Complementary Perfect Triple Connected Domination Number of a Graph

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Abstract:

The concept of triple connected graphs with real life application was introduced in [9] by considering the existence of a path containing any three vertices of G. A graph *G* is said to be *triple connected* if any three vertices lie on a path in *G*. In [3], the concept of triple connected dominating set was introduced. A set S ⊆ V is a triple connected dominating set if S is a dominating set of G and the induced sub graph <S> is triple connected. The triple connected domination number $\gamma_{tc}(G)$ is the minimum cardinality taken over all triple connected dominating sets in G. In this paper, we introduce a new domination parameter, called Complementary perfect triple connected domination number of a graph. A set S ⊆ V is a complementary perfect triple connected dominating set if S is a triple connected dominating set of G and the induced sub graph <V - S> has a perfect matching. The complementary perfect triple connected domination number $\gamma_{cptc}(G)$ is the minimum cardinality taken over all complementary perfect triple connected dominating sets in G. We determine this number for some standard classes of graphs and obtain some bounds for general graph. Their relationships with other graph theoretical parameters are investigated.

Key words: Domination Number, Triple connected graph, Complementary perfect triple connected domination number.
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1 Introduction

By a ***graph*** we mean a finite, simple, connected and undirected graph *G (V, E)*, where *V* denotes its vertex set and *E* its edge set. Unless otherwise stated, the graph *G* has *p* vertices and *q* edges. ***Degree*** of a vertex *v* is denoted by *d (v)*, the ***maximum degree*** of a graph *G* is denoted by $\Delta(G)$. We denote a ***cycle*** on *p* vertices by C_p, a ***path*** on *p* vertices by P_p, and a ***complete graph*** on *p* vertices by K_p. A graph *G* is ***connected*** if any two vertices of *G* are connected by a path. A maximal connected subgraph of a graph *G* is called a ***component*** of *G*. The number of components of *G* is denoted by $\omega(G)$. The ***complement*** $\bar{G}$ of *G* is the graph with vertex set *V* in which two vertices are adjacent if and only if they are not adjacent in *G*. A ***tree*** is a connected acyclic graph. A ***bipartite graph*** (or ***bigraph***) is a graph whose vertices can be divided into two disjoint sets *U* and *V* such that every edge connects a vertex in *U* to one in *V*. A ***complete bipartite graph*** is a special kind of bipartite graph where every vertex of the first set is connected to every vertex of the second set. The complete bipartite graph with partitions of order $|V_1|=m$ and $|V_2|=n$, is denoted $K_{m,n}$. A ***star***, denoted by $K_{1,p-1}$ is a tree with one root vertex and p – 1 pendant vertices. A ***bistar***, denoted by *B(m, n)* is the graph obtained by joining the root vertices of the stars $K_{1,m}$ and $K_{1,n}$.

The ***friendship graph***, denoted by F_n can be constructed by identifying *n* copies of the cycle C_3 at a common vertex. A ***wheel graph***, denoted by W_p is a graph with *p* vertices, formed by connecting a single vertex to all vertices of an *(p-1)* cycle. A ***helm graph***, denoted by H_n is a graph obtained from the wheel W_n by joining a pendant vertex to each vertex in the outer cycle of W_n by means of an edge. ***Corona*** of two graphs G_1 and G_2, denoted by $G_1 \circ G_2$ is the disjoint union of one copy of G_1 and $|V_1|$ copies of G_2 ($|V_1|$ is the number of vertices in G_1) in which i^{th} vertex of G_1 is joined to every vertex in the i^{th} copy of G_2. For any real number x, $\lfloor x \rfloor$ denotes the largest integer less than or equal to x. If *S* is a subset of *V*, then *<S>* denotes the vertex induced subgraph of *G* induced by *S*. The ***open neighbourhood*** of a set *S* of vertices of a graph G, denoted by *N(S)* is the set of all vertices adjacent to some vertex in *S* and $N(S) \cup S$ is called the ***closed neighbourhood*** of *S*, denoted by *N[S]*. The ***diameter*** of a connected graph is the maximum distance between two vertices in G and is denoted by *diam (G).* A ***cut – vertex (cut edge)*** of a graph G is a vertex (edge) whose removal increases the number of components. A ***vertex cut***, or ***separating set*** of a connected graph *G* is a set of vertices whose removal renders *G* disconnected. The ***connectivity*** or ***vertex connectivity*** of a graph G, denoted by $\kappa(G)$ (where G is not complete) is the size of a smallest vertex cut. A connected subgraph *H* of a connected graph G is called a **H -cut** if $\omega(G – H) \geq 2$. The ***chromatic number*** of a graph G, denoted by $\chi(G)$ is the smallest number of colors needed to colour all the vertices of a graph G in which adjacent vertices receive different colour. Terms not defined here are used in the sense of [2].

