Chemical Reaction And Radiation Effects On MHD Free Convective Flow Past An Exponentially Accelerated Vertical Plate With Variable Temperature And Variable Mass Diffusion

K.Rajasekhar *, G.V.Ramana Reddy ** and B.D.C.N.Prasad***
*(Department of Mathematics, Usha Rama College of Engineering, Telaprolu, Krishna (India)-521109)
** (Department of Mathematics, RVR & JC College of Engineering, Guntur, India -522 019)
***(Department of Mathematics, PVPSIT, College of Engineering, Vijayawada, India-520007)

ABSTRACT

An analytical study is performed to investigate the effects of chemical reaction and radiation on unsteady MHD flow past an exponentially accelerated vertical plate with variable temperature and variable mass diffusion in the presence of applied transverse magnetic field. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. The flow is assumed to be in x' - direction which is taken along the infinite vertical plate in the upward direction and y' - axis is taken normal to the plate. At time t' > 0, the both temperature and species concentration levels near the plate are raised linearly with time t. A general exact solution of the governing partial differential equations is obtained by usual closed analytical method. The velocity, temperature and concentration fields are studied for different physical parameters like thermal Grashof number (Gr), mass Grashof number (Gm), Schmidt number (Sc), Prandtl number (Pr), radiation parameter (R), magnetic field parameter (M), accelerated parameter (a), heat absorption parameter, chemical reaction parameter (K) and time (t) graphically.

Keywords – MHD, thermal radiation, chemical reaction, variable temperature, variable mass diffusion

INTRODUCTION

Free convection flows are of great interest in a number of industrial applications such as fiber and granular insulation, geothermal systems etc. Buoyancy is also of importance in an environment where differences between land and air temperatures can give rise to complicated flow patterns. Magneto hydro-dynamic has attracted the attention of a large number of scholars due to its diverse applications. In astrophysics and geophysics, it is applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere etc. In engineering it finds its applications in MHD pumps, MHD bearings etc. The phenomenon of mass transfer is also very common in theory of stellar structure and observable effects are detectable, at least on the solar surface. The study of effects of magnetic field on free convection flow is important in liquid metals, electrolytes and ionized gases. The thermal physics of hydro -magnetic problems with mass transfer is of interest in power engineering and metallurgy.

Diffusion rates can be altered tremendously by chemical reactions. The effect of a chemical reaction depend whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of the order n, if the reaction rate is proportional to the nth power of the concentration. In particular, a reaction is said to be first order, if the rate of reaction is directly proportional to the concentration itself. A few representative fields of interest in which combined heat and mass transfer along with chemical reaction play an important role in chemical process industries such as food processing and polymer production.

Chambre and Young [1] have analyzed a first order chemical reaction in the neighbourhood of a horizontal plate. Gupta [2] studied free convection on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis [3] extended this problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen kumar [4]. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo [5]. Basant kumar Jha [6] studied MHD free convection and mass transform flow through a porous medium. Later Basant kumar Jha et al. [7] analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion. Das et al. [8] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das et al. [9]. The dimensionless governing equations were solved by the usual Laplace Transform technique. Muthucumaraswamy and Senthil Kumar [10] studied heat and mass transfer effects on moving vertical plate in the presence of thermal radiation. Recently Muthucumaraswamy et al. [11] studied mass transfer effects on exponentially accelerated isothermal vertical plate.

The object of the present paper is to study the effects chemical reaction and a uniform transverse magnetic field (fixed relative to the plate) on the free convection and mass transform flow past an exponentially accelerated vertical plate with variable temperature and mass diffusion. The dimensionless governing equations are solved using the closed analytical method.

2. Formulation of the Problem

We consider the unsteady hydro magnetic radiative flow of viscous incompressible fluid past an exponentially accelerated infinite vertical plate with variable temperature and also with variable mass diffusion and heat source parameter in the presence of chemical reaction of first order and applied transverse magnetic field. Initially, the plate and the fluid are at the same temperature T'_∞ in the stationary condition with concentration level C'_∞ at all the points. At time $t'>0$, the plate is exponentially accelerated with a velocity $u=u_0\exp(a't')$ in its own plane and at the same time the temperature of the fluid near the plate is raised linearly with time t and species concentration level near the plate is also increased linearly with time. All the physical properties of the fluid are considered to be constant except the influence of the body-force term. A transverse magnetic field of uniform strength B_0 is assumed to be applied normal to the plate. Then under usual Boussinesq's approximation, the unsteady flow is governed by the following set of equations.

