A Modified Coding Technique For High Peak-To-Average Power Ratio Reduction

Charanjit Singh[1], Gurpreet Singh Walia[2]
[1](ECE, University College of Engineering, Punjabi University, Patiala, India)
[2](ECE, University College of Engineering, Punjabi University, Patiala, India)

ABSTRACT
Orthogonal Frequency Division Multiplexing System (OFDM) suffers from the problem of high peak to average power ratio reduction (PAPR). It is studied that Golay complementary codes (GCC) can provide codewords with low PAPR. However, the usefulness of this coding technique is limited to the OFDM with a small number of subcarriers. With the help of certain theorems and definitions, this paper discusses a modified GCC technique to combat the effect of high PAPR and achieve higher code rate and information rate for 16-QAM OFDM system.

Keywords- GCC, OFDM, PA, PAPR, QAM, RGCC.

1. INTRODUCTION
OFDM system is mainly designed to combat the effect of frequency selective fading, by dividing the wide-band frequency selective fading channel into many narrow-band flat fading sub-channels [1]. OFDM has many advantages such as high spectral efficiency, robustness to channel fading, immunity to impulse interference, uniform average spectral density capacity of handling very strong echoes and non-linear distortion, it suffers from major drawback, i.e., high PAPR. The result of this high PAPR is that, the transmitted signal may suffer significant spectral spreading and in-band distortion as a consequence of inter-modulation effects induced by a non-linear power amplifier (PA) [2]. One possible way to combat this problem is the use of a large power back-off which allows the amplifier to operate in its linear region. However, this results into large decrease in power efficiency of the PA.

This gives a clear motivation to find other methods of minimizing the PAPR of the transmitted OFDM signal. A method which has been introduced in [3] and developed in [4] uses block coding to transmit only those codewords which produces OFDM symbols with PAPR below some threshold level. In [3], it is observed that for eight channels, a code rate of 3/4 exists that provides a maximum PAPR of 3 dB. However for a large number of subcarriers, a reasonable coding rate larger than 3/4 can be achieved for a PAPR level of 4dB. It is concluded that as the desired PAPR level decreases, achievable code rate also decreases. There is no structured way of getting these codewords having low PAPR or what the minimum distance properties of the code has been discussed. However in [4], it is observed that most of codes found, are Golay complementary sequences (GCS), which define a structured way of generating PAPR reduction codes. Golay complementary sequences are sequence pairs for which the sum of autocorrelation functions is zero for all delay shifts not equal to zero [5–8]. Davis and Jedwab in [9] obtained a large set of length 2^m binary Golay Complementary Pairs (GCP) from certain second order cosets of the first order Reed-Muller (RM) codes. Using the correlation property of GCS small PAPR of 3dB can be achieved. The usefulness of these coding techniques is limited to the OFDM system with a small number of subcarriers. The development of a good code for OFDM systems with a large number of subcarriers is difficult, which limits the actual benefits of coding for PAPR reduction in practical OFDM systems.

The rest of the paper has been organized as follows: Section 2 presents PAPR problem in OFDM system. Section 3 describes about Golay complementary sequence. Section 4 has two subsections; first subsection presents important definitions and theorems which helped in the development of Recursive Golay complementary code, second subsection presents the generation of Recursive Golay 16-QAM signal from QPSK pairs. And section 5 is conclusion.

2. PAPR PROBLEM
An OFDM signal is considered as the sum of many independent signals modulated onto sub channels of equal bandwidth. The complex baseband representation of a OFDM signal consisting of N subcarriers in continuous time domain is given by

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n \cdot e^{j2\pi n \Delta f t}, 0 \le t < NT \qquad (1)$$

where $j = \sqrt{-1}$, Δf is the subcarrier spacing, and NT denotes the useful data block period. In OFDM the subcarriers are chosen to be orthogonal $(i.e., \Delta f = 1/NT)$.

The peak to average ratio of a zero-mean signal can be defined in continuous as,

$$PAPR = \frac{\max_{0\le t<NT} |x(t)|^2}{\frac{1}{NT}\cdot\int_0^{NT} |x(t)|^2\, dt}$$

The Complementary CDF (CCDF) is used to measure the probability that the PAPR of a certain data block exceeds the given threshold.