A subset S of V is called a ***dominating set*** of G if every vertex in $V - S$ is adjacent to at least one vertex in S. The ***domination number*** $\gamma(G)$ of G is the minimum cardinality taken over all dominating sets in G. A dominating set S of a connected graph G is said to be a ***connected dominating set*** of G if the induced sub graph <S> is connected. The minimum cardinality taken over all connected dominating set is the ***connected domination number*** and is denoted by γ_c. A subset S of V of a nontrivial graph G is said to be ***Complementary perfect dominating set***, if S is a dominating set and the subgraph induced by <V-S> has a perfect matching. The minimum cardinality taken over all Complementary perfect dominating sets is called the ***Complementary perfect domination number*** and is denoted by γ_{cp}.

One can get a comprehensive survey of results on various types of domination number of a graph in [11].

Many authors have introduced different types of domination parameters by imposing conditions on the dominating set [1,8]. Recently the concept of triple connected graphs was introduced by Paulraj Joseph J. et. al.,[9] by considering the existence of a path containing any three vertices of G. They have studied the properties of triple connected graph and established many results on them. A graph G is said to be *triple connected* if any three vertices lie on a path in G. All paths and cycles, complete graphs and wheels are some standard examples of triple connected graphs.

A set S ⊆ V is a triple connected dominating set if S is a dominating set of G and the induced sub graph <S> is triple connected. The triple connected domination number γ_{tc}(G) is the minimum cardinality taken over all triple connected dominating sets in G.

A set S ⊆ V is a complementary perfect triple connected dominating set if S is a triple connected dominating set of G and the induced sub graph <V - S> has a perfect matching. The complementary perfect triple connected domination number γ_{cptc}(G) is the minimum cardinality taken over all complementary perfect triple connected dominating sets in G.

In this paper we use this idea to develop the concept of complementary perfect triple connected dominating set and complementary perfect triple connected domination number of a graph.

Previous Results

Theorem 1.1 [9] A tree T is triple connected if and only if $T \cong P_p$; $p \geq 3$.

Theorem 1.2 [9] A connected graph G is not triple connected if and only if there exists a H -cut with $\omega(G – H) \geq 3$ such that $|V(H) \cap N(C_i)| = 1$ for at least three components C_1, C_2, and C_3 of $G – H$.

Notation 1.3 Let G be a connected graph with m vertices $v_1, v_2, …., v_m$. The graph $G(n_1P_{l_1}, n_2P_{l_2}, n_3P_{l_3}, …., n_mP_{l_m})$ where $n_i, l_i \geq 0$ and $1 \leq i \leq m$, is obtained from G by attaching n_1 times a pendant vertex of P_{l_1} on the vertex v_1, n_2 times a pendant vertex of P_{l_2} on the vertex v_2 and so on.

Example 1.4 Let v_1, v_2, v_3, v_4, be the vertices of K_4, the graph $K_4(2P_2, P_3, P_4, P_3)$ is obtained from K_4 by attaching 2 times a pendant vertex of P_2 on v_1, 1 time a pendant vertex of P_3 on v_2, 1 times a pendant vertex of P_4 on v_3 and 1 time a pendant vertex of P_3 on v_4 and is shown in Figure 1.1.