$$\frac{\partial u'}{\partial t'} = g\beta(T'-T'_\infty)+g\beta^*(C'-C'_\infty)+\nu\frac{\partial^2 u'}{\partial y'^2}-\frac{\sigma B_0^2}{\rho}u' \quad (1)$$

$$\rho C_P\frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2}-\frac{\partial q_r}{\partial y'}-Q_0(T'-T'_\infty) \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2}-K'_r(C'-C'_\infty) \quad (3)$$

with the following initial and boundary conditions:

$$t\le 0: u'=0, T=T'_\infty, C'=C'_\infty \quad \text{forall } y'$$

$$t'>0,\ \begin{cases} u'=u_0\exp(a't'),\ T'=T'_\infty+(T'_w-T'_\infty)At', \\ C'=C'_\infty+(C'_w-C'_\infty)At' \quad \text{at } y'=0 \\ u'=0,\ T'\to T'_\infty, C'\to C'_\infty \quad \text{as } y'\to\infty \end{cases} \quad (4)$$

where $A=\frac{u_0^2}{\nu}$, u' - velocity of the fluid in the $x-$direction, y' - the dimensional distance along the co-ordinate axis normal to the plate, $g-$the gravitational acceleration, ρ - the fluid density, β and β^* are the thermal and concentration expansion coefficients respectively, B_0 - the magnetic induction, T' - temperature of the fluid in the $x-$direction, $C'-$ the corresponding concentration, $C'-$concentration of the fluid far away from the plate, σ is the electric conductivity, C_p - the specific heat at constant pressure, D -the diffusion coefficient, q_r- the heat flux, Q_0 -the dimensional heat absorption coefficient, and K'_r is the chemical reaction parameter.

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y'} = -4a^*\sigma\left(T'^4_\infty-T'^4\right) \quad (5)$$

It is assumed that the temperature differences within the flow are sufficiently small and that T'^4 may be expressed as a linear function of the temperature. This is obtained by expanding T'^4 in a Taylor series about T'_∞ and neglecting the higher order terms, thus we get

$$T'^4 \cong 4T'^3_\infty T'-3T'^3_\infty \quad (6)$$

From equations (5) and (6), equation (2) reduces to

$$\rho C_p\frac{\partial T'}{\partial y'} = \kappa\frac{\partial^2 T'}{\partial y'^2}+16a^*\sigma T'^3_\infty(T'_\infty-T') \quad (7)$$

On introducing the following non-dimensional quantities

$$u=\frac{u'}{u_0}, t=\frac{t'u_0^2}{\nu}, y=\frac{y'u_0}{\nu}, \theta=\frac{T'-T'_\infty}{T'_w-T'_\infty}, \ \phi=\frac{C'-C'_\infty}{C'_w-C'_\infty},$$

$$a=\frac{a'\nu}{u_0^2}, Gr=\frac{g\beta\nu(T'_w-T'_\infty)}{u_0^3}, Gm=\frac{g\beta^*\nu(C'_w-C'_\infty)}{u_0^3},$$

$$\Pr=\frac{\mu C_p}{k}, Sc=\frac{\nu}{D}, M=\frac{\sigma B_0^2\nu}{\rho u_0^2}, R=\frac{16a^*\nu^2\sigma T'^3_\infty}{\kappa u_0^2},$$

$$K_r=\frac{K'_r\nu}{u_0^2}, Q=\frac{\nu Q_0}{\rho C_p u_0^2}, \quad (8)$$

we get the following governing equations which are dimensionless

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gm\phi - Mu \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} - \frac{R}{\Pr}\theta - Q\theta \tag{10}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} - K_r\phi \tag{11}$$

The initial and boundary conditions in dimensionless form are as follows:

$$t \le 0: \quad u = 0,\ \theta = 0,\ \phi = 0 \quad \text{forall} \quad y$$

$$t > 0, \quad \begin{cases} u = \exp(at), \quad \theta = t,\ \phi = t \ \text{ at } y = 0 \\ u \to 0,\ \theta \to 0, \phi \to 0 \ \text{ as } y \to \infty \end{cases} \tag{12}$$

where $M, Gr, Gm, R, \Pr, Q, Sc$ and K_r denote the magnetic field parameter, thermal Grashof number, mass Grashof number, radiation parameter, Prandtl number, heat absorption parameter, Schmidt number and chemical reaction parameter respectively.

