frac{1}{L})u_a$$

$$i_b = -(R_a/L)i_b + (\lambda/L)\omega_r \cos\theta_r + (\frac{1}{L})u_b$$

$$\omega_r = -(3\lambda/2J)\, i_a \sin\theta_r + (3\lambda/2J)\, i_b \cos\theta_r - (F/J)\omega_r - ({}^{1}/_{J})T_L$$

(2.1)

$$\theta_r = \omega_r$$

where i_a and i_b are the currents through the two windings, Ra and L are the resistance and inductance of the windings, θ_r and ω_r are the angular position and velocity of the rotor, λ is the flux constant of the motor, u_a and u_b are the voltages applied across the two windings, J is the moment of inertia of the rotor and its load, F is the viscous friction of the rotor, and T_L is the load torque [4, 5].

To avoid convergence problems at startup and to simplify the motor equations, the rotor reference frame is chosen for evaluation of the Kalman filters [6]. The motor nonlinear state equations can be expressed in the form:

$$\dot{x} = Ax + Bu + MT_L \quad (2.2)$$

$$y = Cx \quad (2.3)$$

Where

$x = [i_a \;\; i_b \;\; \omega_r \;\; \theta_r\,]^T$ is a state vector.

$y = [i_a \;\; i_b\,]^T$ is a output vector.

With this definition, "equation (2.1)" can be written compactly[7] as $\dot{x} = [\dot{x}_1\ \dot{x}_2\ \dot{x}_3\ \dot{x}_4\ ]^T$

$$A=\begin{bmatrix} \frac{-R_a}{L} & 0 & -\frac{\lambda}{L}\sin x_4 & 0 \\ 0 & \frac{-R_a}{L} & \frac{\lambda}{L}\cos x_4 & 0 \\ \frac{-3\lambda}{2J}\sin x_4 & \frac{3\lambda}{2J}\cos x_4 & \frac{-F}{j} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B=\begin{bmatrix} (\frac{1}{L}) & 0 \\ 0 & (\frac{1}{L}) \\ 0 & 0 \\ 0 & 0 \end{bmatrix} ,M=\begin{bmatrix} 0 \\ 0 \\ \frac{-1}{J} \\ 0 \end{bmatrix} \text{ and } C=\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T$$

3. ESTIMATION OF SPEED AND ROTOR POSITION USING ROBUST EXTENDED KALMANFILTER

The Kalman filter is often applied during dissolving state estimation of dynamical system, disturbed by the known signals. Kalman filter algorithm is used for estimating the parameters of linear system, but the PMSM model is non-linear, so we cannot use that filter in this case. Extended Kalman Filter is generalized algorithm, which can be used for non-linear systems

The EKF is an optimal estimator in the least square sense for the estimation of nonlinear dynamic systems. It is derived from the Kalman filter based on successive linearization of the signal process and observation map [8].

For this application the motor nonlinear state "equations (2.1)" are expressed in the discretized form

$$x_{k+1} = A_k x_k + B_k u_k + M_k T_L \quad (3.1)$$

$$y_k = C_k x_k \quad (3.2)$$

A_k and B_k are the discretized system and input matrices, respectively. They are [9,2,10]

$$A_k = e^{AT} = I + AT + \frac{(AT)^2}{2!} + \cdots \cong I+AT \quad (3.3)$$

$$B_k = \int_0^T e^{A\zeta} B d\zeta = [e^{AT} - I]A^{-1}B = BT + \frac{ABT^2}{2!} + \cdots \cong BT \quad (3.4)$$

$$M_k \cong MT \quad (3.5)$$

$$Ck = C \quad (3.6)$$

where T is the sampling time and I is an identity (4X4) matrix.

If the noises Δu_a , Δu_b have corrupted the inputs u_a and u_b respectively, and the noise $\Delta\alpha$ has been admitted to account for uncertainties in the load torque[11], then a noise vector will arise in "equation.(3.1)".

$$x_{k+1} = A_k x_k + B_k \begin{bmatrix} u_a + \Delta u_a \\ u_b + \Delta u_b \end{bmatrix}_k + M_k(T_L + \Delta T_L)$$

OR

$$x_{k+1} = A_k x_k + B_k u_k + M_k T_L + w_k \quad (3.7)$$

where,

$$w_k = \begin{bmatrix} T/L\,\Delta u_a \\ T/L\,\Delta u_b \\ -T/J\,\Delta T_L \\ 0 \end{bmatrix} \quad (3.8)$$

Similarly, if the measurements i_a and i_b are distorted by noises Δi_a and Δi_b respectively, then "equation (3.2)" becomes

$$y_k = C_k \begin{bmatrix} i_a + \Delta i_a \\ i_b + \Delta i_b \\ \omega_r \\ \theta_r \end{bmatrix} = C_k x_k + \begin{bmatrix} \Delta i_a \\ \Delta i_b \end{bmatrix} = C_k x_k + v_k \quad (3.9)$$

where,

$$v_k = \begin{bmatrix} \Delta i_a \\ \Delta i_b \end{bmatrix} \quad (3.10)$$

where the vectors w_k and v_k are called the process and measurement noises respectively[12].

