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Flow of Single-Phase Water and Two-Phase Air-Water Mixtures through Sudden Contractions in Mini Channels

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Abstract

Pressure drop through sudden contraction in small circular pipes have been numerically investigated, using air and water as the working fluids at room temperature and near atmospheric pressure. Two-phase computational fluid dynamics (CFD) calculations, using Eulerian–Eulerian model with the air phase being compressible, are employed to calculate the pressure drop across sudden contraction. The pressure drop is determined by extrapolating the computed pressure profiles upstream and downstream of the contraction. The larger and smaller tube diameters are 1.6 mm and 0.84 mm, respectively. Computations have been performed with single-phase water and air, and two-phase mixtures in a range of Reynolds number (considering all-liquid flow) from 1000 to 12000 and flow quality from 1.9×10^{-3} to 1.6×10^{-2}**. The contraction loss coefficients are found to be different for single-phase flow of air and water. The numerical results are validated against experimental data from the literature and are found to be in good agreement. Based on the numerical results as well as experimental data, a correlation is developed for two-phase flow pressure drop caused by the flow area contraction.**

Keywords: Two-phase flow; pressure drop; loss coefficients; flow quality; two-phase multiplier.

1. Introduction

In the recent years, several papers have been published on flow of two-phase gas/liquid mixtures through pipe fittings. Schmidt and Friedel [1] studied experimentally two-phase pressure drop across sudden contractions using mixtures of air and liquids, such as water, aqueous glycerol, calcium nitrate solution and Freon 12 for a wide range of conditions. Salcudean et al. [2] studied the effect of various flow obstructions on pressure drops in horizontal air-water flow and derived pressure loss coefficients and two-phase multipliers. Many of the published studies have assumed the occurrence of the vena-contracta phenomenon in analogy with single-phase flow and have assumed that dissipation occurs downstream of the vena-contracta point [3-4]. Al'Ferov and Shul'zhenko [3], Attou and Bolle [4] have attempted to develop mechanistic models for two-phase pressure drop across sudden contractions. They assumed the occurrence of the vena-contracta phenomenon and a dispersed droplet flow pattern downstream of the vena-contracta point. However, Schmidt and Friedel [1], based on their experimental results indicated that the vena-contracta phenomenon did not occur in their system at all. Applications of mini and micro channels in advanced and high performance systems have been rapidly increasing in recent years. Some two-phase hydrodynamics and heat transfer processes in mini and micro channels are different from larger channels [5-7], indicating that the available vast literature associated with flow and heat transfer in larger channels may not be directly applicable to micro channels. Significant velocity slip occurs at the vicinity of the flow disturbance in case of mini and micro channels (Abdelall et al. [7]). They observed that homogeneous flow model over predicts the data monotonically and significantly and with the slip ratio expression of Zivi [8], on the other hand, their experimental data and theory were in relatively good agreement. Pressure drops associated with single-phase flow through abrupt flow area changes in commonly-applied large systems have been extensively studied in the past. But little has been reported with respect to two-phase flow through mini and micro channels. In the present work, an attempt has been made to simulate the flow through sudden contraction in mini channels using two phase flow models in an Eulerian scheme. The flow field is assumed to be axisymmetric and solved in two dimensions. Before, we can rely on CFD models to study the two-phase pressure drop through such sections; we need to establish whether the model yields valid results. For the validation of results, we have referred to the experimental studies conducted by Abdelall et al. [7], who measured pressure drops resulting from an abrupt flow area changes in horizontal pipes.

2. Mathematical formulation

The Eulerian-Eulerian mixture model has been used as the mathematical basis for the two phase simulation through sudden contraction. The volume fractions α_q and α_p for a control volume can therefore be equal to any value between 0 and 1, depending on the space occupied by phase q and phase p. The mixture model allows the phases to move at different velocities, using the concept of slip velocities.

Continuity Equation:
Continuity equation for the mixture is given by:

$$\frac{\partial}{\partial t}(\rho_m)+\nabla.(\rho_m\vec{v}_m)=0 \qquad (1)$$

where, $\vec{v}_m$ is the mass averaged velocity:

$$\vec{v}_m=\frac{\alpha_q\rho_q\vec{v}_q+\alpha_p\rho_p\vec{v}_p}{\rho_m} \qquad (2)$$

and ρ_m is the mixture density:

$$\rho_m=\alpha_q\rho_q+\alpha_p\rho_p \qquad (3)$$

and α_q is the volume fraction of phase q.