3. SOLUTION OF THE PROBLEM:

In order to reduce the above system of partial differential equations to a system of ordinary equations in dimensionless form, we may represent the velocity, temperature and concentration as

$$u(y,t) = u_0(y)e^{i\omega t} \tag{13}$$

$$\theta(y,t) = \theta_0(y)e^{i\omega t} \tag{14}$$

$$\phi(y,t) = \phi_0(y)e^{i\omega t} \tag{15}$$

Substituting Eqns (13), (14) and (15) in Eqns (9), (10) and (11), we obtain:

$$u_0'' - N_3 u_0 = -\left[Gr\theta_0 + Gm\phi_0\right] \tag{16}$$

$$\theta_0'' - N_2^2\theta_0 = 0 \tag{17}$$

$$C_0'' - N_1^2 C_0 = 0 \tag{18}$$

Here the primes denote the differentiation with respect to y.

The corresponding boundary conditions can be written as

$$\begin{aligned} &u_0 = e^{(a-i\omega)t}, \quad \theta_0 = te^{-i\omega t}, \quad \phi_0 = te^{-i\omega t} \quad \text{at} \quad y = 0 \\ &u_0 \to 0, \quad \theta_0 \to 0, \quad \phi_0 \to 0 \quad \text{as} \quad y \to \infty \end{aligned} \tag{19}$$

The analytical solutions of equations (16) – (18) with satisfying the boundary conditions (19) are given by

$$u_0(y) = A_3 e^{-N_3 y} e^{-i\omega t} + \left(A_1 e^{-N_2 y} + A_4 e^{N_1 y}\right) te^{-i\omega t} \tag{20}$$

$$\theta_0(y) = \left(te^{-N_2 y}\right)e^{-i\omega t} \tag{21}$$

$$\phi_0(y) = \left(t\,e^{-N_1 y}\right)e^{-i\omega t} \tag{22}$$

In view of the above solutions, the velocity, temperature and concentration distributions in the boundary layer become

$$u(y,t) = A_3 e^{-N_3 y} + \left(A_1 e^{-N_2 y} + A_2 e^{-N_1 y}\right)t \tag{23}$$

$$\theta(y,t) = \left(te^{-N_1 y}\right) \tag{24}$$

$$\phi(y,t) = \left(t\,e^{-N_2 y}\right) \tag{25}$$

It is now important to calculate the physical quantities of primary interest, which are the local wall shear stress, the local surface heat, and mass flux. Given the velocity field in the boundary layer, we can now calculate the local wall shear stress (i.e., skin-friction) is given by

$\tau_w^* = \mu\left(\frac{\partial u'}{\partial y'}\right)_{y'=0}$, and in dimensionless form, we obtain

$$C_f = -\left(\frac{\partial u}{\partial y}\right)_{y=0} = \left(N_3 A_3 + N_2 A_1 + N_1 A_2\right)t$$

From temperature field, now we study the rate of heat transfer which is given in non - dimensional form as:

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = tN_1$$

From concentration field, now we study the rate of mass transfer which is given in non - dimensional form as:

$$Sh = -\left(\frac{\partial \phi}{\partial y}\right)_{y=0} = tN_2$$

where

$$N_1 = \sqrt{Sc\left(K_r + i\omega\right)},\ N_2 = \sqrt{\Pr i\omega + R},$$

$$N_3 = \sqrt{M + i\omega}\ ,\ A_1 = -Gr/\left(N_2^2 - N_3^2\right),$$

$$A_2 = -Gm/\left(N_1^2 - N_3^2\right), A_3 = e^{at} - t\left(A_1 + A_2\right)$$

Results and Discussion

In order to get physical insight into the problem, we have plotted velocity profiles for different values of the physical parameters a(accelerated parameter), M(magnetic field parameter) R(radiation parameter), Gr(thermal Grashof number), Gm(mass Grashof number), Pr(Prandtl number), Sc(Schmidt number), t(time) and the chemical reaction parameter (K) is presented graphically. The value of Sc (Schmidt number) is taken to be 0.6 which corresponds to the water-vapor. Also, the value of Pr (prandtl number) are chosen such that they represent air (Pr=0.71).