Let $Z_{\max}$ denote the PAPR (i.e., $(Z_{\max} = \max_{n=0,1,2,3.........N-1} Z_n)$. The CDF of $Z_{\max}$ for the peak power per OFDM symbol having N subcarriers is given by,

$$F_{Z_{\max}}(z) = P(Z_{\max} < Z) = \left(1-\exp(-Z^2)\right)^N \quad (3)$$

The probability of exceeding the PAPR from the value '*PAPR₀*' is given by Complementary Cumulative Distribution Function (CCDF). Usually (CCDF) can be used to evaluate the performance of any PAPR reduction schemes,

$$\breve{F}_{Z_{\max}}(PAPR_0) = P(Z_{\max} > PAPR_0) = 1 - P(Z_{\max} \le PAPR_0) = 1-\left(1-\exp(-PAPR_0^2)\right)^N \quad (4)$$

3. GOLAY COMPLEMENTARY SEQUENCE

Golay complementary sequence is defined as a pair of two sequences x *and* y whose aperiodic autocorrelations sum equal to zero in all out-of-phase positions [10].

$$A_{xx}(l) + A_{yy}(l) = \begin{cases} 2N & \text{if } l = 0 \\ 0 & \text{if } l \ne 0 \end{cases} \quad (5)$$

In [11] the possibility of using complementary codes for both PAPR reduction and forward error correction has been shown. Also, [9] shows that a large set of binary length 2^m Golay complementary pairs can be obtained from Reed-Muller codes. If the total number of bits assigned to one OFDM symbol is *Nm*, where *N* is the number of subchannels, then N-valued sequence can be chosen from a codebook and this sequence is fed into the FFT block.

4. MODIFIED GOLAY COMPLEMETARY CODING SCHEME

The need for low PAPR [12], high coding rate has motivated us to investigate the non-equal power codes that achieve low PAPR. To do this we define a special case of GCSs using the recursive construction. Using (5), many recursive constructions for GCPs can be obtained by simply performing the algebraic manipulation. By doing so, it is observed that 2*N*-valued Recursive Golay pairs can be created from *N*-valued GCP. As Golay complementary code has been modified using recursive construction schemes it is named as Recursive Golay complementary code (RGCC). Mathematically, (2)

$$(x, y) \to (x \mid y, x \mid \bar{y})$$

4.1 Recursive Golay Complementary Codes for Low PAPR

Definition 1. Two N-valued complex sequences x and y are called Recursive Golay Complementary Pairs (RGCP) if

- They are Golay complementary pairs.
- If P_{av} is the average power of the constellation,

$$P_x + P_y \le 2NP_{av}$$

where $x \approx y$

Definition 2. The Cross-Correlation of two N-valued complex codes x and y with replacement $-N \le l \le N-1$ is defined as

$$C_l(x,y) = \begin{cases} \sum_{i=}^{N-l-1} x_{i+l} y_i^* & \text{if } 0 \le l \le N-1 \\ \sum_{i}^{N+l-1} x_i y_{i-l}^* & \text{if } -N \le l - 1 \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

Definition 3. The Auto-Correlation of an N-valued complex codes x with nonzero replacement l is defined as

$$A_l(x) = \begin{cases} \sum_{i=0}^{N-l-1} x_{i+l} x_i^* & \text{for } l \ge 0 \\ A_{-l}^*(x), & \text{for } l \le 0 \end{cases} \quad (7)$$

Theorem 1. The PAPR achieved by any RGCS is bounded up by 3dB.

Proof. By virtue of Theorem 1 and the fact that the instantaneous power of each code is always non-negative,

$$PAPR(x) = \frac{\max_t \{p_x(t)\}}{P_x} \le \frac{p_x(t) + p_y(t)}{NP_{av}} = \frac{P_x + P_y}{NP_{av}} \le 2 = 3dB \quad (8)$$

Following theorems represent a way to develop Recursive Golay Codes.

Theorem 2. The property of being Recursive Golay complementary pairs is invariant under the following transformations:

a. Reflection w.r.t the origin
b. Reflection w.r.t both axes.
c. Multiplication of one or both sequences by a complex number with magnitude 1.
d. Reflection w.r.t the bisectors of all regions.
e. Rotation.