Momentum Equation:
The momentum equation for the mixture can be obtained by summing the individual momentum equations for all phases. It can be expressed as (Drew [9]; Wallis[10])

$$\frac{\partial}{\partial t}(\rho_m\vec{v}_m)+\nabla.(\rho_m\vec{v}_m\vec{v}_m)=-\nabla p+\nabla.\left[\mu_m\left(\nabla\vec{v}_m+\nabla\vec{v}_m^T\right)\right] +\rho_m\vec{g}+\vec{F}+\nabla.\left(\left(\alpha_q\rho_q\vec{v}_{dr,q}\vec{v}_{dr,q}\right)+\left(\alpha_p\rho_p\vec{v}_{dr,p}\vec{v}_{dr,p}\right)\right) \qquad (4)$$

where, μ_m is the viscosity of the mixture:

$$\mu_m=\alpha_q\mu_q+\alpha_p\mu_p \qquad (5)$$

$\vec{v}_{dr,p}$ is the drift velocity for the secondary phase p:

$$\vec{v}_{dr,p}=\vec{v}_p-\vec{v}_m \qquad (6)$$

Energy Equation:
The energy equation for the mixture takes the following form:

$$\frac{\partial}{\partial t}\left(\alpha_q\rho_qE_q+\alpha_p\rho_pE_p\right)+\nabla.\left(\alpha_q\vec{v}_q\left(\rho_qE_q+p\right)\right) +\nabla.\left(\alpha_p\vec{v}_p\left(\rho_pE_p+p\right)\right)=\nabla.\left(k_{eff}\nabla T\right)+S_E \qquad (7)$$

where k_{eff} is the effective conductivity:

$$k_{eff}=\left(\alpha_q\left(k_q+k_t\right)+\alpha_p\left(k_p+k_t\right)\right) \qquad (8)$$

and k_t is the turbulent thermal conductivity. The first term on the right hand side of Equation (7) represents energy transfer due to conduction. S_E includes any other volumetric heat sources. For the compressible phase,

$$E_q=h_q-\frac{p}{\rho_q}+\frac{v_q^2}{2} \qquad (9)$$

and $E_p=h_p$ for the incompressible phase, where h_q is the sensible enthalpy for phase q.

Relative (Slip) Velocity and the Drift Velocity:
The slip velocity is defined as the velocity of a secondary phase p relative to the velocity of the primary phase q:

$$\vec{v}_{pq}=\vec{v}_p-\vec{v}_q \qquad (10)$$

The mass fraction for any phase p is defined as

$$c_p=\frac{\alpha_p\rho_p}{\rho_m} \qquad (11)$$

The drift velocity and the relative velocity ($\vec{v}_{pq}$) are connected by the following expression:

$$\vec{v}_{dr,p}=\vec{v}_{pq}-c_p\vec{v}_{qp} \qquad (12)$$

The basic assumption of the algebraic slip mixture model is to prescribe an algebraic relation for the relative velocity so that a local equilibrium between the phases should be reached over short spatial length scale.

$$\vec{v}_{pq}=\frac{\tau_p}{f_{drag}}\frac{\left(\rho_p-\rho_m\right)}{\rho_p}\vec{a} \qquad (13)$$

where τ_p is the particle relaxation time and is given by:

$$\tau_p=\frac{\rho_pd_p^2}{18\mu_p} \qquad (14)$$

where d_p is the diameter of the particles (or droplets or bubbles) of secondary phase p, $\vec{a}$ is the secondary-phase particle's acceleration. The drag function f_{drag} is taken from Schiller and Naumann:

$$f_{drag}=\begin{cases}1+0.15\,\mathrm{Re}^{0.687} & \mathrm{Re}\le 1000\\ 0.0183\,\mathrm{Re} & \mathrm{Re}>1000\end{cases} \qquad (15)$$

and the acceleration $\vec{a}$ is of the form:

$$\vec{a}=\vec{g}-(\vec{v}_m.\nabla)\vec{v}_m-\frac{\partial\vec{v}_m}{\partial t} \qquad (16)$$

In turbulent flows the relative velocity should contain a diffusion term due to the dispersion appearing in the momentum equation for the dispersed phase.

$$\vec{v}_{pq}=\frac{\left(\rho_p-\rho_m\right)d_p^2}{18\mu_qf_{drag}}\vec{a}-\frac{\upsilon_m}{\alpha_p\sigma_D}\nabla\alpha_q \qquad (17)$$

where (υ_m) is the mixture turbulent viscosity and (σ_D) is a Prandtl dispersion coefficient. It may be noted that, if the slip velocity is not solved, the mixture model is reduced to a homogeneous multiphase model.