The effect of the magnetic parameter M is shown in Fig.1. It is observed that the tangential velocity of the fluid decreases with the increase of the magnetic field number values. The decrease in the tangential velocity as the magnetic parameter M increases is because the presence of a magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow if the magnetic field is applied in the normal direction, as in the present study. This resistive force slows down the fluid velocity component as shown in Fig.1.

Fig.2 shows the effect of the accelerated parameter a on the velocity distribution. It is found that the velocity increases with an increase in an accelerated parameter a .

For various values of the thermal Grashof number Gr and mass Grashof number Gm, the velocity profiles 'u' are plotted in Figs. (3) and (4). The thermal Grashof number Gr signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Also, as Gr increases, the peak values of the velocity increases rapidly near to the plate and then decays smoothly to the free stream velocity. The mass Grashof number Gm defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach the free stream value. It is noticed that the velocity increases with increasing values of the mass Grashof number. Fig.5 shows the effect of the time t on the velocity field. It is found that the velocity increases with an increase in time t. Fig.6. illustrates the velocity profiles for different values of radiation parameter R, the numerical results show that the effect of increasing values of radiation parameter result in a decreasing velocity. For different values of the Prandtl number $\Pr$ the velocity profiles are plotted in Fig.7. It is obvious that an increase in the Prandtl number $\Pr$ results in decrease in the velocity within the boundary layer. Fig.8 illustrates the behavior velocity for different values of chemical reaction parameter K_r. It is observed that an increase in leads to a decrease in the values of velocity. For different values of the Schmidt number Sc the velocity profiles are plotted in Fig.9. It is obvious that an increase in the Schmidt number Sc results in decrease in the velocity within the boundary layer.

For different values of frequency of excitation ω on the temperature profiles are shown in Fig.9. It is noticed that an increase in the frequency of excitation results a decrease in the temperature field. Fig.10 shows the behavior of the temperature for different values Prandtl number $\Pr$. It is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of $\Pr$ are equivalent to increase in the thermal conductivity of the fluid and therefore, heat is able to diffuse away from the heated surface more rapidly for higher values of $\Pr$. Hence in the case of smaller Prandtl number as the thermal boundary later is thicker and the rate of heat transfer is reduced.

For different values of thermal radiation R the temperature profiles are shown in Fig.11. It is observed that an increase in the thermal radiation parameter, the decrease of the temperature profiles. The effect of the Schmidt number Sc on the concentration is shown in Fig.12. As the Schmidt number increases, the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. A reduction in the concentration distribution is accompanied by simultaneous reduction in the concentration boundary layers. The effect of chemical reaction parameter K_r on the concentration profiles as shown in Fig.13. It is observed that an increase in leads to a decrease in the values of concentration.

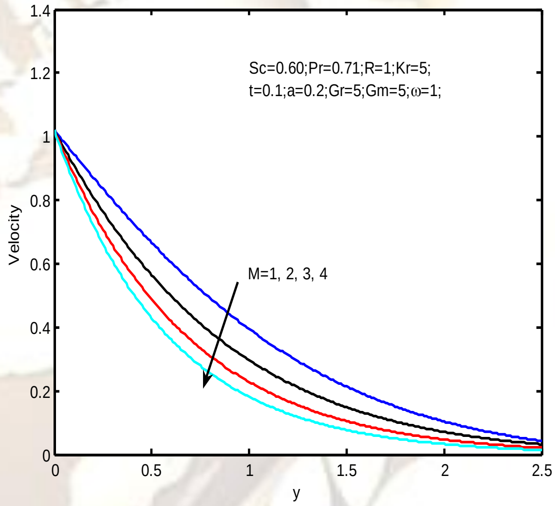

Fig. 1. Effects of magnetic parameter on velocity profiles.

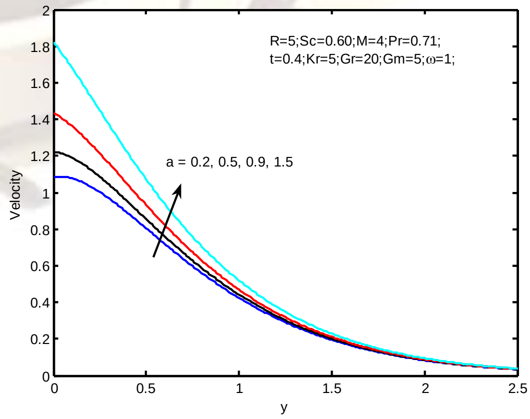

Fig. 2. Effects of accelerated parameter on velocity profiles.

