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Method For Solving Hungarian Assignment Problems Using Triangular And Trapezoidal Fuzzy Number

Kadhirvel.K, Balamurugan.K
Assistant Professor in Mathematics, T.K.Govt. Arts College, Vriddhachalam – 606 001.
Associate Professor in Mathematics, Govt. Arts College, Thiruvannamalai.

ABSTRACT

In this paper, we proposed the fuzzy job and workers by Hungarian assignment problems. In this problem c_{ij} denotes the cost for assigning the n jobs to the n workers. The cost is usually deterministic in nature. In this paper $\widetilde{C}_{ij}$ has been considered to be trapezoidal and triangular numbers denoted by $\widetilde{C}_{ij}$ which are more realistic and general in nature. For finding the optimal assignment, we must optimize total cost this problem assignment. In this paper first the proposed assignment problem is formulated to the crisp assignment problem in the linear programming problem form and solved by using Hungarian method and using Robust's ranking method (3) for the fuzzy numbers. Numerical examples show that the fuzzy ranking method offers an effective tool for handling the fuzzy assignment problem.

Key words:
Fuzzy sets, fuzzy assignment problem, Triangular fuzzy number, Trapezoidal fuzzy number ranking function.

I. INTRODUCTION

The assignment problem (AP) is a special type of linear programming problem (LPP) in which our objective is to assign number of jobs to number of workers at a minimum cost (time). The mathematical formulation of the problem suggests that this is a 0-1 programming problem and is highly degenerate all the algorithms developed to find optimal solution of transportation problem are applicable to assignment problem. However, due to its highly degeneracy nature a specially designed algorithm, widely known as Hungarian method proposed by kuhn [1], is used for its solution.

In this paper, we investigate more realistic problem & namely the assignment problem, with fuzzy costs $\widetilde{C}_{ij}$ Since the objectives are to minimize the total cost or to maximize the total profit, subject to some crisp constraints, the objective function is considered also as a fuzzy number. The method is to rank the fuzzy objective values of the objective function by some ranking method for fuzzy number to find the best alternative. On the basis of idea the Robust's ranking method (3) has been adopted to transform the fuzzy assignment problem to a crisp one so that the conventional solution metyhods may be applied to solve assignment problem. The idea is to transform a problem with fuzzy parameters to a crisp version in the LPP from and solve it by the simplex method. other than the fuzzy assignment problem other applications & this method can be tried in project scheduling, maximal flow, transportation problem etc.

Lin and wen solved the Ap with fuzzy interval number costs by a labeling algorithm (4) in the paper by sakawa et.al (2), the authors dealt with actual problems on production and work force assignment in a housing material manufacturer and a sub construct firm and formulated tow kinds of two level programming problems. Chen (5) proved some theorems and proposed a fuzzy assignment model that considers all individuals to have same skills. Wang (6) solved a similar model by graph theory. Dubois and for temps (7) surveys refinements of the ordering of solutions supplied by the max-min formulation, namely the discrimin partial ordering and the leximin complete preordering. Different kinds of transportation problem are solved in the articles (8,10,12,14,15). Dominance of fuzzy numbers can be explained by many ranking methods (9,11,13,16) of these, Robust's ranking method (3) which satisfies the properties of compensation, linearity and additivity. In this paper we have applied Robust's ranking technique (3).

II. PRELIMINARIES

In this section, some basic definitions and arithmetic operations are reviewed. Zadeh (16) in 1965 first introduced fuzzy set as a mathematical way of representing impreciseness or vagueness in every day life.

Definition: 2.1

A fuzzy set is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval [0,1], is A={x,μ_A (x): x $\in$ X}, Here : μ_A : X$\rightarrow$ [0,1] is a mapping called the degree of membership function of the fuzzy set A and μ_A(X) is called the membership value of x $\in$ X in the fuzzy set A. these membership grades are often represented by real numbers ranging from [0,1].