Volume Fraction Equation for the Secondary Phase:

From the continuity equation for the secondary phase p, the volume fraction equation for the secondary phase p can be obtained:

$$\frac{\partial}{\partial t}\left(\alpha_p \rho_p\right) + \nabla.\left(\alpha_p \rho_p \vec{v}_m\right) = -\nabla.\left(\alpha_p \rho_p \vec{v}_{dr,p}\right) \quad (18)$$

Turbulence modeling:

The k and ε equations used in this model are as follows (FLUENT 6.2 Manual [11]):

$$\frac{\partial}{\partial t}(\rho_m k) + \nabla \cdot (\rho_m \vec{v}_m k) = \nabla \cdot \left(\frac{\mu_{t,m}}{\sigma_k} \nabla k\right) + G_{k,m} - \rho_m \varepsilon \quad (19)$$

and

$$\frac{\partial}{\partial t}(\rho_m \varepsilon) + \nabla \cdot (\rho_m \vec{v}_m \varepsilon) = \nabla \cdot \left(\frac{\mu_{t,m}}{\sigma_\varepsilon} \nabla \varepsilon\right) + \frac{\varepsilon}{k}\left(C_{1\varepsilon} G_{k,m} - C_{2\varepsilon} \rho_m \varepsilon\right) \quad (20)$$

where the mixture density and velocity ρ_m and $\vec{v}_m$, are computed from

$$\rho_m = \alpha_p \rho_p + \alpha_q \rho_q \quad (21)$$

$$\vec{v}_m = \frac{\alpha_p \rho_p \vec{v}_p + \alpha_q \rho_q \vec{v}_q}{\alpha_p \rho_p + \alpha_q \rho_q} \quad (22)$$

The turbulent viscosity $\mu_{t,m}$ is computed from

$$\mu_{t,m} = \rho_m C_\mu \frac{k^2}{\varepsilon} \quad (23)$$

and the production of turbulent kinetic energy, $G_{k,m}$ is computed from

$$G_{k,m} = \mu_{t,m}\left(\nabla \vec{v}_m + (\nabla \vec{v}_m)^T\right) : \nabla \vec{v}_m \quad (24)$$

The constants in the above equations are

$C_{1\varepsilon} = 1.44$; $C_{2\varepsilon} = 1.92$; $C_\mu = 0.09$; $\sigma_k = 1.0$; $\sigma_\varepsilon = 1.3$

3. Results and discussion

3.1. Single phase flow pressure drop

Idealized course of boundary stream lines and pressure profile for single phase flow of water through sudden contraction is shown in Fig. 1. Computational domain with boundary conditions is shown in Fig. 2. The flow field is assumed to be axisymmetric and solved in two dimensions. The Pressure profiles for the single-phase flow of water and air through sudden contraction is depicted in Figs. 3 and 4 respectively, for different mass flow rates. It can be seen that the pressure profiles are nearly linear up to 5 pipe diameters, both upstream and downstream from the contraction plane. Because there is a change in pipe cross-section and hence a change in mean velocity, the slopes of the pressure profiles before and after contraction are different. The gradients are greater in the smaller diameter pipe. It can be observed from figs.3 and 4 that very close to the contraction plane the static pressure in the inlet line decreases more rapidly than in fully developed flow region. It attains the (locally) smallest value at a