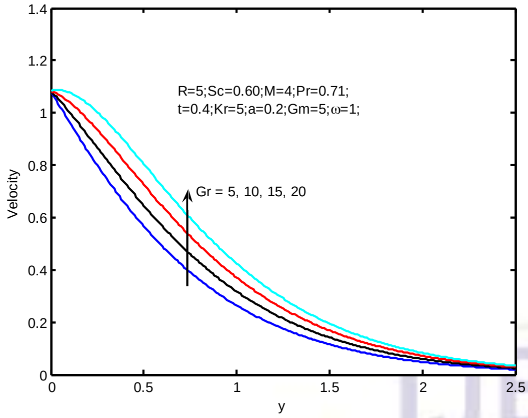

Fig. 3. Effects of Thermal Grashof number on velocityprofiles.

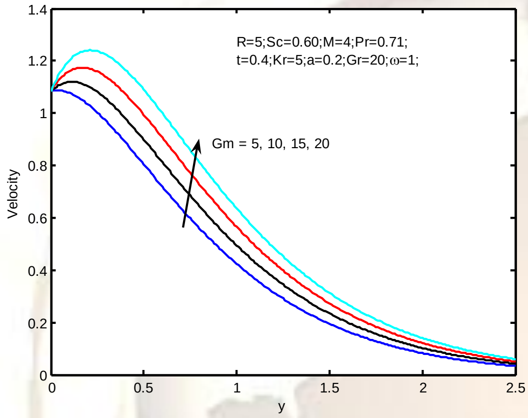

Fig. 4. Effects of mass Grashof number on velocity profiles.

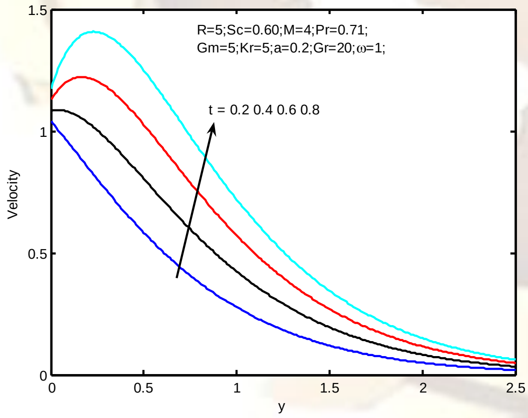

Fig.5. Effects of time on velocity profiles.

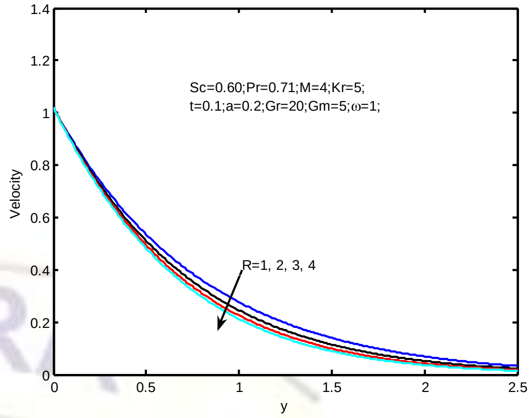

Fig.6. Effects of radiation parameter on velocity profiles.

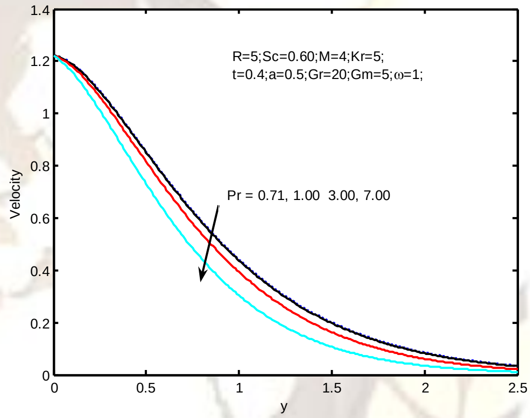

Fig.7. Effects of Prandtl number on velocity profiles.