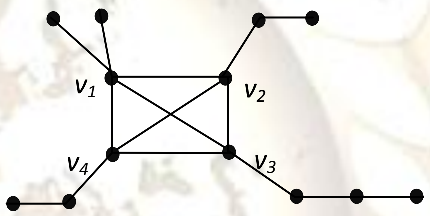

Figure 1.1: $K_4(2P_2, P_3, P_4, P_3)$

2 Complementary Perfect Triple Connected Domination Number

Definition 2.1 A set S ⊆ V is a complementary perfect triple connected dominating set if S is a triple connected dominating set of G and the induced sub graph <V - S> has a perfect matching. The complementary perfect triple connected domination number γ_{cptc}(G) is the minimum cardinality taken over all complementary perfect triple connected dominating sets in G. Any triple connected dominating set with γ_{cptc} vertices is called a γ_{cptc} -set of G.

Example 2.2 For the graph G_1 in Figure 2.1, $S = \{v_1, v_2, v_5\}$ forms a γ_{cptc} -set of G_1. Hence $\gamma_{cptc}(G_1) = 3$.

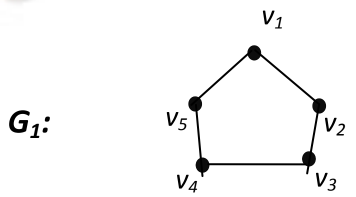

Figure 2.1

Observation 2.3 Complementary perfect triple connected dominating set does not exists for all graphs and if exists, then $\gamma_{cptc} \geq 3$.

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Example 2.4 For the graph G_2 in Figure 2.2, any minimum dominating set must contain the supports and any connected dominating set containing these supports is not Complementary perfect triple connected and hence γ_{cptc} does not exists.

Figure 2.2

Remark 2.5 Throughout this paper we consider only connected graphs for which complementary perfect triple connected dominating set exists.

Observation 2.6 The complement of the complementary perfect triple connected dominating set need not be a complementary perfect triple connected dominating set.

Example 2.7 For the graph G_3 in Figure 2.3, $S = \{v_1, v_2, v_3\}$ forms a Complementary perfect triple connected dominating set of G_3. But the complement $V – S = \{v_4, v_5, v_6, v_7\}$ is not a Complementary perfect triple connected dominating set.

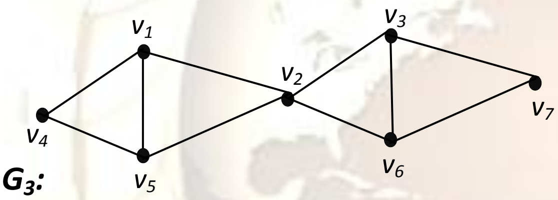


Figure 2.3

Observation 2.8 Every Complementary perfect triple connected dominating set is a dominating set but not the converse.

Example 2.9 For the graph G_4 in Figure 2.4, $S = \{v_1\}$ is a dominating set, but not a complementary perfect triple connected dominating set.

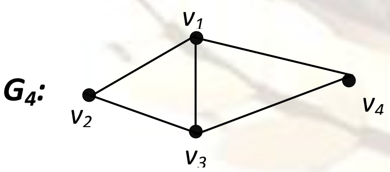


Figure 2.4

Observation 2.10 For any connected graph G, $\gamma(G) \le \gamma_{cp}(G) \le \gamma_{cptc}(G)$ and the inequalities are strict.

Example 2.11 For K_6 in Figure 2.5, $\gamma(K_6) = \{v_1\} = 1$, $\gamma_{cp}(K_6) = \{v_1, v_2\} = 2$ and $\gamma_{cptc}(K_6) = \{v_1, v_2, v_3, v_4\} = 4$. Hence $\gamma(G) \le \gamma_{cp}(G) \le \gamma_{cptc}(G)$.

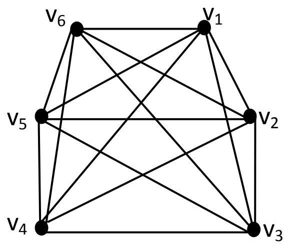


Figure 2.5

Theorem 2.12 If the induced subgraph of each connected dominating set of G has more than two pendant vertices, then G does not contain a Complementary perfect triple connected dominating set.

Proof This theorem follows from *Theorem 1.2*.

Example 2.13 For the graph G_6 in Figure 2.6, $S = \{v_6, v_2, v_3, v_4\}$ is a minimum connected dominating set so that $\gamma_c(G_6) = 4$. Here we notice that the induced subgraph of S has three pendant vertices and hence G does not have a Complementary perfect triple connected dominating set.