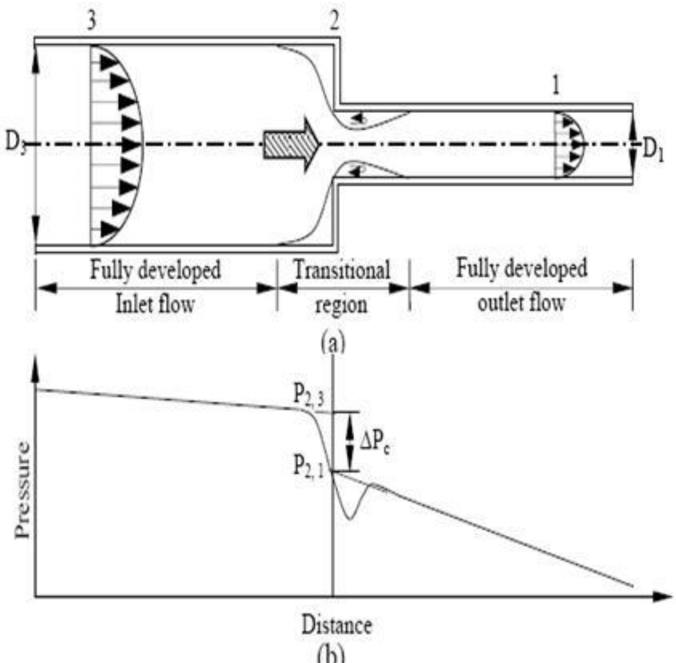


Fig. 1: (a) Idealized course of boundary stream lines and (b) pressure profile for a sudden contraction.