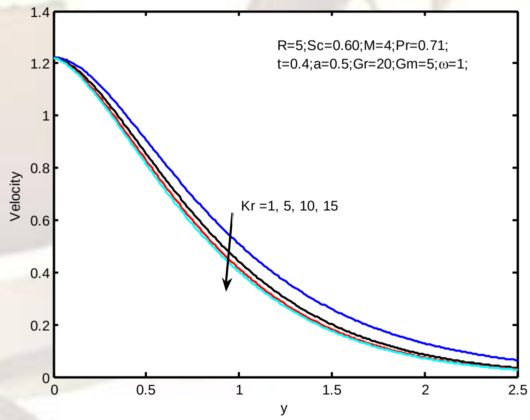

Fig.8. Effects of chemical reaction parameter on velocity profiles.

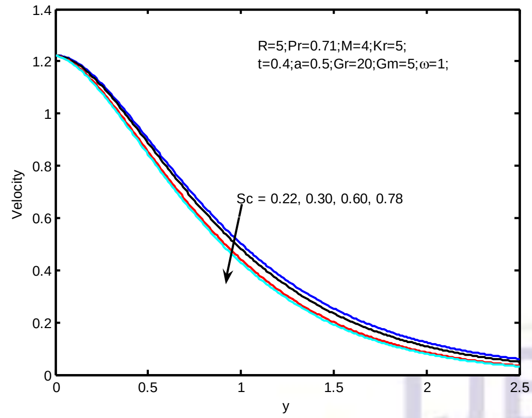

Fig.9. Effects of Schmidt number on velocity profiles.

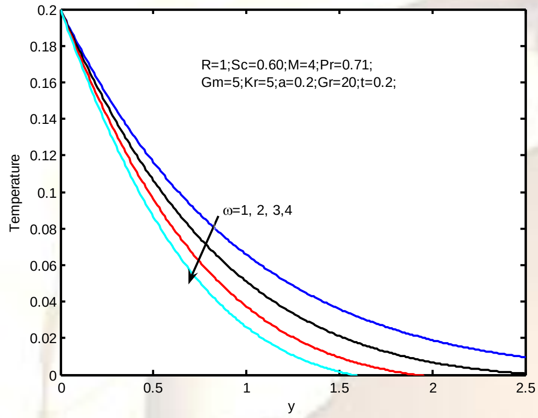

Fig.10. Effects of frequency of excitation on temperature profiles.

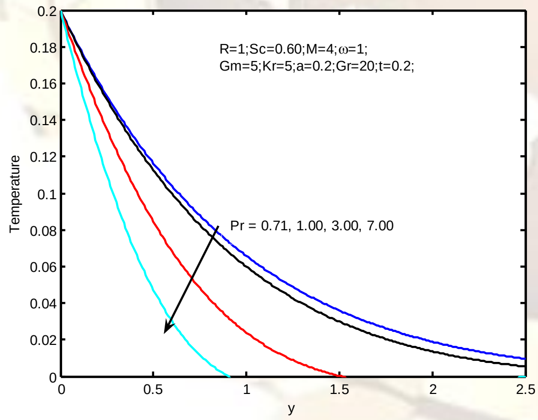

Fig.11. Effects of Prandtl number on temperature profiles.

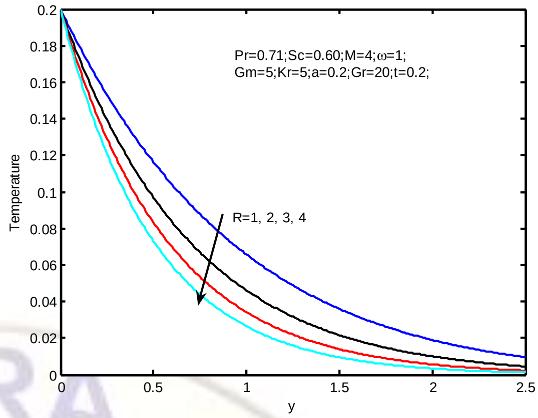

Fig.12. Effects of radiation parameter on temperature profiles.

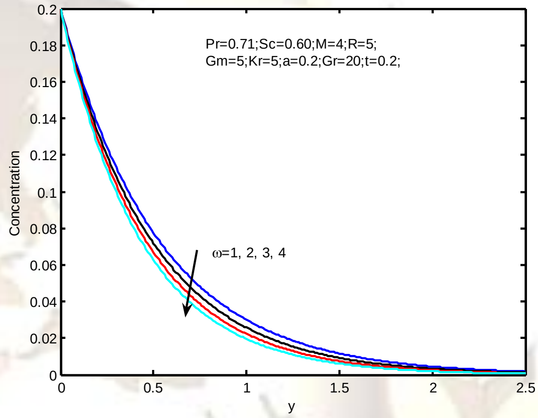

Fig.13. Effects of frequency of excitation on concentration profiles.