Proof. Each item will be proved separately;

a. Using Definition 3,

$$A_l(-x) = A_l(x)$$

b. The reflection of x w.r.t the real axis is x^*. Using Definition 3.

$A_l(x^*) = [A_l(x)]^*$

The reflection of a sequence x w.r.t the imaginary axis is $-x^*$

c. If α is an arbitrary complex number, then

$A_l(\alpha x) = |\alpha|^2 A_l(x)$

Therefore if $x \approx y$ and $|\alpha| = 1$, then $x \approx \alpha y$ and $\alpha x \approx \alpha y$.

d. The reflection of x w.r.t the bisector of the first and third regions is jx^*. Also, the reflection of x w.r.t the bisector of the second and fourth regions is $-jx^*$.

e. Rotation of a sequence with the angle θ is equivalent to multiplying the sequence by $e^{j\theta}$.

Note that, in all of these cases, the power of the sequences is preserved.

Theorem 3. *If* $x \approx y$ then

$a.\ x' \approx y' \quad b.\ \hat{x} \approx \hat{y} \quad c.\ x \approx \hat{y}^*$

Proof. If $x \approx y$ then

a. The k^{th} of the sequence x' is $x'_k = (-1)^k x_k$, therefore using Definition 3.

$$A_l(x') = (-1)^l A_l(x) \tag{12}$$

b. The kth member of the sequence $\hat{x}$ is $\hat{x}_k = x_{N-1-k}$, therefore using Definition 3.

$$A_l(\hat{x}) = \sum_{i=0}^{N-l-1} \hat{x}_{i+l}\hat{x}_i^* = \sum_{i=0}^{N-l-1} x_{N-l-1}x^*_{N-i-1}$$
$$= \sum_{k=0}^{N-l-1} x^*_{k+l}x_k = (A_l(x))^* \tag{13}$$

c. Using (10) and (13) the statement is concluded.

Theorem 4. If $x \approx y$ then

$a.\ x|y \approx x|-y \qquad b.\ x\downarrow y \approx x \downarrow -y$

Proof. The items are proved separately:

a. Concatenation Property

$$A_l(x|y) = A_l(x) + A_l(y) + \sum_{i=0}^{l-1} x^*_{N-1-i}y_{l-1-i}$$

and therefore

$$A_l(x|-y) + A_l(x|-y) = 2(A_l(x) + A_l(y)) = 0 \tag{14}$$

b. Interleaving Property

If $l = 2k$ then

$$A_l(x\downarrow y) = A_{\frac{l}{2}}(x) + A_{\frac{l}{2}}(y) = 0 \tag{15}$$

Using (9) and since $x \approx -y$, $A_l(x \downarrow y) = 0$ .Therefore, $A_l(x \downarrow y) + A_l(x \downarrow -y) = 0$ .If $l = 2k+1$ then

(10)

$$A_l(x\downarrow y) = \sum_{i=0}^{N-1-\frac{l-1}{2}} x_i^* y_{i+\frac{l-1}{2}} \tag{16}$$

therefore $A_l(x \downarrow y) + A_l(x \downarrow -y) = 0$

If x and y are N-valued sequence then,

$\hat{x}$ *represents the inverse of* x (11)

x^* *represents the conjugate of* x

$x \mid y$ *represents the concatenation of* x *and* y

$x \downarrow y$ *represents the interleaving of* x *and* y

By applying the transformations defined in Theorems 2 and 3 to the statements of Theorem 4, a set of structures can be generated that create 2*N*-valued Recursive Golay pairs from *N*-valued ones. Specifically, if $x \approx y$ each with size *N*, then the following sequences are Recursive Golay pairs:

1) $\pm[j](x|y) \approx \pm[j](x|-y)$
2) $\pm[j](x\downarrow y) \approx \pm[j](x\downarrow -y)$
3) $\pm[j](x|y) \approx \pm[j](\hat{y}|-\hat{x}^*)$
4) $\pm[j](x\downarrow y) \approx \pm[j](\hat{y}^*\downarrow \hat{x}^*)$
5) $\pm[j](x\downarrow -y) \approx \pm[j](\hat{y}^*\downarrow \hat{x}^*)$

However, because of the special structure of 16-QAM constellation, many of these constructions yield similar sequences. For example reversing the role of *x* and *y* will not yield new pairs. If the number of *N*-valued pairs is *M*, the first structure yields 4*M* of 2*N*-valued Recursive Golay pairs and this is true for the second structure too. It has been found that in general each pair with size *N* yields 32 pairs each with size 2*N*.