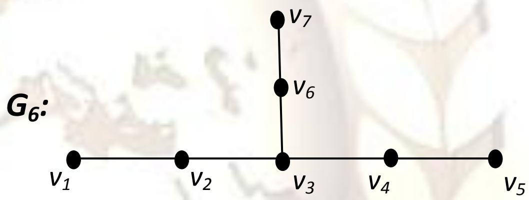


Figure 2.6

Complementary perfect triple connected domination number for some standard graphs are given below.

1) For any cycle of order $p \ge 5$, $\gamma_{cptc}(C_p) = p – 2$.

2) For any complete bipartite graph of order $p \ge 5$,

$$\gamma_{cptc}(K_{m,n}) = \begin{cases} 3, \text{if } p \text{ is odd} \\ 4, \text{if } p \text{ is even.} \end{cases}$$

(where $m, n \ge 2$ and $m + n = p$.

3) For any complete graph of order $p \ge 5$,

$$\gamma_{cptc}(K_p) = \begin{cases} 3, \text{if } p \text{ is odd} \\ 4, \text{if } p \text{ is even.} \end{cases}$$

4) For any wheel of order $p \ge 5$,

$$\gamma_{cptc}(W_p) = \begin{cases} 3, \text{if } p \text{ is odd} \\ 4, \text{if } p \text{ is even.} \end{cases}$$

5) For any Fan graph of order $p \ge 5$,

$$\gamma_{cptc}(F_p) = \begin{cases} 3, \text{if } p \text{ is odd} \\ 4, \text{if } p \text{ is even.} \end{cases}$$

6) For any Book graphs of order $p \ge 6$, $\gamma_{cptc}(B_p) = 4$.

7) For any Friendship graphs of order $p \ge 5$, $\gamma_{cptc}(F_n^{\prime}) = 3$.

Observation 2.14 If a spanning sub graph H of a graph G has a Complementary perfect triple connected dominating set, then G also has a complementary perfect triple connected dominating set.

Example 2.16 For any graph G and H in Figure 2.7, S = {v_1, v_4, v_5} is a complementary perfect triple connected dominating set and so $\gamma_{cptc}(G) = 3$. For the spanning subgraph H, S = {v_1, v_4, v_5} is a complementary perfect triple connected dominating set and so $\gamma_{cptc}(H) = 3$.

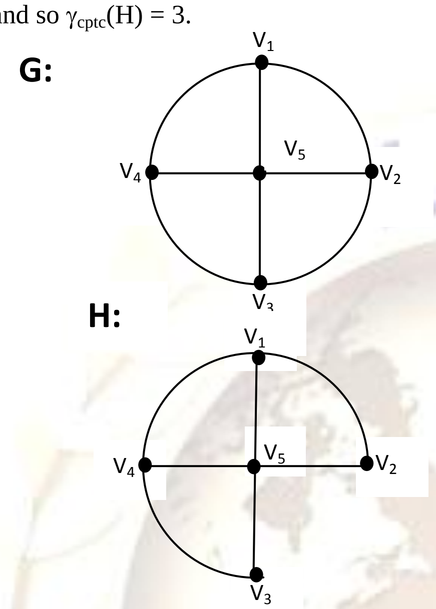


Figure 2.7

Observation 2.17 Let G be a connected graph and H be a spanning sub graph of G. If H has a complementary perfect triple connected dominating set, then $\gamma_{cptc}(G) \leq \gamma_{cptc}(H)$ and the bound is sharp.

Example 2.18 For the graph G in Figure 2.8, S = {v_1, v_2, v_7, v_8} is a complementary perfect triple connected dominating set and so $\gamma_{cptc}(G) = 4$. For the spanning subgraph H of G, S = {v_1, v_2, v_3, v_6, v_7, v_8} is a complementary perfect triple connected dominating set and so $\gamma_{cptc}(H) = 6$.