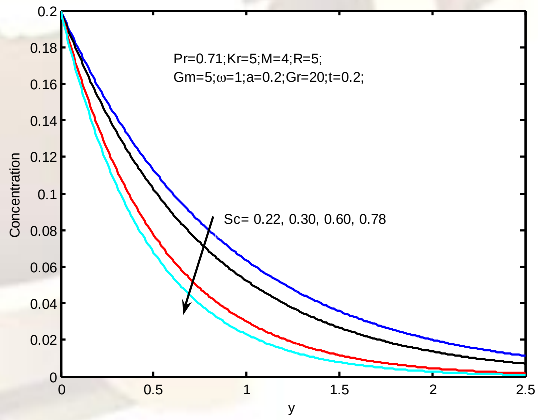

Fig.14. Effects of Schmidt number on concentration profiles.

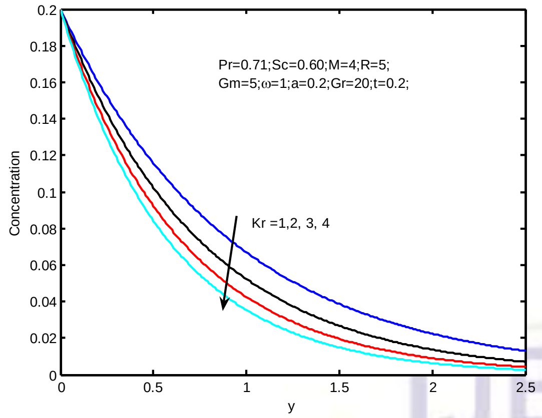

Fig.15. Effects of chemical reaction parameter on concentration profiles.

REFERENCES

[1]. P. L. Chambre, J. D. Young, On the diffusion of a chemically reactive species in a laminar boundary layer flow, The Physics of Fluids, Vol.1, pp.48-54, 1958.

[2]. A. S. Gupta, I. Pop, V. M. Soundalgekar, Free convection effects on the flow past an accelerated vertical plate in an incompressible dissipative fluid, Rev. Roum. Sci. Techn. -Mec. Apl., 24, pp.561-568, 1979.

[3]. N. G. Kafousias, A.A. Raptis, Mass transfer and free convection effects on the flow past an accelerated vertical infinite plate with variable suction or injection, Rev. Roum. Sci. Techn.- Mec. Apl., 26, pp.11-22, 1981.

[4]. A. K. Singh, N. Kumar, Free convection flow past an exponentially accelerated vertical plate, Astrophysics and Space science, 98, pp. 245-258, 1984.

[5]. M. A. Hossain, L. K. Shayo, The Skin friction in the unsteady free convection flow past an accelerated plate, Astrophysics and Space Science, 125, pp. 315-324, 1986.

[6]. B.K. Jha, MHD free-convection and mass transform flow through a porous medium, Astrophysics and Space science, 175, 283-289, 1991.

[7]. B.K. Jha, R. Prasad, S. RAI, Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux, Astrophysics and Space Science, 181, pp.125-134, 1991.

[8]. U. N. Das, R. K. Deka, V. M. Soundalgekar, Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction, Forschung in Ingenieurwsen, Vol.60, pp.284-287, 1994.

[9]. U. N. Das, R. K. Deka, V. M. Soundalgekar, Effects of mass transfer on flow past an impulsively started infinite vertical plate with chemical reaction, The Bulletin of GUMA, Vol.5, pp.13-20, 1999.

[10]. R. Muthucumaraswamy, G. Senthil kumar, Heat and Mass transfer effects on moving vertical plate in the presence of thermal radiation, Theoret. Appl. Mech., Vol.31, No.1, pp.35-46, 2004.

[11]. R. Muthucumaraswamy, K.E. Sathappan, R. Natarajan, Mass transfer effects on exponentially accelerated isothermal vertical plate, Int. J. of Appl. Math. and Mech., 4(6), 19-25, 2008.