4.2 Recursive Golay 16 QAM pairs from QPSK pairs [13]

To generate 16-QAM Recursive Golay sequences from QPSK Golay sequences, let's define QPSK symbols as

$$QPSK = \left\{ \exp\left[j\left(\frac{k\pi}{2} + \frac{\pi}{4} \right) \right], \middle| k \in Z_{2^h} \right\}$$

Using Definitions 3 and 2, the following theorem can be proved easily,

Theorem 5. For any two sequences x and y any two complex number α and β

$$A_l(\alpha x + \beta y) = |\alpha|^2 A_l(x) + |\beta|^2 A_l y + \alpha\beta^* C_1(x, y)$$
$$+ \alpha^* \beta C_l(y, x)$$

Theorem 6. If x and y are N- valued QPSK Golay complementary pairs, and α and β are two arbitrary complex numbers with $|\alpha| = |\beta|$, then each of the following pairs are 16-QAM Recursive Golay sequences:

1. $c = \alpha(x + 2y)$ *and* $t = \beta(-2x + y)$

2. $c = \alpha(x-2y)$ *and* $t = \beta(2x+y)$
3. $c = \alpha(x+2jy)$ *and* $t = \beta(2jx+y)$
4. $c = \alpha(x-2jy)$ *and* $t = \beta(-2jx+y)$

Proof. Here the result for the third item only has been proved and by using similar method other results can be proved. Using Theorem 5, for each nonzero *l*,

$$\begin{aligned} A_l(c)+A_l(t) &= |\alpha|^2 A_l(x)+4|\alpha|^2 A_l(y)-2j|\alpha|^2 C_l(x,y)+2j|\alpha|^2 C_l(y,x) \\ &\quad +4|\beta|^2 A_l(x)+|\beta|^2 A_l(y) \\ &\quad +2j|\beta|^2 C_l(x,y)-2j|\beta|^2 C_l(y,x) \\ &= 5|\alpha|^2\left(A_l(x)+A_l(y)\right)=0 \end{aligned}$$

Therefore c and t are Golay complementary pairs. It is easy to see that each of these sequences is actually a 16-QAM sequence, when the average power of the constellation is $P_{av} = 5|\alpha|^2$. If the Hermitian of *x* is denoted by x^H, and considering the same power *N* both for *x* and *y*, then

$$\begin{aligned} P_c+P_t &= |\alpha|^2\left(\|x+2jy\|^2+\|2jx+y\|^2\right) \\ &= |\alpha|^2\begin{bmatrix} \left(\|x\|^2+4\|y\|^2+2jx^H y-2jy^H x\right) \\ +\left(\|y\|^2+4\|x\|^2-2jx^H y+2jy^H x\right) \end{bmatrix} \\ &= 5|\alpha|^2\left(\|x\|^2+\|y\|^2\right)=10N|\alpha|^2 \\ &= 2NP_{av} \end{aligned}$$

Therefore, by Definition 1, *c* and *t* are Recursive Golay sequences.

For generation of 16-QAM with $P_{av} = 10$, α and β can be chosen from the set $\{\sqrt{2}, -\sqrt{2}, j\sqrt{2}, -j\sqrt{2}\}$, and therefore for each of the $2^{h(m+2)\frac{m!}{2}}$ Golay complementary pairs over Z_{2^h} of length 2^m, there are 64 Recursive Golay 16-QAM pairs. Starting from 4-valued codes, code rate achieved for N subcarrier OFDM is given by,

$$R = \frac{12+5\log_2 N/4}{4N}$$

For *N* = 128, the coding rate and information rate of 7.23% has been achieved while the PAPR remains bounded up to 3.6.

5 CONCLUSION

In this paper generation of Recursive Golay complementary sequence has been proposed. 16-QAM symbol has been generated by taking the weighted sum of two QPSK symbols. Results show that the PAPR of the constructed sequences is bounded up to 3.6 dB, and the information rate is twice as the rate for Golay QPSK codes. For *N* = 128 the coding rate of 7.23% has been achieved while the PAPR remains bounded up to 3.6 dB.

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