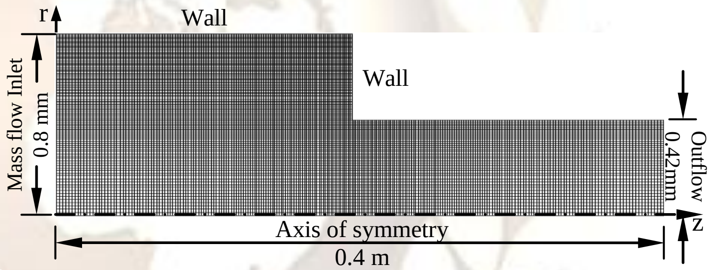


Fig. 2: Computational domain for contraction section

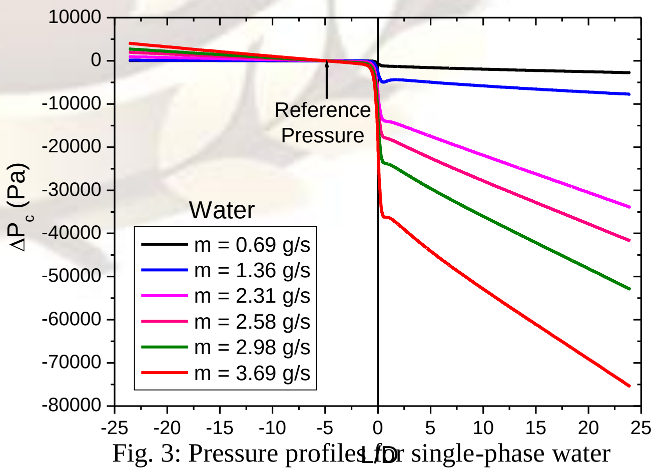


Fig. 3: Pressure profiles for single-phase water flow through sudden contraction

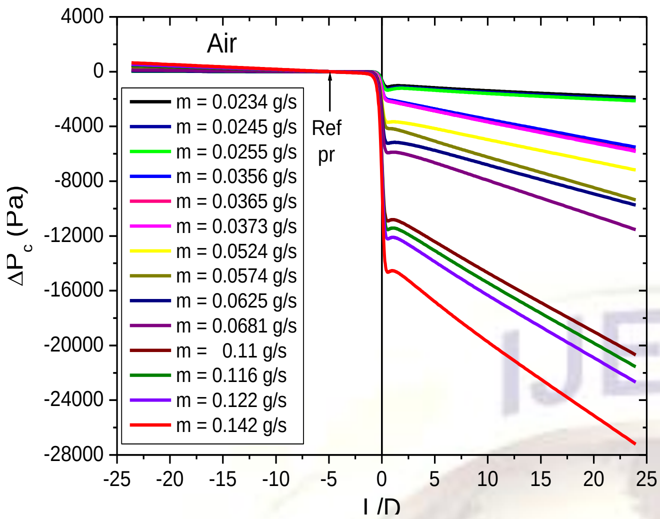


Fig. 4: Pressure profiles for single-phase air flow through sudden contraction

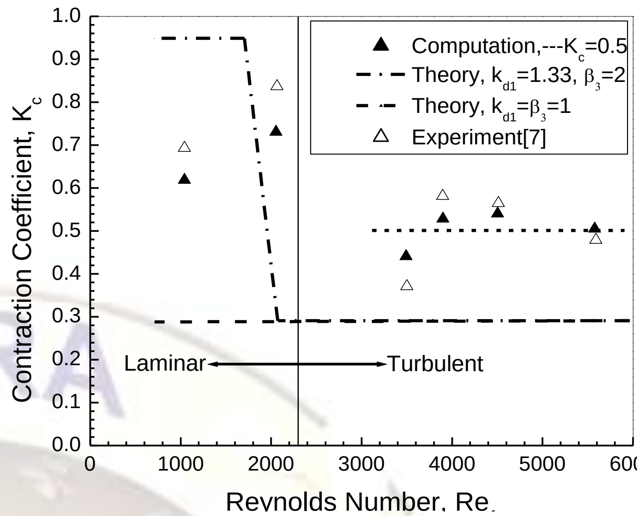
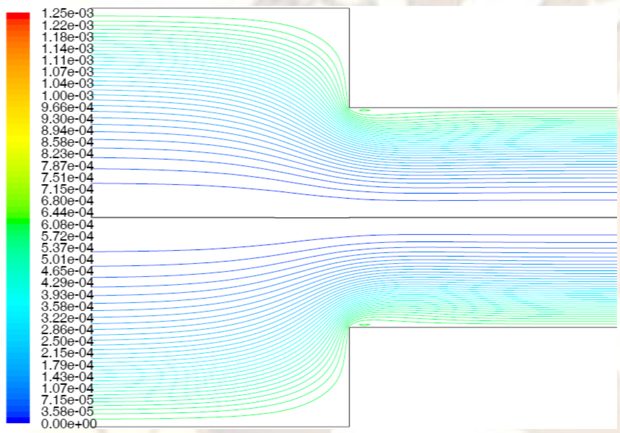
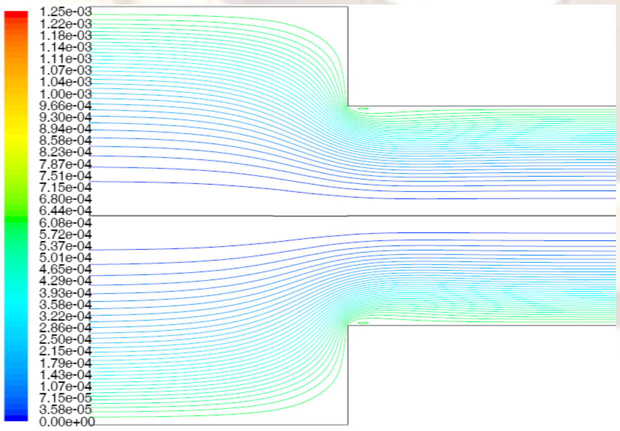


Fig. 6: Contraction loss coefficient for single phase water flow



(a)



(b)

Fig. 5: (a) Velocity vectors & (b) Stream lines for single-phase flow through sudden contraction ($\dot{m}_L = 3.69$ g/s)

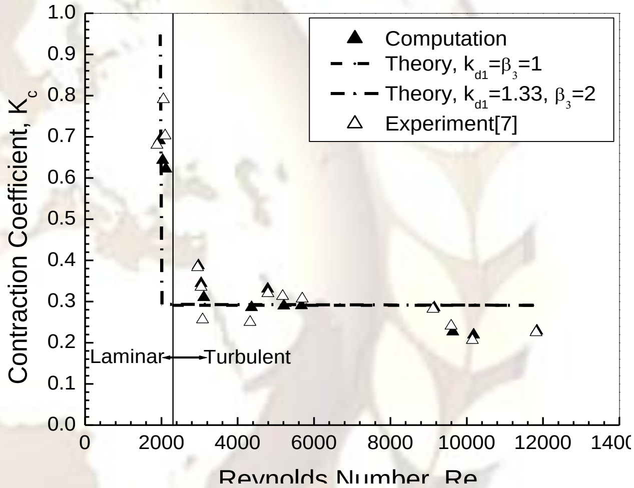


Fig. 7: Contraction loss coefficient for single phase air flow

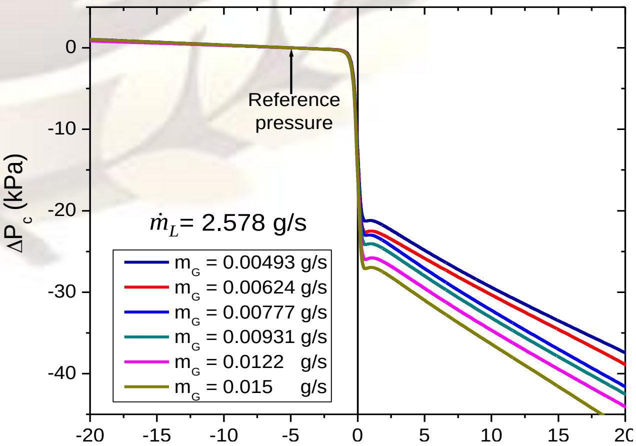


Fig. 8: Pressure profiles for flow of two-phase air-water through sudden contraction