The Kalman filter solution does not apply unless certain assumptions about the noise that affects the system under study must be satisfied[13]:

It is firstly to assume that the average

1. It is firstly to assume that the average value of both w_k and v_k are zero.
2. One has to further assume that no correlation exists between w_k and v_k . That is, at any time k, w_k and v_k are independent random variables. Then the noise covariance matrices S_w and S_v are defined as:

Process noise covarianc[6]:

$$S_w = E(w_k w_k^T) \quad (3.11)$$

Measurement noise covariance:

$$S_v = E(v_k v_k^T) \quad (3.12)$$

where w_T and v_T indicate the transpose of w and v random noise vectors, and E(.) means the expected value.

Substituting "equation (3.8) "into" equation (3.11)" and "equation (3.10)" into "equation (3.12)",[14] one can get the following process and measurement noise covariance matrices:

$$S_w = \begin{bmatrix} \left(\frac{T}{L}\right)^2 (\Delta u_a)^2 & \left(\frac{T}{L}\right)^2 \Delta u_a \Delta u_b & -\left(\frac{T}{JL}\right)^2 \Delta u_a \Delta T_L & 0 \\ \left(\frac{T}{L}\right)^2 \Delta u_a \Delta u_b & \left(\frac{T}{L}\right)^2 (\Delta u_b)^2 & -\left(\frac{T}{JL}\right)^2 \Delta u_b \Delta T_L & 0 \\ -\left(\frac{T}{JL}\right)^2 \Delta u_a \Delta T_L & -\left(\frac{T}{JL}\right)^2 \Delta u_b \Delta T_L & \left(\frac{T}{J}\right)^2 (\Delta T_L)^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.13)$$

$$S_v = E\begin{bmatrix}(\Delta i_a)^2 & \Delta i_a \Delta i_b \\ \Delta i_a \Delta i_b & (\Delta i_b)^2\end{bmatrix}$$
(3.14)

If the noises Δu_a (Δu_b), ΔT_L , Δi_a (Δi_b) are white, zero mean, uncorrelated, and have known variances σ_i^2, σ_T^2 and σ_M^2, 2Ts and 2Ms , respectively, then the covariance matrices S_w and S_v will become

$$S_w = \begin{bmatrix}\left(\frac{T}{L}\right)^2 \sigma_i^2 & 0 & 0 & 0 \\ 0 & \left(\frac{T}{L}\right)^2 \sigma_i^2 & 0 & 0 \\ 0 & 0 & \left(\frac{T}{J}\right)^2 \sigma_T^2 & 0 \\ 0 & 0 & 0 & 0\end{bmatrix}$$
(3.15)

$$S_v = \begin{bmatrix}\sigma_M^2 & 0 \\ 0 & \sigma_M^2\end{bmatrix} \quad (3.16)$$

The timing diagram of the various quantities involved in the discrete optimal filter equations is shown in “Fig.(3.1)”.

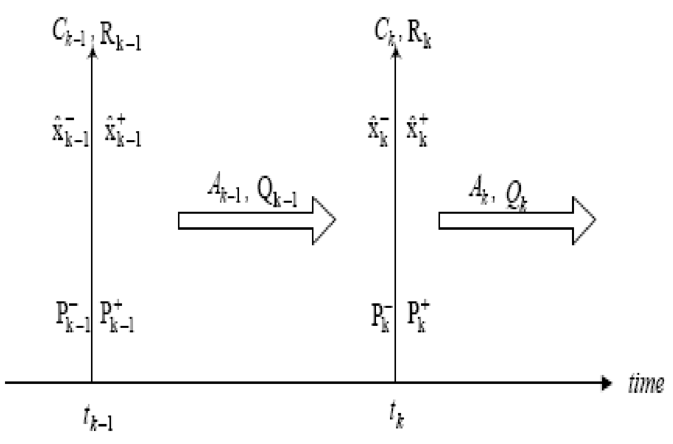

Figure 3.1 timeline showing a priori and a posteriori state estimates and estimation- error covariance.

The “Fig (3.1)” shows that after we process the measurement at time (k-1), we have an estimate of x_{k+1} (denoted $\hat{x}_{k-1}^{+}$) and the covariance of that estimate (denoted P_{k-1}^{+})[15]. When time k arrives, before we process the measurement at time k we compute an estimate of x_k (denoted $\hat{\mathbf{x}}_k^{-}$) and the covariance of that estimate (denoted $\mathbf{P}_k^{-}$). Then the measurement is processed at time k to refine our estimate of x_k . The resulting estimate of x_k is denoted $\hat{\mathbf{x}}_k^{+}$ and its covariance is denoted $\mathbf{P}_k^{+}$.