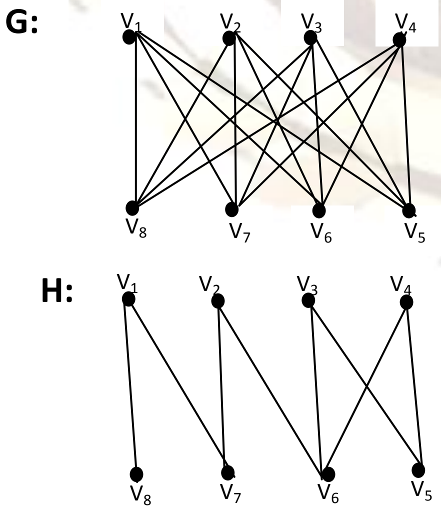


Figure 2.8

Theorem 2.19 For any connected graph G with $p \geq 5$, we have $3 \leq \gamma_{cptc}(G) \leq p - 2$ and the bounds are sharp.

Proof The lower and upper bounds trivially follows from *Definition 2.1*. For C_5, the lower bound is attained and for C_9 the upper bound is attained.

Theorem 2.20 For a connected graph G with 5 vertices, $\gamma_{cptc}(G) = p - 2$ if and only if G is isomorphic to C_5, W_5, K_5, $K_{2,3}$, F_2, $K_5 - \{e\}$, $K_4(P_2)$, $C_4(P_2)$, $C_3(P_3)$, $C_3(2P_2)$ or any one of the graphs shown in Figure 2.9.

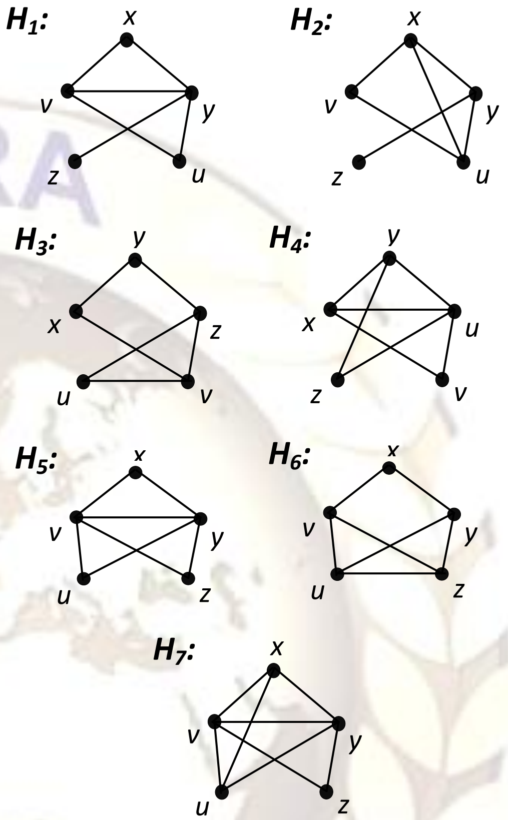


Figure 2.9

Proof Suppose G is isomorphic to C_5, W_5, K_5, $K_{2,3}$, F_2, $K_5 - \{e\}$, $K_4(P_2)$, $C_4(P_2)$, $C_3(P_3)$, $C_3(2P_2)$ or any one of the graphs H_1 to H_7 given in Figure 2.9, then clearly $\gamma_{cptc}(G) = p - 2$. Conversely, let G be a connected graph with 5 vertices and $\gamma_{cptc}(G) = 3$. Let $S = \{x, y, z\}$ be a γ_{cptc} -set, then clearly $<S> = P_3$ or C_3. Let $V - S = V(G) - V(S) = \{u, v\}$, then $<V - S> = K_2 = uv$.

Case (i) $<S> = P_3 = xyz$.

Since G is connected, there exists a vertex say x (or y, z) in P_3 is adjacent to u (or v) in K_2, then γ_{cptc} -set of G does not exists. But on increasing the degrees of the vertices of S, let x be adjacent to u and z be adjacent to v. If d(x) = d(y) = d(z) = 2, then G $\cong C_5$. Now by increasing the degrees of the vertices, by the above argument, we have G $\cong$ K_5, $K_5 - \{e\}$, $K_4(P_2)$, $C_4(P_2)$, $C_3(P_3)$ or any one of the graphs H_1 to H_7 given in Figure 2.9. Since G is connected, there exists a vertex say y in P_3 is adjacent to u (or v) in K_2, then γ_{cptc} -set of G does not exists. But on increasing the degrees of the vertices of S, let y be adjacent to v, x be adjacent to u and v and z be adjacent to u and v. If d(x) = 3,

$d(y) = 3$, $d(z) = 3$, then $G \cong W_5$. Now by increasing the degrees of the vertices, by the above argument, we have $G \cong K_{2,3}, C_3(2P_2)$. In all the other cases, no new graph exists.