distance of about L/D = 0.5 after the contraction section and depends only slightly on the mass flow rates. The same is also demonstrated by the velocity vectors and streamline contours in fig. 5, which shows that the flow has minimum cross-sectional area at about 0.5D downstream of pipe contraction, D being the diameter of the smaller pipe. Then, the pressure gradually increases and, after reaching its maximum, it merges into the curve of the pipe frictional pressure drop downstream of contraction. This local minimum value of pressure corresponds to the vena-contracta position. So, it can be concluded that vena-contracta is always obtained in the single-phase flow of water and air through sudden contraction at a distance of 0.5D from the contraction plane in the downstream direction. Simulated velocity vectors are shown in fig. 5(a), which clearly shows that eddy zones are formed in the separated flow region. The pressure change at the contraction plane (ΔP_c) is obtained by extrapolating the computed pressure profiles upstream and downstream of the pipe contraction (in the region of fully developed flow) to the contraction plane. It is observed that the pressure drop increases with the mass flow rates for single phase flow of both water and air through sudden contraction.

Assuming uniform velocity profiles at cross-section 1 and 3, the contraction loss coefficient can be found by applying one-dimensional momentum and mechanical energy conservation equations (Abdelall et al.[7]; Kays [12]):

$$K_c\left(\frac{1}{2}\rho\langle u_1\rangle^2\right)=\left(P_{2,3}-P_{2,1}\right)-\frac{1}{2}\rho\langle u_1\rangle^2\left(1-\sigma^2\right) \quad (25)$$

Where $(P_{2,3}-P_{2,1})$ is obtained numerically. The contraction loss coefficients for single-phase water and air flows are depicted in Figs. 6 and 7, respectively, where the experimental data [7] and theoretical predictions are also shown. The data points for water, although few and scattered, are higher than that of the theoretical predictions and in relatively close agreement with the experimental data. For turbulent flow in the smaller channel, contraction loss coefficient for single phase water flow (K_c) is found to be 0.5 (Fig. 6). The contraction loss coefficient obtained for single phase air flow (depicted in Fig. 7), on the other hand, agree with the aforementioned theoretical predictions as well as experimental data [7] well.

3.2 Two-phase flow pressure drop

Fig. 8 depicts the two-phase pressure drop through sudden contraction for different mass flow rates of air keeping the mass flow rate of water constant. Similarly for different mass flow rates of water several numerical experiments are performed and it is observed that the pressure drop through sudden contractions increases with increasing the mass flow rates of either water or air. The velocity vectors and stream lines for a particular mass flow rate of water and air are illustrated in Fig.9. It clearly demonstrates that unlike single phase flow, the two-phase flow does not contract behind the edge of transition and there is no change in the flow direction i.e; a zone of recirculation is not observed. Thus vena-contracta is not a relevant term in two-phase flow through sudden contractions. The computed two-phase flow pressure drops caused by flow area contractions are displayed in Figs. 10 (a) and (b) as functions of Reynolds number ($\mathrm{Re}_{L0,1}$).

$$\text{Where } \mathrm{Re}_{L0,1}=(GD)_1/\mu_L, \quad (26)$$

Here G_1 is the total mass flux corresponding to the smaller diameter pipe. The computed as well as the experimental data are compared with analytical model calculations assuming homogeneous flow, and slip flow (Abdelall et al.[7]) with the aforementioned slip ratio of $S=(\rho_L/\rho_G)^{\frac{1}{3}}$. The homogeneous flow and the slip flow models are considered assuming a vena-contracta, and no vena-contracta (i.e., C_c=l). It is observed that calculations with the homogeneous flow assumption over predict the data monotonically and significantly. The computational and the experimental data ([7]) are found to be in relatively good agreement with the predictions of slip flow model..