Definition: 2.2

A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exist at least one x ∈ X such that $\mu_A(X)=1$.

Definition: 2.3

The fuzzy set A is convex if and only if, for any $x_1, x_2 \in X$, the membership function of A satisfies the inequality $\mu_A\{\lambda x_1 + (1-\lambda)x_2\} \geq \min\{\mu_A(x_1), \mu_A(x_2)\}$, $0 \leq \lambda \leq 1$

Definition: 2.4 (Triangular fuzzy number)

For a triangular fuzzy number A(x), it can be represented by A(a,b,c;1) with membership function μ(x) given by

$$\mu(x) = \begin{cases} (x-a)/(b-a) & : a \leq x \leq b \\ 1 & : x = b \\ (c-x)/(c-b) & : b \leq x \leq c \\ 0 & : \text{otherwise} \end{cases}$$

Definition: 2.5 (Trapezoidal fuzzy number)

For a trapezoidal number A(x). it can be represented by A (a,b,c,d;1) with membership function μ(x) given by

$$\mu(x) = \begin{cases} (x-a)/(b-a) & : a \leq x \leq b \\ 1 & : b \leq x \leq c \\ (d-x)/(d-c) & : c \leq x \leq d \\ 0 & : \text{otherwise} \end{cases}$$

Definition: 2.6 (k-cut of a trapezoidal fuzzy number)

The k-cut of a fuzzy number A(x) is defined as $A(k) = \{x: \mu(x) \geq k, k \in [0,1]\}$

Definition: 2.7 (Arithmetic Operations)

Addition of two fuzzy numbers can be performed as

$$(a_1, b_1, c_1) + (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$$

Addition of two trapezoidal fuzzy numbers can be performed as

$$(a_1, b_1, c_1, d_1) + (a_2, b_2, c_2, d_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$$

III. Robust's Ranking Techniques – Algorithms

The Assignment problem can be stated in the form of nxn cost matrix $[c_{ij}]$ of real numbers as given in the following

	Job 1	Job 2	Job 3	-------	Job j	-------	Job N
Worker 1	C_{11}	C_{12}	C_{13}	-------	C_{1j}	-------	C_{1n}
Worker 2	C_{21}	C_{22}	C_{23}	-------	C_{2j}	-------	C_{2n}
Worker i	C_{i1}	C_{i2}	C_{i3}	-------	C_{ij}	-------	C_{in}
Worker N	C_{n1}	C_{n2}	C_{n3}	-------	C_{nj}	-------	C_{nn}

Mathematically assignment problem can be stated as

minimize

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to

$$\sum_{i=1}^{n} Xij = 1$$

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad x_{ij} \in [0,1] \longrightarrow 1$$

Where $x_{ij} = \begin{cases} 1; & \text{if the } i^{th} \text{ worker is assigned the } j^{th} \text{ job} \\ 0; & \text{otherwise.} \end{cases}$

is the decision variable denoting the assignment of the worker i to job j. $\widetilde{cij}$ is the cost of assigning the j^{th} job to the i^{th} worker. The objective is to minimize the total cost of assigning all the jobs to the available persons (one job to one worker).

When the costs or time $\widetilde{cij}$ are fuzzy numbers, then the total costs becomes a fuzzy number

$$\overset{V*}{z} = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$$

Hence it cannot be minimized directly for solving the problem. we defuzzify the fuzzy cost coeffients into crisp ones by a fuzzy number ranking method.