Case (ii) $<S> = C_3 = xyzx$.

Since G is connected, there exists a vertex say x (or y, z) in C_3 is adjacent to u (or v) in K_2, then $S = \{x, u, v\}$ forms a γ_{cptc} -set of G so that $\gamma_{cptc}(G) = p - 2$. If $d(x) = 3$, $d(y) = d(z) = 2$, then $G \cong C_3(P_3)$. If $d(x) = 4$, $d(y) = d(z) = 2$, then $G \cong F_2$. In all the other cases, no new graph exists.

Nordhaus – Gaddum Type result:

Theorem 2.21 Let G be a graph such that G and $\bar{G}$ have no isolates of order $p \geq 5$, then

(i) $\gamma_{cptc}(G) + \gamma_{cptc}(\bar{G}) \leq 2(p-2)$

(ii) $\gamma_{cptc}(G) \cdot \gamma_{cptc}(\bar{G}) \leq (p-2)^2$ and the bounds are sharp.

Proof The bounds directly follow from *Theorem 2.19*. For the cycle C_5, $\gamma_{cptc}(G) + \gamma_{cptc}(\bar{G}) = 2(p-2)$ and $\gamma_{cptc}(G) \cdot \gamma_{cptc}(\bar{G}) \leq (p-2)^2$.

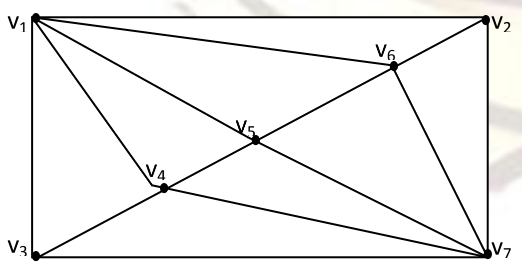
Theorem 2.22 $\gamma_{cptc}(G) \geq \lceil p/\Delta+1 \rceil$

Proof Every vertex in V-S contributes one to degree sum of vertices of S. Then $|V - S| \leq \sum_{u \in S} d(u)$ where S is a complementary perfect triple connected dominating set. So $|V - S| \leq \gamma_{cptc} \Delta$ which implies $(|V| - |S|) \leq \gamma_{cptc}\Delta$. Therefore $p - \gamma_{cptc} \leq \gamma_{cptc}\Delta$, which implies $\gamma_{cptc}(\Delta+1) \geq p$. Hence $\gamma_{cptc}(G) \geq \lceil p/\Delta+1 \rceil$

Theorem 2.23 Any complementary perfect triple connected dominating set of G must contains all the pendant vertices of G.

Proof Let S be any complementary perfect triple connected dominating set of G. Let v be a pendant vertex with support say u. If v does not belong to S, then u must be in S, which is a contradiction S is a complementary perfect triple connected dominating set of G. Since v is a pendant vertex, so v belongs to S.

Observation 2.24 There exists a graph for which $\gamma_{cp}(G) = \gamma_{tc}(G) = \gamma_{cptc}(G)$ is given below.



H:

Figure 2.10

For the graph H_i in figure 2.10, $S = \{v_1, v_3, v_7\}$ is a triple connected dominating set complementary perfect and complementary perfect triple connected dominatin set. Hence $\gamma_{cp}(G) = \gamma_{tc}(G) = \gamma_{cptc}(G) = 3$.

3. Complementary Perfect Triple Connected Domination Number and Other Graph Theoretical Parameters

Theorem 3.1 For any connected graph G with $p \geq 5$ vertices, $\gamma_{cptc}(G) + \kappa(G) \leq 2p - 3$ and the bound is sharp if and only if $G \cong K_5$.