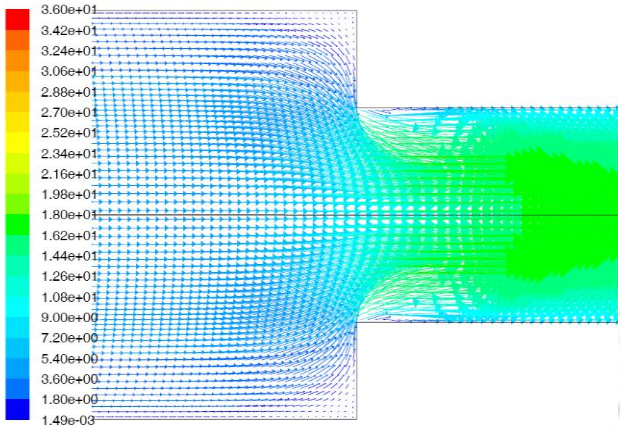
An attempt is made to correlate the two-phase pressure change data in terms of the Martinelli factor:

$$X_M=\left(\frac{\mu_L}{\mu_G}\right)^{0.1}\left(\frac{1-x}{x}\right)^{0.9}\left(\frac{\rho_G}{\rho_L}\right)^{0.5}, \quad (27)$$

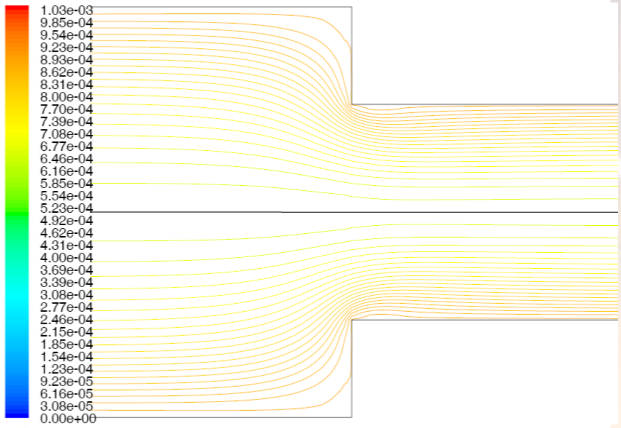
Fig. 11 shows the graph $\Phi_{L0,c}/X_M$ verses $X_M\,\mathrm{Re}_{L0,1}$ considering both computational as well as the experimental data for sudden contraction. The resulting correlation for two-phase multiplier for sudden contraction is given by:

$$\frac{\Phi_{L0,c}}{X_M}=160\left(X_M\,\mathrm{Re}_{L0,1}\right)^{-0.6877} \quad (28)$$

The comparison between the predicted values of the two-phase pressure drops (calculated from Eq. 28) with the computed as well as experimental data [7] are depicted in Fig. 12. The agreement is found to be quite good. It is notable that the correlation can predict the two-phase pressure drop quite well within ±16% error. The correlation is valid in the range of 1000 < $Re_{L0,1}$ < 12000.

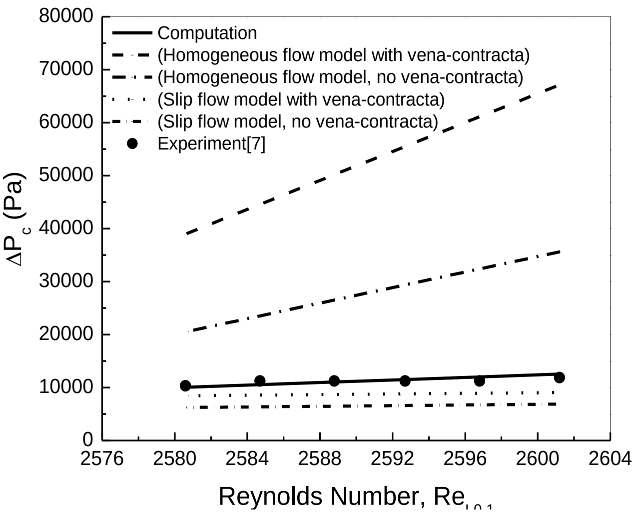


(a)



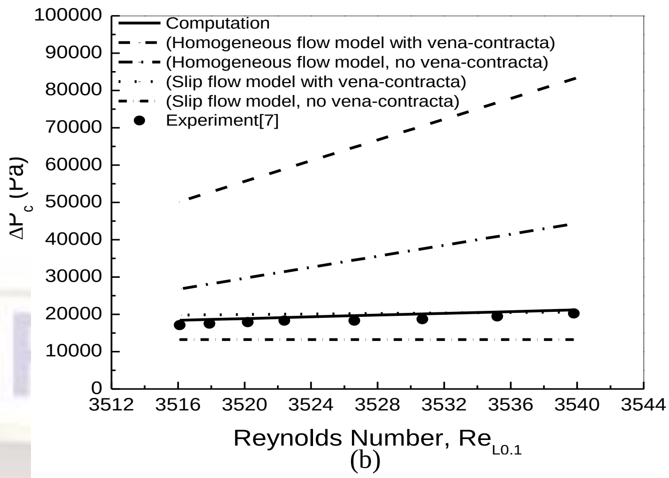
(b)