Robust's ranking technique (3) which satisfies compensation, linearity, and additivity properties and provides results which are consistent with human intuition. Give a covex fuzzy

number $\tilde{c}$, the Robust's Ranking index is defined by

$$R(\tilde{c}) = \int_0^1 0.5\,(C_k^l, C_k^u)dk \text{ where } (C_k^l, C_k^u) \text{ is the } k-\text{level cut of the fuzzy number } \tilde{c}.$$

In this paper we use this method for ranking the objective values. The Robust's ranking index R ($\tilde{c}$) gives the representative value of the fuzzy number $\tilde{c}$, it satisfies the linearity and additive property:

If $\hat{p} = \tilde{E} + m\tilde{y}$ & $\tilde{Q} = S\tilde{c} - t\tilde{N}$ where l, m, s, t are constant then we have

R($\tilde{p}$) =l R($\tilde{E}$) + n R($\tilde{y}$) and R ($\tilde{Q}$) =S R($\tilde{c}$) - tR ($\tilde{N}$) on the basis of this property the fuzzy assignment problem can be transformed in to a crisp assignment problem linear programming problem from. The ranking technique of the Robust's is

If R ($\hat{G}$) ≤ R ($\overset{v}{H}$) then $\hat{G} \leq \overset{v}{H}$ (ie) min {$\hat{G}$, $\overset{v}{H}$} = $\hat{G}$ from the assignment problem (1), with fuzzy objective function

$$\min \overset{V*}{z} = \sum_{i=1}^{n}\sum_{j=1}^{n} R(\tilde{c}_{ij}x_{ij})$$

We apply Robust's ranking method (3) (using the linearity and assistive property) to get the minimum objective value $\overset{V*}{z}$ from the formulation

$$R\left(\overset{V*}{z}\right) = \min Z = \sum_{i=1}^{n}\sum_{j=1}^{n} R(\widetilde{C_{ij}})x_{ij}$$

Subject to

$$\sum_{i=1}^{n} x_{ij} = 1$$

Min
{

$$\sum_{j=1}^{n} x_{ij} = 1, \quad x_{ij} \in [0,1] \quad \rightarrow (2)$$

Where x_{ij} = { 1 ; if the i[th] Worker is assigned j[th] job
0 ; Otherwise

is the decision variable denoting the assignment of the worker i to j[th] job. $\widetilde{C_{ij}}$ is the cost of designing the j[th] job to the i[th] worker. The objective is to minimize the total cost of assigning all the jobs to the available workers.

Since $R(\widetilde{C_{ij}})$ are crisp values, this problem (2) is obviously the crisp assignment problem of the form (1) which can be solved by the conventional methods, namely the Hungarian method or simplex method to solve the linear programming problem form of the problem. Once the optimal solution x^* of model (2) is found,the optimal fuzzy objective value $\overset{V*}{z}$ of the original problem can be calculated as

$$\overset{V*}{z} = \sum_{i=1}^{n}\sum_{j=1}^{n} (\widetilde{C_{ij}})\, x_{ij}^{*}$$

Numerical Example

Let us consider a fuzzy assignment problem with rows representing 4 workers A,B,C,D and Columns representing the jobs, Job 1,Job 2,Job3,and Job 4.the cost matrix $(\widetilde{C_{ij}})$ is given whose elements are **Triangular Fuzzy numbers**. The problem is to find the optimal assignment so that the total cost of job assignment becomes minimum

$$[\widetilde{C_{ij}}] = \begin{pmatrix} (10,20,30) & (10,20,40) & (10,30,40) & (10,20,30) \\ (10,20,40) & (10,30,40) & (10,20,30) & (20,30,40) \\ (10,20,40) & (20,30,40) & (20,30,40) & (10,20,30) \\ (20,30,50) & (10,20,30) & (20,40,60) & (20,30,50) \end{pmatrix}$$

Solution:

In conformation to model (2) the fuzzy assignment problem can be formulated in the following mathematical programming from.

$$R(10,20,30)\,x_{11} + R(10,20,40)x_{12} + R(10,30,40)x_{13}$$

$$+R(10,20,30)x_{14} + R(10,20,40)x_{21} + R(10,30,40)x_{22}$$

$$+R(10,20,30)x_{23} + R(20,30,40)x_{24} + R(10,20,40)x_{31}$$

$$+R(20,30,40)x_{32} + R(20,30,40)x_{33} + R(10,20,30)x_{34}$$

$$+R(20,30,50)x_{41} + R(10,20,30)x_{42} + R(20,40,60)x_{43} + R(20,30,50)x_{44}$$