Proof Let G be a connected graph with $p \geq 5$ vertices. We know that $\kappa(G) \leq p - 1$ and by *Theorem 2.19*, $\gamma_{cptc}(G) \leq p - 2$. Hence $\gamma_{cptc}(G) + \kappa(G) \leq 2p - 3$. Suppose G is isomorphic to K_5. Then clearly $\gamma_{cptc}(G) + \kappa(G) = 2p - 3$. Conversely, Let $\gamma_{cptc}(G) + \kappa(G) = 2p - 3$. This is possible only if $\gamma_{cptc}(G) = p - 2$ and $\kappa(G) = p - 1$. But $\kappa(G) = p - 1$, and so $G \cong K_p$ for which $\gamma_{cptc}(G) = 3 = p - 2$ so that $p = 5$. Hence $G \cong K_5$.

Theorem 3.2 For any connected graph G with $p \geq 5$ vertices, $\gamma_{cptc}(G) + \chi(G) \leq 2p - 2$ and the bound is sharp if and only if $G \cong K_5$.

Proof Let G be a connected graph with $p \geq 5$ vertices. We know that $\chi(G) \leq p$ and by *Theorem 2.19*, $\gamma_{cptc}(G) \leq p - 2$. Hence $\gamma_{cptc}(G) + \chi(G) \leq 2p - 2$. Suppose G is isomorphic to K_5. Then clearly $\gamma_{cptc}(G) + \chi(G) = 2p - 2$. Conversely, Let $\gamma_{cptc}(G) + \chi(G) = 2p - 2$. This is possible only if $\gamma_{cptc}(G) = p - 2$ and $\chi(G) = p$. But $\chi(G) = p$, and so G is isomorphic to K_p for which $\gamma_{cptc}(G) = 3 = p - 2$ so that $p = 5$. Hence $G \cong K_5$.

Theorem 3.3 For any connected graph G with $p \geq 5$ vertices, $\gamma_{cptc}(G) + \Delta(G) \leq 2p - 3$ and the bound is sharp if and only if G is isomorphic to W_5, K_5, $C_3(2)$, $K_5 - \{e\}$, $K_4(P_2)$, $C_3(2P_2)$ or any one of the graphs shown in Figure 3.1.

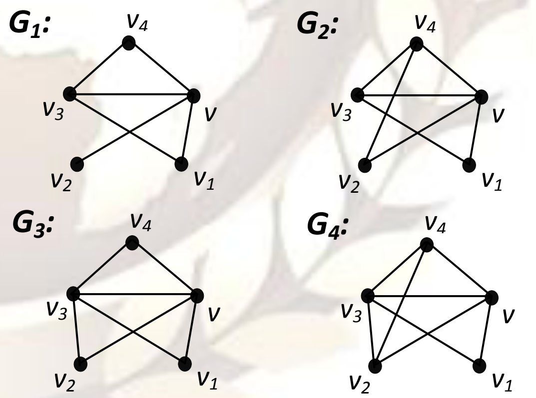


Figure 3.1

Proof Let G be a connected graph with $p \geq 5$ vertices. We know that $\Delta(G) \leq p - 1$ and by *Theorem 2.19*, $\gamma_{cptc}(G) \leq p - 2$. Hence $\gamma_{cptc}(G) + \Delta(G) \leq 2p - 3$. Let G be isomorphic to W_5, K_5, $C_3(2)$, $K_5 - \{e\}$, $K_4(P_2)$, $C_3(2P_2)$ or any one of the graphs G_1 to G_4 given in Figure 3.1, then clearly $\gamma_{cptc}(G) + \Delta(G) = 2p - 3$. Conversely, Let $\gamma_{cptc}(G) + \Delta(G) = 2p - 3$. This is possible only if $\gamma_{cptc}(G) = p - 2$ and $\Delta(G) = p - 1$. But $\gamma_{cptc}(G) = p - 2$ and $\Delta(G) = p - 1$, by *Theorem 2.20*, we have $G \cong W_5$, K_5, $C_3(2)$, $K_5 - \{e\}$, $K_4(P_2)$, $C_3(2P_2)$ and the graphs G_1 to G_4 given in Figure 3.1.

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