4. EXPERIMENTAL RESULTS & SIMULATIONS

Simulation of the given PMSM has been carried out using MATLAB[12]. The three phase stator currents are taken from the motor model, and given to the robust extended Kalman filter. The Robust EKF estimates the instantaneous motor speed, rotor position, quadrature and direct axis currents and voltages.

The parameters of the motor are listed in “Table - 4.1”,[16]

TABLE-4.1: PARAMETERS OF TWO-PHASE MOTOR

Winding Resistance	R_a	2Ω
Winding Inductance	L	3mH
Motor flux constant	λ	0.1
Standard deviation of control input noises	σ_i	0.001A
Standard deviation of load torque noise	σ_T	0.05 rad/ sec^2
Standard deviation of measurement noise	σ_M	0.1A
Moment of Inertia	J	0.002
Frequency	f	1Hz
Coefficient of viscous friction	B	0.001

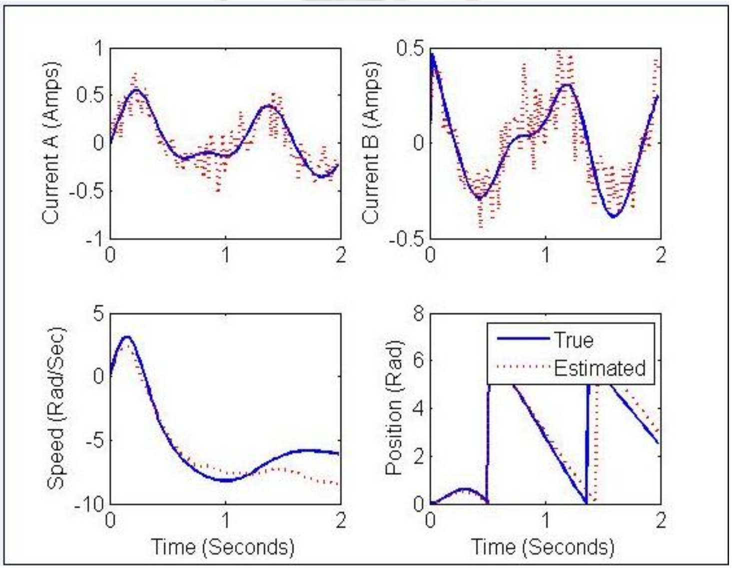

Figure-4.2 extended Kalman filter simulation results for a two-phase permanent magnet synchronous motor

“Figure 4.2” show, respectively, the simulation results of EKF state estimation performance for the sensor less PMSM drive. The system was simulated at sampling time (T=1 ms).

One can easily notice that the EKF estimator could successively estimate the motor states.

5. CONCLUSION

The intent of this paper was to show the utility of the EKF as a fundamental method for solving range of problems in sensor less control of PMSM. There was presented the design and implementation of high-performance servo motor drive with speed, position and torque estimation. The described control system is a solution without any mechanical sensor for a wide range of applications where good steady-state and dynamical properties are required.

Even though our measurements consist only of the winding currents, we can use an EKF to estimate the rotor position and velocity. We used MATLAB to simulate the motor system and the EKF.

The simulation results are shown in "Figure 4.2". We see that the rotor position and velocity are estimated quite well. Practically speaking, this means that we can figure out the rotor position and velocity without using an encoder. Instead of an encoder to get rotor position, we just need a couple of sense resistors, and a Kalman filter in our microcontroller. This is a good example of a trade-off between instrumentation and mathematics.

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Munmun Dutta, born in Rajasthan, India, in 1985. He received the Bachelor degree in Electrical Engineering from the NIT Raipur, India, in 2007 and pursuing Master degree in Power System Engineering with the Department of Electrical & Electronic Engineering from the DIMAT,Raipur, India. Her main area of interest are **Control system engineering, Power system engineering , Testing & commissioning of electrical installations.**

Prof. Balram Timande, Born in 1970, currently working as Reader in department of E&TC at DIMAT Raipur. He is having 9.5 years Industrial experience and 7.5 years teaching experience. He obtained B.E. (Electronics) from Nagpur University (INDIA) in 1993 and M. Tech. form B.I.T. Durg (affiliated to C.S.V.T.U. Bhilai) in 2007. He published 02 papers in National Journal LAB EXPERIMENTS, (LE) ISSN no. 09726055KARENG/2001/8386 and 02 Papers in International Journal IJECSE – ISSN-2277-1956. His area of research/interest is **Embedded system design and Digital Image Processing.**