Fig. 9: (a) Velocity vectors and (b) Stream lines for $\left(\dot{m}_L = 2.319 \text{ g/s and } \dot{m}_G = 0.0203 \text{ g/s}\right)$



(a)

Fig. 10(a): Two-phase flow pressure drop caused by flow area contraction



(b)

Fig. 10(b): Two-phase flow pressure drop caused by flow area contraction

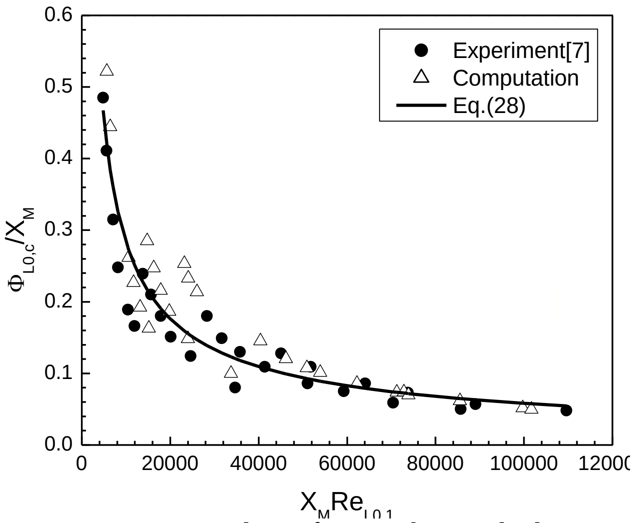


Fig. 11: Correlation for two-phase multiplier through sudden contraction

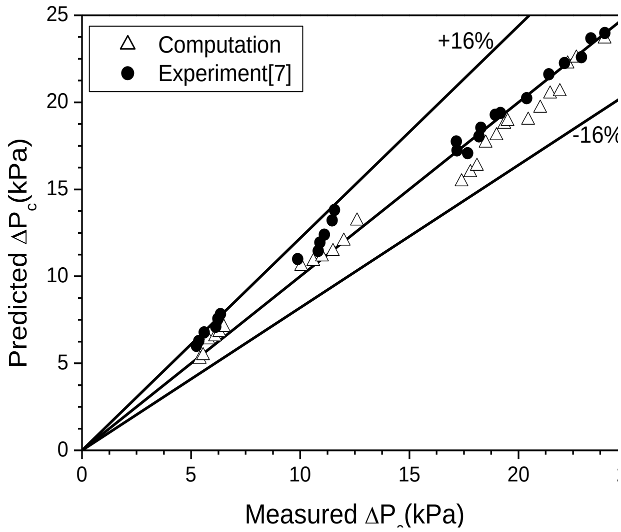


Fig. 12: Comparison between predicted and measured ΔP_c

4. Conclusions

Pressure drops caused by abrupt flow area contraction in mini channels using water and air mixtures have been numerically investigated by using two-phase flow model in an Eulerian scheme. For single phase flow of water and air through sudden contraction, the vena contracta is always established at a distance of L/D = 0.5 after the contraction section and its position depends only slightly on mass flow rates. With turbulent flow in the smaller channel, approximately constant contraction loss coefficients are obtained for single phase flow of water and air. The contraction loss coefficients for water are found to be slightly larger than the theoretical predictions and that for air is in good agreement with the theoretical predictions. The computed values of two-phase flow pressure drops caused by sudden flow area contraction are found to be significantly lower than the predictions of the homogeneous flow model and are in relatively close agreement with the predictions of slip flow model. It is observed that there is significant velocity slip in the vicinity of the flow area change. The data suggest that widely-applied methods for pressure drops due to flow area changes, particularly for two-phase flow is not applicable to mini and micro channels. It is also observed that unlike single phase flow, the two-phase flow does not contract behind the edge of transition and there is no change in the flow direction i.e; a zone of recirculation is not observed. Thus vena-contracta is not a relevant term in two-phase flow through sudden contractions. Based on the computational as well as the experimental data, correlations have been developed (Eq.28) for two-phase flow pressure drop caused by the flow area contraction.

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