}

Subject to

$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$
$$x_{11} + x_{21} + x_{31} + x_{41} = 1$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$
$$x_{12} + x_{22} + x_{33} + x_{42} = 1$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{44} = 1$$
$$x_{14} + x_{24} + x_{34} + x_{44} = 1$$
$$x_{ij} \in [0,1]$$

Now we calculate R(10,20,30) by applying Robust's ranking method. The membership function of the triangular fuzzy number (10,20,30) is

$$\mu(x) = \begin{cases} (x-10)/10 & ;\ 10 \le x \le 20 \\ 1 & ;\ x = 20 \\ (30-x)/10 & ;\ 20 \le x \le 30 \\ 0 & ;\ \text{Otherwise.} \end{cases}$$

The K-Cut of the fuzzy number (10,20,30) is

$(c_k^L, c_k^U) = (10k+10,\ 30-10k)$ for which

$$R\ [\widetilde{C_{11}}] = R(10,20,30) = \int_0^1 0.5\ (C_k^L, C_k^U)dk$$

$$= \int_0^1 (0.5)\ (40)dk$$

$$= 20$$

Proceeding similarly, the Robust's ranking indices for the fuzzy costs $\widetilde{C_{ij}}$ are calculated as:

$R(C_{12}) = 22.5$, $R(C_{13}) = 27.5$, $R(C_{14}) = 20$, $R(C_{21}) = 22.5$, $R(C_{22}) = 27.5$, $R(C_{23}) = 20$, $R(C_{24}) = 30$, $R(C_{31}) = 22.5$, $R(C_{32}) = 30$, $R(C_{33}) = 30$, $R(C_{34}) = 20$, $R(C_{41}) = 32.5$, $R(C_{42}) = 20$, $R(C_{43}) = 40$, $R(C_{44}) = 32.5$

We replace these values for their corresponding $\widetilde{C_{ij}}$ in (3) which results in a convenient assignment problem in the linear programming problem. We solve it by Hungarian method to get the following optimal solution. → 3

$$x_{11}^* = x_{23}^* = x_{34}^* = x_{42}^* = 1$$

$$x_{12}^* = x_{13}^* = x_{14}^* = x_{21}^* = x_{22}^* = x_{24}^* = x_{31}^* = x_{32}^* = x_{33}^* = x_{41}^* = x_{43}^* = x_{44}^* = 0$$

with the optimal objective value $R\ (\overset{V*}{z}) = 80$

The optimal assignment

$$A \to 1 \quad B \to 3 \quad C \to 4 \quad D \to 2$$

The optimal Solution

$$R\ (\overset{V*}{z}) = 20 + 20 + 20 + 20 = 80$$

The fuzzy optimal total cost = $\widetilde{C_{11}} + \widetilde{C_{23}} + \widetilde{C_{34}} + \widetilde{C_{42}}$

$$= R(10,20,30) + R(10,20,30) + R(10,20,30) + R(10,20,30)$$

$= R(40,80,120)$

Also we find that $R\left(\begin{smallmatrix} V* \\ z \end{smallmatrix}\right) = 80$

CONCLUSIONS

In this paper, the assignment costs are considered as imprecise numbers described by fuzzy numbers which are more realistic and general in nature. Moreover, the fuzzy assignment problem has been transformed into crisp assignment problem using Robust's ranking indices (3). Numerical examples show that by using method we can have the optimal assignment as well as the crisp and fuzzy optimal total cost. By using Robust's(3) ranking methods we have shown that the total cost obtained is optimal. Moreover, one can conclude that the solution of fuzzy problems can be obtained by Robust's ranking methods effectively. This technique can also be used in solving other types of problems like, project schedules, transportation problems and network flow problems.

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